Summary & Course Review

ELEC 3004: **Systems**: Signals & Controls

Dr. Surya Singh (& Timothy Sherry via Skype)

Lecture 25

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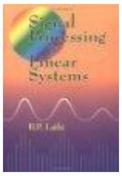
http://robotics.itee.uq.edu.au/~elec3004/

May 31, 2019

Lecture Schedule:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Systems as Maps & Signals as Vectors
	8-Mar	Systems: Linear Differential Systems
3	13-Mar	Sampling Theory & Data Acquisition
	15-Mar	Aliasing & Antialiasing
4	20-Mar	Discrete Time Analysis & Z-Transform
	22-Mar	Second Order LTID (& Convolution Review)
5	27-Mar	Frequency Response
	29-Mar	Filter Analysis
6	3-Apr	Digital Filters (IIR) & Filter Analysis
	5-Apr	PS 1: Q & A
7	10-Apr	Digital Windows
	12-Apr	Digital Filter (FIR)
8	17-Apr	Active Filters & Estimation
	19-Apr	Holiday
	24-Apr	
	26-Apr	
9	1-May	Introduction to Feedback Control
	3-May	Servoregulation & PID Control
10	8-May	State-Space Control
	10-May	Guest Lecture: FFT
11	15-May	Advanced PID & & FFT Processes
	17-May	State Space Control System Design
12	22-May	Shaping the Dynamic Response
	24-May	Stability and Examples
13	29-May	System Identification & Information Theory & Information Space
	31-May	**Summary and Course Review**

Follow Along Reading:



B. P. Lathi
Signal processing and linear systems
1998
TK5102.9.L38 1998

G. Franklin,
J. Powell,
M. Workman
Digital Control of Dynamic Systems
1990

TJ216.F72 1990
[Available as UQ Ebook]

Today

- Everything in Lectures! ☺
- Lathi:
  - Ch. 5: **Sampling**
  - Ch. 7: Frequency Response and Analog Filters
  - Ch. 8: Discrete-Time Signals and Systems
  - Ch. 12: Frequency Response and Digital Filters
  - Ch. 13: State-Space Analysis
- FPW:
  - Ch. 2: Linear, Discrete, Dynamic-Systems Analysis: The **z-Transform**
  - Ch. 3: Sampled-Data Systems
  - Ch. 4: Discrete Equivalents to Continuous Transfer Functions: The Digital Filter
  - Ch. 5: Design of Digital Control Systems Using Transform Techniques
  - Ch. 6: Design of Digital Control Systems Using State-Space Methods
- Final Exam 2017

- Final Exam 2012-2018 ☺

  Review Materials at:
  http://robotics.itee.uq.edu.au/~elec3004/tutes.html#Final

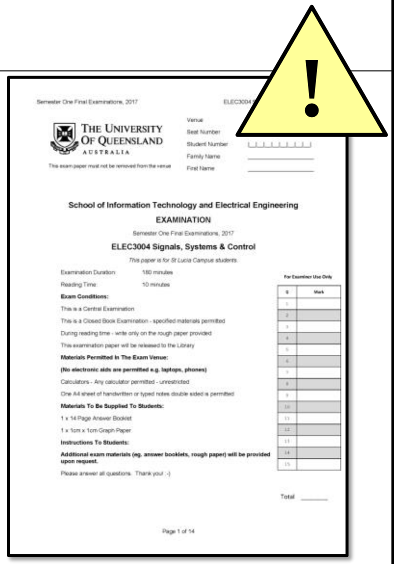
Next Time

Final Exam Information

- Date: **Monday**, **June/10**
- **Time:**
  2:30-5:30 pm
- **Location:**
  Advanced Engineering Building (49) - Room 301
  Michie Building (9) - Room 217
- **Parking:**
  Try Conifer Knoll (maybe!)
  (It's harder than the exam! ☺)
- UQ Exams are now "ID Verified"
  ➔ Please remember your ID! 🡐

ELEC 3004 Final[2] Review

Thursday, June 6
- 12-2 pm
- In: 7-234
  (Here ☺)


Semester One Final Examinations, 2017

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue
Seat Number
Student Number
Family Name
First Name

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2017

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes
Reading Time: 10 minutes

Exam Conditions:
This is a Central Examination
This is a Closed Book Examination - specified materials permitted
During reading time - write only on the rough paper provided
This examination paper will be released to the Library

Materials Permitted In The Exam Venue:
(No electronic aids are permitted e.g. laptops, phones)
Calculators - Any calculator permitted - unrestricted
One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:
1 x 14 Page Answer Booklet
1 x 1cm x 1cm Graph Paper

Instructions To Students:
Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.
Please answer all questions. Thank you! :-)

For Examiner Use Only

Q	Mark
1	
2	
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11	
12	
13	
14	
15	

Total ______

Page 1 of 14

Wed, June 5, 2019
- EBESS – Loc: 67-146
- 2-4pm

- Review Notes (Summarized from Course Textbooks)

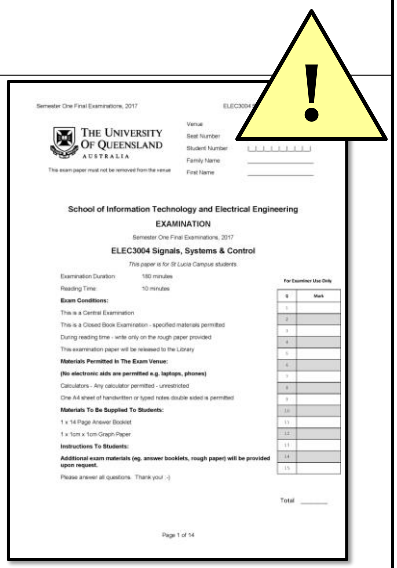
➔ http://robotics.itee.uq.edu.au/~elec3004/tutes.html

Final Exam Information

- Section 1:
  - Digital Linear Dynamical **Systems**
  - 4 Questions
  - 60 Points (33 %)
- Section 2:
  - Digital Processing / Filtering of **Signals**
  - 6 Questions
  - 60 Points (33 %)
- Section 3:
  - Digital & State-Space **Control**
  - 5 Questions
  - 60 Points (33 %)
- Online materials:

➔ Supplied Equation Sheet

➔ Some Review Notes



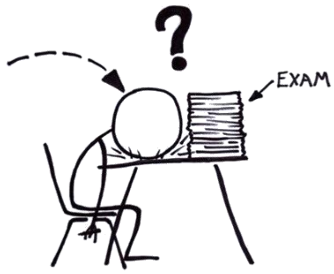
Fun Fact: With this, 45% of the exam is already "public" ☺

Announcements

ELEC 3004 <u>Grading:</u>

- We're working on it!

- You can preview grades by completing peer reviews.

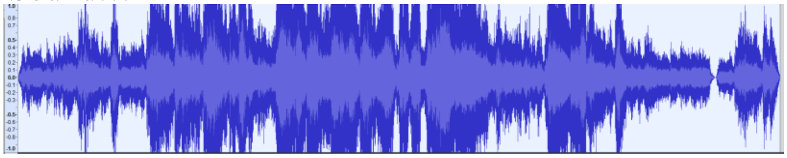


Please don't make this our fate in ~2 weeks!

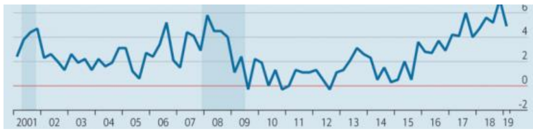
Why are we here ?

Signals are everywhere…

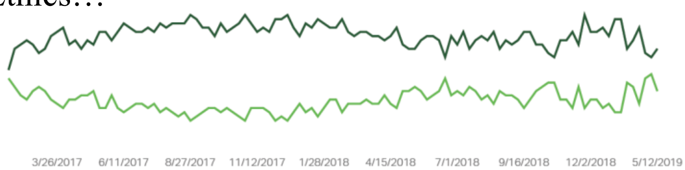
- Sound…



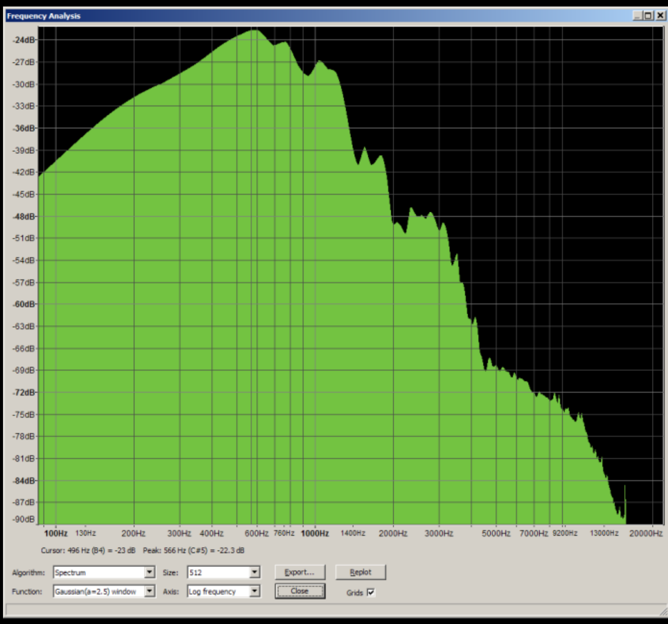
- Economics…



- Ethics…



What is … a **Signal**?


Frequency Analysis
-24dB
-27dB
-30dB
-33dB
-36dB
-39dB
-42dB
-45dB
-48dB
-51dB
-54dB
-57dB
-60dB
-63dB
-66dB
-69dB
-72dB
-75dB
-78dB
-81dB
-84dB
-87dB
-90dB
100Hz 130Hz 200Hz 300Hz 400Hz 600Hz 760Hz 1000Hz 1400Hz 2000Hz 3000Hz 5000Hz 7000Hz 9200Hz 13000Hz 20000Hz
Cursor: 496 Hz (B4) = -23 dB Peak: 566 Hz (C#5) = -22.3 dB
Algorithm: Spectrum
Size: 512
Export...
Replot
Function: Gaussian(a=2.5) window
Axis: Log frequency
Close
Grids

Why are we here ?

AKA ELEC 3004: **What do I need to know about *.* ???**

PS 1

ELEC 3004 / 7312 – Systems: Signals & Controls **2019**

Problem Set 1: An Introduction to Signals and Systems

Total marks: 100 **Due Date:** Sunday, **April 7, 2019** at 23:59 AEST [*end of week 6*]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and **submitted directly in Platypus** (preferred) or as **PDF**. Note that Microsoft **Word** documents and scanned images of **handwritten pages** are specifically **disallowed**. The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Please double-check that your name is not in the solution directly or via the associated metadata.

Also, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

Finally, a note of remembrance for our colleagues and compatriots in Christchurch.
For, no matter the signal, we are but one system -- United.

Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. Linearity: Starting Straight Away **[10 points]**

This question explores some of these interesting properties of Linear systems, notably superposition. This gives that if several inputs are acting on a linear system, then the total response of this system is the sum of the outputs from each input on its own.

Please **determine** and **justify** if these equations are linear

(a) $w(t) = 300t + 4$

(b) $y(t) = 300\frac{dx}{dt} + 4x(t)$

(c) $z(t) = 300 \cdot t \cdot \frac{dx}{dt} + 4 \cdot t^2 \cdot x(t)$

Please **determine** and then **generally prove** (or **disprove**) if these statements about linearity are true.

(d) Linearity and the Converse. Consider $f(x) = [A]x$.
For this case, is matrix multiplication a linear operation?
The Converse: Can **any** linear function f **always** be written as $f(x) = [A]x$?

(e) **Uniqueness**.
For any linear function f there is **only** one matrix $[A]$ for which $f(x) = [A]x$ for all x.

1

PS 2

ELEC 3004 / 7312 – Systems: Signals & Controls **2019**

Problem Set 2: Sampling & Filters

Total marks: 100 **Due Date:** Sunday, **May 12, 2019** at 23:59 AEST [*end of week 10*]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and **submitted directly in Platypus** (preferred) or as **PDF**. Note that Microsoft **Word** documents and scanned images of **handwritten pages** are specifically **disallowed**. The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Please double-check that your name is not in the solution directly or via the associated metadata.

Also, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

Finally, a note of remembrance for our colleagues and compatriots in Paris.
Where there is noise, there is also a new, indelible, signal.

Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. A Quick Sample[1] **[10 points]**

Consider a continuous time signal $s(t)$ sampled at f_s and is bandlimited to a frequency less than $f_n = f_s/2$. It has a Nyquist rate of $\omega_n = 2\pi f_s = 4\pi f_n$. **Determine** and **briefly justify** the required **Nyquist rate** (ω_n) of the following variations of this signal.

(a) $a(t) = s(t) + s(t-3004)$

(b) $b(t) = \frac{ds(t)}{dt}$

(c) $c(t) = s^2(t)$

(d) $d(t) = s(t) \cdot (\cos(\omega_n t))$

[1] Correction: ωn was defined incorrectly with respect to fn (instead of the correct fs). See also, Lathi, p.321. However, the answers are relative to the given ωn so they would have all scaled across. (Also, an apology, it seems that Google Doc does not allow equations in Footnotes).

1

PS 3

ELEC 3004 / 7312 – Systems: Signals & Controls **2019**

Problem Set 3: Digital Feedback Control

Total marks: 100 **Due Date:** Sunday, **June 2, 2019** at 23:59 AEST [*end of week 13*]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and **submitted directly in Platypus** (preferred) or as **PDF**. Note that Microsoft **Word** documents and scanned images of **handwritten pages** are specifically **disallowed**. The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Please double-check that your name is not in the solution directly or via the associated metadata.

Also, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

Finally, a note of remembrance for our colleagues and compatriots in Sri Lanka.
A disturbance, no matter how extreme, may always be rejected via an integral effort.

Thank you. :-)

For Questions Q1 to Q3: Please answer **2 out of the 3** questions (your choice).
If all three questions are submitted, *the tutors will only mark two chosen at random.*

For Questions Q6 and Q7: These questions are optional and for extra credit only.
There is no cap on the total extra credit for the problem set.

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. The Stately State Transition Matrix, Φ **[20 points]**

Consider a state transition matrix, $\mathbf{\Phi}$, for a SISO LTI system $\mathbf{A}$[1].

$$\mathbf{A} = \begin{bmatrix} 16 & 2 & 3 & 13 \\ 5 & 11 & 10 & 8 \\ 9 & 7 & 6 & 12 \\ 4 & 14 & 15 & 1 \end{bmatrix}$$

$$\mathbf{\Gamma} = \begin{bmatrix} 3 & 1 & 4 & 1 \\ 1 & 3 & 1 & 4 \\ 4 & 1 & 3 & 1 \\ 1 & 4 & 1 & 3 \end{bmatrix}$$

Please determine and justify :

(a) $\mathbf{\Phi}(t)$ for this system $\mathbf{A}$

(b) $\mathbf{\Phi}(s)$ for this system $\mathbf{A}$

(c) System $\mathbf{A}$'s characteristic polynomial

(d) $\widehat{\Phi}(z)$ for this system $\mathbf{A}$ via Tustin's method (i.e. trapazoid-rule)

(e) A difference equation assuming: (1) a step input at the first step ($u(k)$), followed by (2) ZOH sampling, with (3) **H=I**, and **Γ** as given.
[**Hint:** What is z-transform of the ZOH, i.e., $G(z)$ for a system $G(s)$?]

[1] **Note:** In Matlab, **A**=`magic(4);` Γ=`toeplitz([3 1 4 1]);`

1

To Review:
Back to the **Beginning…Lecture 1 Slide 27**

- Systems
- Signal Abstractions
- Signals as Vectors / Systems as Maps

- Linear Systems and Their Properties
- LTI Systems
- Autonomous Linear Dynamical Systems

- Convolution
- FIR & IIR Systems
- Frequency domain
- Fourier Transform (CT)
- Fourier Transform (DT)

- Even and Odd Signals
- Likelihood
- Causality

- Impulse Response
- Root Locus
- Bode Functions

- Left-hand Plane

- Frequency Response

- Discrete Time
- Continuous Time

- Laplace Transformation
- Feedback and Control
- Additional Applications

- Linear Functions
- Linear Algebra Review
- Least Squares
- Least Squares Problems
- Least Squares Applications
- Matrix Decomposition and Linear Algebra
- Regularized Least Squares

- Least-squares
- Least-squares applications
- Orthonormal sets of vectors
- Eigenvectors and diagonalization
- Linear dynamical systems with inputs and outputs
- Symmetric matrices, quadratic forms, matrix norm, and SVD

- Controllability and state transfer
- Observability and state estimation

- And that, of course, **Linear Systems are Cool!** ☺

Lots of Stuff To Cover…

- Systems
- ✔ Signal Abstractions
- ✔ Signals as Vectors / Systems as Maps

- ✔ Linear Systems and Their Properties
- ✔ LTI Systems
- ✔ Autonomous Linear Dynamical Systems

- Convolution
- ✔ FIR & IIR Systems
- ✔ Frequency domain
- ✔ Fourier Transform (CT)
- ✗ Fourier Transform (DT)

- ✔ Even and Odd Signals
- ✔ Likelihood
- ✔ Causality

- ✔ Impulse Response
- ✔ Root Locus
- ✔ Bode Functions

- ✔ Left-hand Plane

- Frequency Response

- ✔ Discrete Time
- ✔ Continuous Time

- ✔ Laplace Transformation
- ✔ Feedback and Control
- ✔ Additional Applications

- ✔ Linear Functions
- ✔ Linear Algebra Review
- ✔ Least Squares
- ✔ Least Squares Problems
- ✗ Least Squares Applications
- ✔ Matrix Decomposition and Linear Algebra
- ✗ Regularized Least Squares

- ✗ Least-squares
- ✔ Least-squares applications
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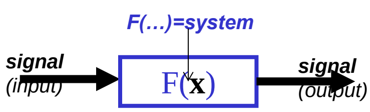
- ✗ Eigenvectors and diagonalization
- ✔ Linear dynamical systems with inputs and outputs
- ✔ Symmetric matrices, quadratic forms, matrix norm, and SVD

- ✔ Controllability and state transfer
- Observability and state estimation

- ✔ And that, of course, **Linear Systems are Cool!** ☺ ✔

Linear Signals

Linear Systems in 1-Slide

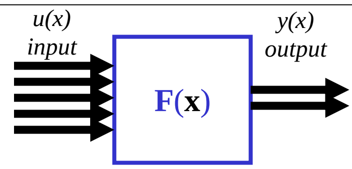


- Signals Are Vectors

- Systems Are Matricies

Linear Systems

- Model describes the relationship between the input **u**(x) and the output **y**(x)



- If it is a <u>Linear</u> System (wk 3):

$$y(t) = \int_0^t F(t-\tau)\, u(\tau)\, d\tau$$

- If it is also a (Linear and) <u>**lumped**</u>, it can be expressed <u>**algebraically**</u> as:

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t)$$
$$y(t) = C(t)\, x(t) + D(t)\, u(t)$$

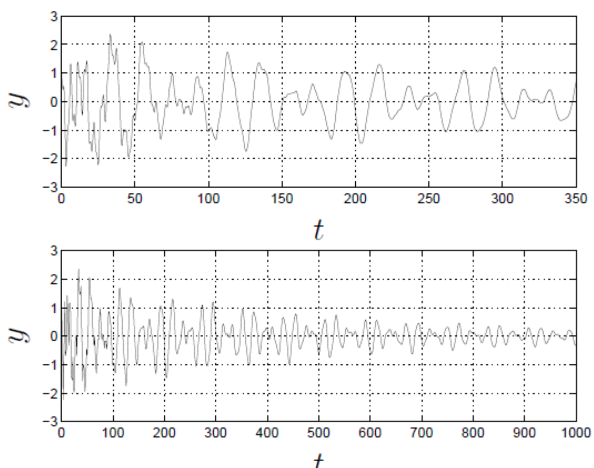
- If it is also (Linear and) <u>**time invariant**</u> the matrices can be reduced to:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$\mathscr{L}$aplacian: $$y(s) = F(s)\, u(s)$$

WHY? This can help simplify matters…

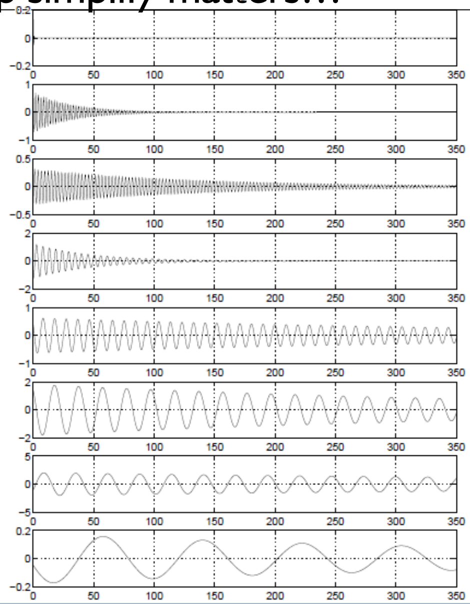
For Example: Consider the following system:



- How to model and predict (and control the output)?

Source: EE263 (s.1-13)

This can help simplify matters…



Source: EE263 (s.1-13)

This can help simplify matters…

- Consider the following system:

$$\dot{x} = Ax, \qquad y = Cx$$

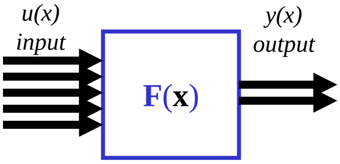
- $x(t) \in \mathbb{R}^8$, $y(t) \in \mathbb{R}^1$ → 8-state, single-output system
- Autonomous: No input yet! ($u(t) = 0$)

Source: EE263 (s.1-13)

System Classifications/Attributes

1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Recap: Linear Systems



- Model describes the relationship between the input **u**(x) and the output **y**(x)
- If it is a <u>Linear</u> System (wk 3):

$$y(t) = \int_0^t F(t-\tau)\, u(\tau)\, d\tau$$

- If it is also a (Linear and) <u>**lumped**</u>, it can be expressed <u>**algebraically**</u> as:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

- If it is also (Linear and) <u>**time invariant**</u> the matrices can be reduced to:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$\mathscr{L}$aplacian: $$y(s) = F(s)\,u(s)$$

Equivalence Across Domains

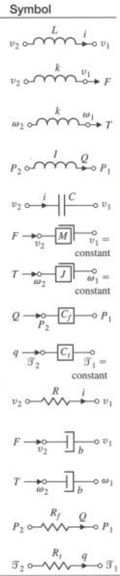
Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2^2$	
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2^2$	
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}^2$	
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}^2$	
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}^2$	
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	

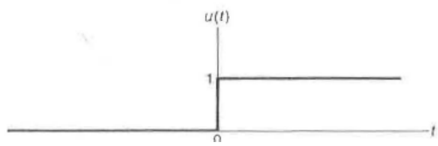


Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 74

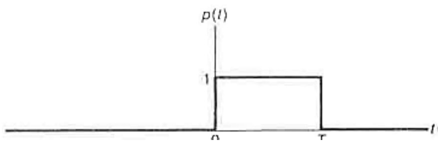
Signal Terminology

Unit Step Function

- $u(t) = \begin{cases} 0, t < 0 \\ 1, t > 0 \end{cases}$

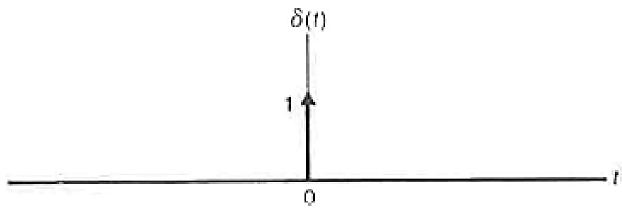


- "Rectangular Pulse"
- $p(t) = u(t) - u(t - T)$



Unit-Impulse Function

1. $\delta(t) = 0$ for $t \neq 0$.
2. $\delta(t)$ undefined for $t = 0$.
3. $\int_{t_1}^{t_2} \delta(t)\, dt = \begin{cases} 1, & \text{if } t_1 < 0 < t_2 \\ 0, & \text{otherwise.} \end{cases}$



Sampling Theorem

- The Nyquist criterion states:

To prevent aliasing, a ***bandlimited*** *signal of bandwidth* w_B *rad/s must be sampled at a rate greater than* $2w_B$ *rad/s*

$$w_s > 2w_B$$

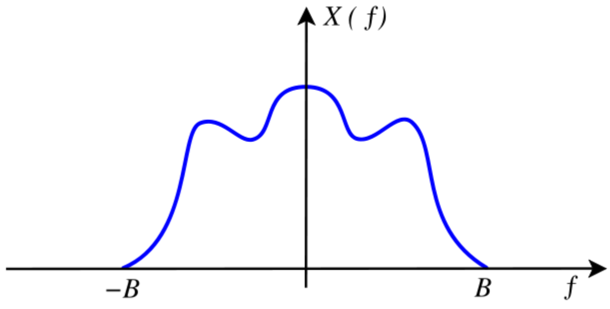
Note: this is a > sign not a ≥

Also note: Most real world signals require band-limiting with a lowpass (anti-aliasing) filter

Reconstruction

- Whittaker–Shannon interpolation formula

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot \mathrm{sinc}\left(\frac{t-nT}{T}\right)$$



Practical Reconstruction

Two stage process:

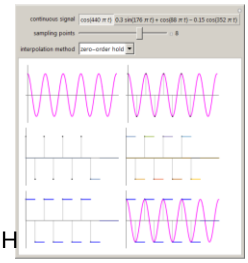
1. Digital to analogue converter (D/A)

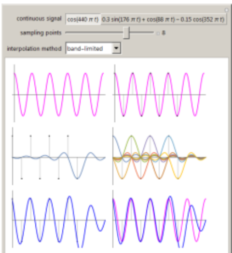
- zero order hold filter
- produces 'staircase' analogue output

2. Reconstruction filter

- non-ideal filter: $\omega_c = \frac{\omega_s}{2}$
- further reduces replica spectrums
- usually 4th – 6th order (Butterworth)
  - 4^{th}-6^{th} Order: for acceptable phase response

Ex_1:ZOH





Ex_2: Sinc

Filters

Flashback: FIR and Low Pass Filters…

∴

$$H_d(\omega) = \begin{cases} 1 \text{ if } |\omega| \le \omega_c \\ 0 \text{ if } \omega_c < |\omega| < \pi \end{cases}$$

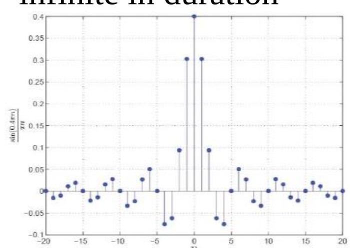
Has impulse response:

$$h_d(n) = \frac{\omega_c}{\pi} \frac{\sin \omega_c n}{\omega_c n}$$

Thus, to filter an impulse train with an ideal low-pass filter use:

$$x(t) = \left(\sum_{n=-\infty}^{\infty} x[n] \cdot \delta(t - nT)\right) * \mathrm{sinc}\left(\frac{t}{T}\right)$$

- **<u>However</u>**!!
  a sinc is non-causal and infinite in duration



And, this **cannot** be implemented **in practice** ☹

∵ we need to know all samples of the input, both in the **past** and in the **future**

Filter Design Using Windows

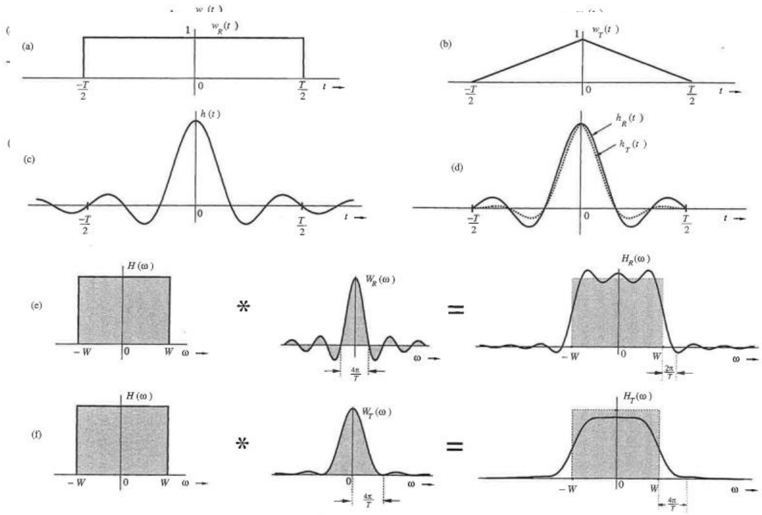
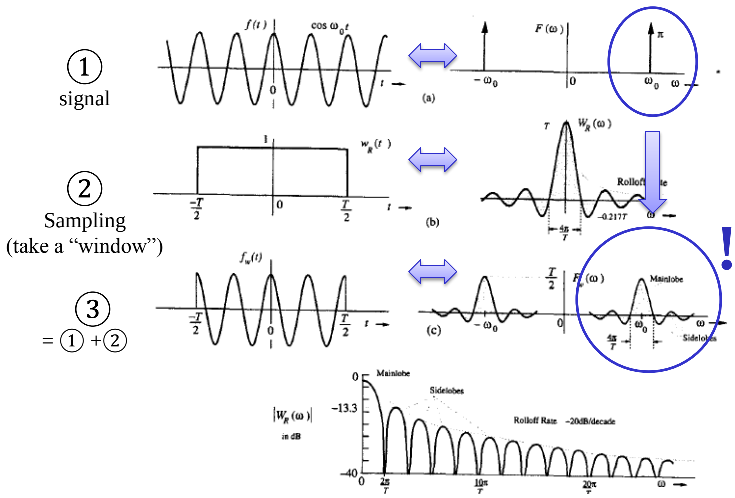


Fig. 4.48 Filter design using windows.

Recall: Windowing for the DFT

$$f_w(t) = f(t)w(t) \quad \text{and} \quad F_w(\omega) = \frac{1}{2\pi}F(\omega) * W(\omega)$$



Source: Lathi, p.303

Remedies for Side Effects of Truncation

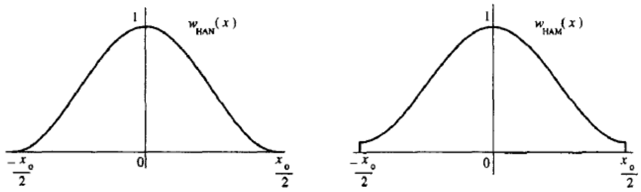
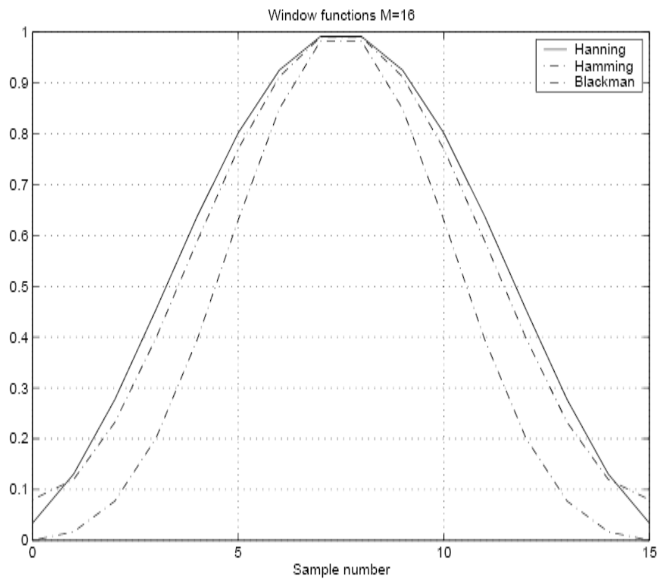


Fig. 4.47 Hanning and Hamming windows.

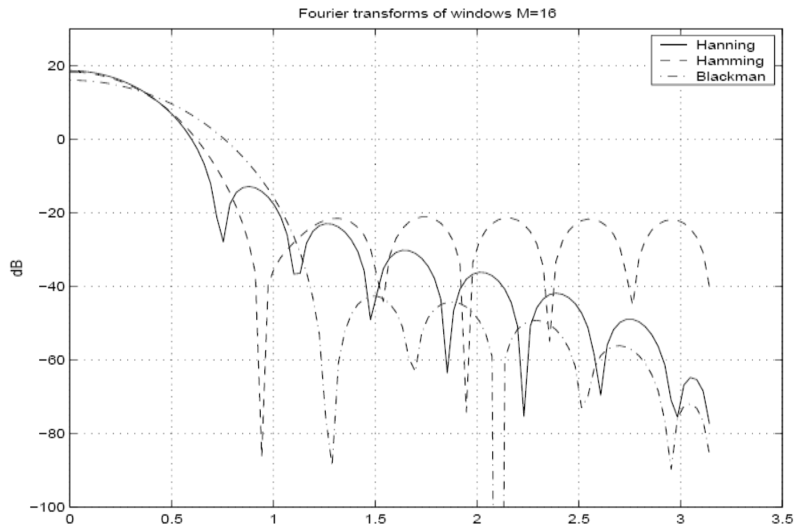
There are hundreds of windows, each with differing characteristics. But the choice depends on a particular application. The rectangular window has the narrowest mainlobe. The Bartlett (triangle) window (also called the Fejer or Cesaro) is inferior in all respects to the Hanning window. For this reason it is rarely used in practice. Hanning is preferred over Hamming in spectral analysis because it has faster sidelobe decay. For filtering applications, on the other hand, the Hamming window is the choice because it has the smallest sidelobe magnitude for a given mainlobe width. The Hamming window is the most widely used, general purpose window. The Kaiser window, which uses $I_0(\alpha)$, the Bessel function of the order 0, is more versatile and adjustable. Selecting a proper value of α $(0 \le \alpha \le 10)$ allows the designer to tailor the window to suit a particular application. The parameter α controls the mainlobe and sidelobe trade-off. When $\alpha = 0$, the Kaiser window is the rectangular window. For $\alpha = 5.4414$, it is the Hamming window, and when $\alpha = 8.885$, it is the Blackman window. As α increases, the mainlobe width increases and the sidelobe level decreases.

Comparison of Alternative Windows –Time Domain



Punskaya, Slide 90

Comparison of Alternative Windows
Frequency Domain



Punskaya, Slide 91

The Kalman Filter

- Question: What does it do?
- Answer: It estimates $x(t)$ based on $y(t)$ from:

$$
\begin{aligned}
x(t+1) &= A\,x(t) + u(t) \\
y(t) &= C\,x(t) + w(t)
\end{aligned}
$$

(Bayesian) Kalman Filter Summary

I. Model:

$$\begin{aligned} \mathbf{x}_{t+1} &= A\mathbf{x}_t + \mathbf{w}_t, \quad \mathbf{w}_t \sim W_t = N(\mathbf{0}, Q) \\ \mathbf{y}_t &= C\mathbf{x}_t + \mathbf{v}_t, \quad \mathbf{v}_t \sim V_t = N(\mathbf{0}, R) \end{aligned}$$

II. Algorithm: Repeat…

– Time update:

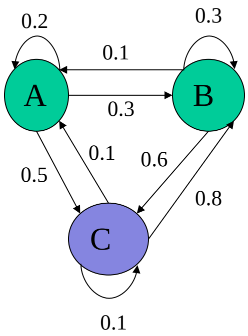
$$\hat{\mathbf{x}}_{t+1|t} = A\hat{\mathbf{x}}_{t|t}$$

$$P_{t+1|t} = AP_{t|t}A^T + Q$$

– Measurement update:

$$\begin{aligned} K_{t+1} &= P_{t+1|t}C^T\left(CP_{t+1|t}C^T + R\right)^{-1} \\ \hat{\mathbf{x}}_{t+1|t+1} &= \hat{\mathbf{x}}_{t+1|t} + K_{t+1}\left(\mathbf{y}_{t+1} - C\hat{\mathbf{x}}_{t+1|t}\right) \\ P_{t+1|t+1} &= P_{t+1|t} - K_{t+1}CP_{t+1|t} \end{aligned}$$

Lets fill in the rest of that transition Matrix



$$T_{X_0 \to X_{-1}} = \begin{matrix} 0.2 & 0.1 & 0.1 \\ 0.3 & 0.3 & 0.8 \\ 0.5 & 0.6 & 0.1 \end{matrix}$$

Probability of transition as a matrix

- We can form a matrix which describes the probability of transition given a particular state described as a "one hot" vector

$$X_{-1} = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

$$T_{X_1 \to X_0} = \begin{matrix} 0.2 & ? & ? \\ 0.3 & ? & ? \\ 0.5 & ? & ? \end{matrix}$$

$$P_{X_0|X_{-1}} = T_{X_{-1} \to X_0} \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

- The columns of T must sum to one (see previous slide)
- Note the transition matrix is often denoted as P

THAT'S AN EIGENVECTOR PROBLEM!

- **The distribution of the symbols are simply the left-eigenvectors of the transition matrix**

$$\mathrm{Eig}(P^T)$$

- To solve, you can solve the characteristic Eq.

$$|P^T - \lambda\, I| = 0$$

- We will go through how to solve the eq above in the tutes for a 2x2 matrix

Linear Systems

Review

- What do you think when you see?

$$\ddot{y} + 2\dot{y} + 3y = u$$

- System?
- ODE?
- Linear Algebra?
- Joy?
- Excitement?
- Shock and Awe??

Linear algebra provides the tools/foundation for working with (linear) differential equations.

Signals & Systems

Linear algebra provides the tools/foundation for working with (linear) differential equations.

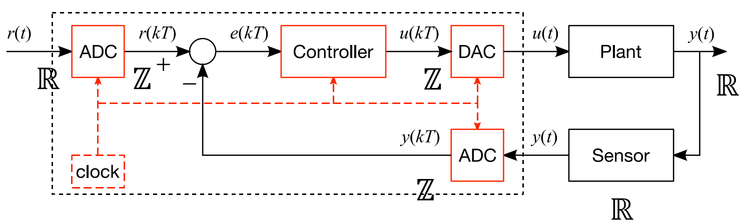
- Signals are vectors. Systems are matrices.

$$\mathbf{y} = F\mathbf{x} + G\mathbf{u}$$

Recap: Digital Systems:

- Implies something "something discrete" or … **that a mapping exists to an "integer set"** $s \in \mathbb{Z}$

- Often the "state-space" and "time" are discretised. *(But they **both** need no be)*



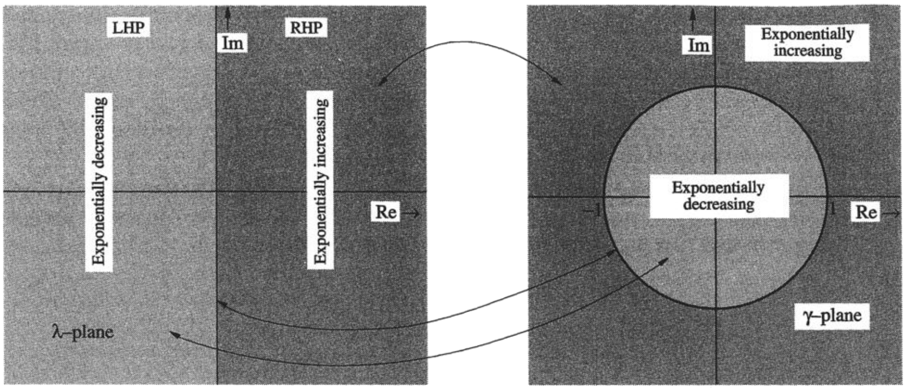
- Why?
  - Beat the noise (e.g., more signal "sharing")
  - Leverage time-keeping (oscillator) precision

Discrete-Time Exponential γ^k

Recall we defined $z = e^{sT}$

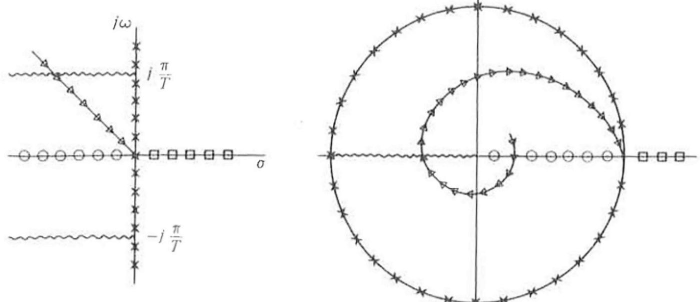
Thus, this is the γ plane ➔

$$e^{\lambda k} = \gamma^k$$



(a) (b)

*Hint: Use γ to Transform s ↔ z: z=e^sT^*

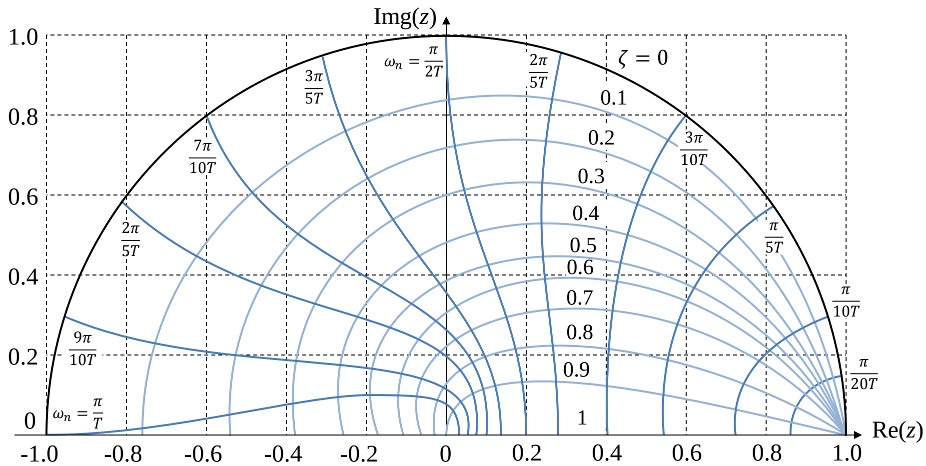


s-plane	Symbol	z-plane
$s = j\omega$ Real frequency axis	× × ×	$\lvert z\rvert = 1$ Unit circle
$s = \sigma \geq 0$	□□□	$z = r \geq 1$
$s = \sigma \leq 0$	○○○	$z = r,\ 0 \leq r \leq 1$
$s = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -a + jb$ Constant damping ratio if ζ is fixed and ω_n varies	△△△	$z = re^{j\theta}$ where $r = \exp(-\zeta\omega_n T) = e^{-aT}$, $\theta = \omega_n T\sqrt{1-\zeta^2} = bT$ Logarithmic spiral
$s = \pm j(\pi/T) + \sigma, \sigma \leq 0$	〰〰	$z = -r$

(a) (b)

z-Plane Response for 2nd Order Systems: Damping (ζ) and Natural frequency (ω)

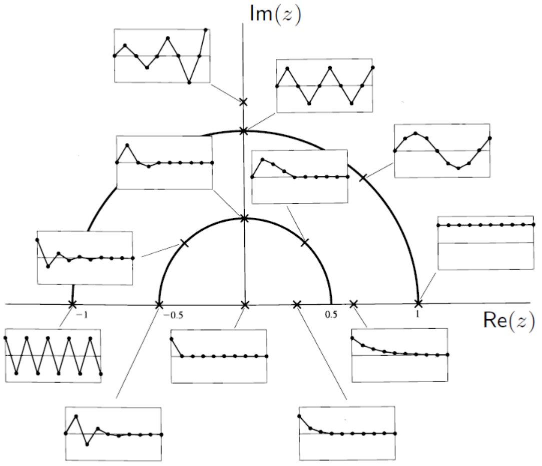
$$z = e^{sT} \text{ where } s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$



[Adapted from Franklin, Powell and Emami-Naeini]

Pole positions in the z-plane

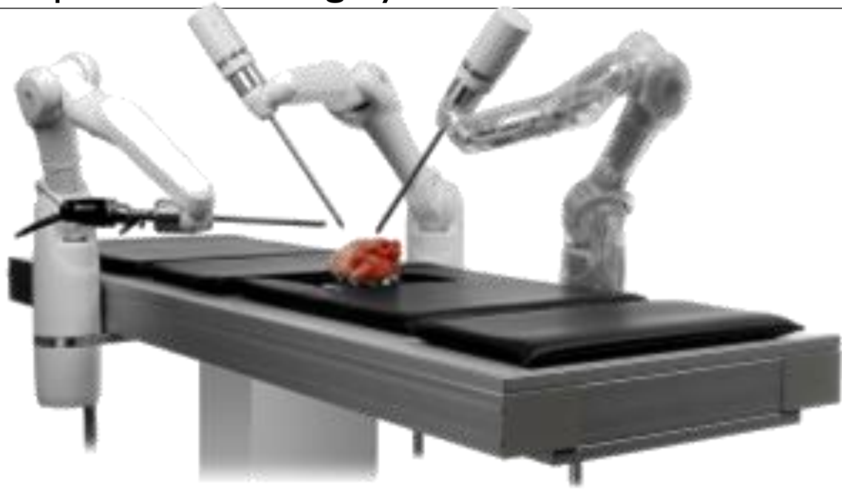
- Poles inside the unit circle are **stable**
- Poles outside the unit circle **unstable**
- Poles on the unit circle are oscillatory
- Real poles at 0 < z < 1 give exponential response
- Higher frequency of oscillation for larger
- Lower apparent damping for larer and r



Break ☺

Now, What's Next?

Computer Aided Surgery: R/C Toolholders?

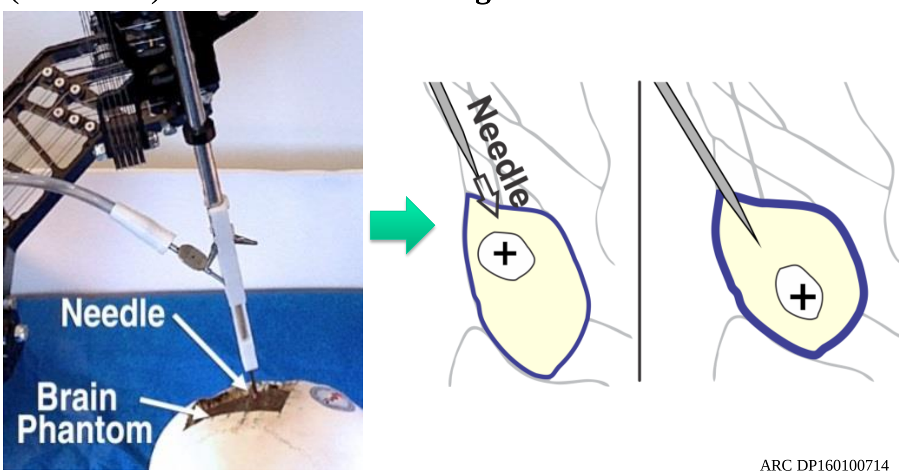


→ Move in tandem with heart: Cardiac procedures without stopping it

- Unstructured environment (patient) makes this harder

Modern (Tele)Surgical Robotics:

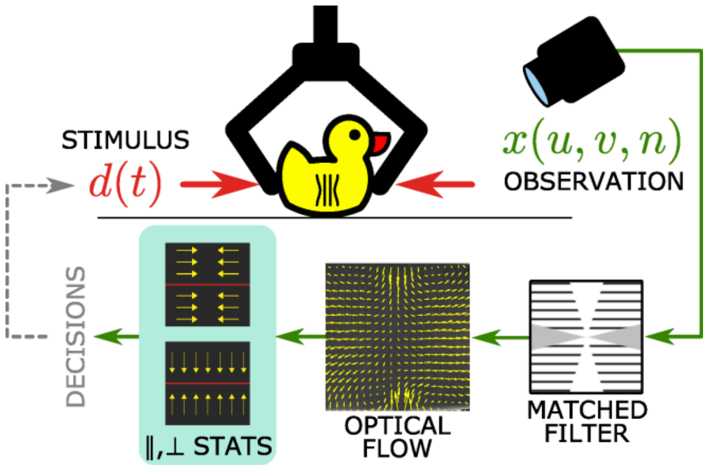
- **Biomechanics approach**: Predict expected tissue trajectories
- (Stochastic) **Robot Motion Planning / Control Methods**!



ARC DP160100714

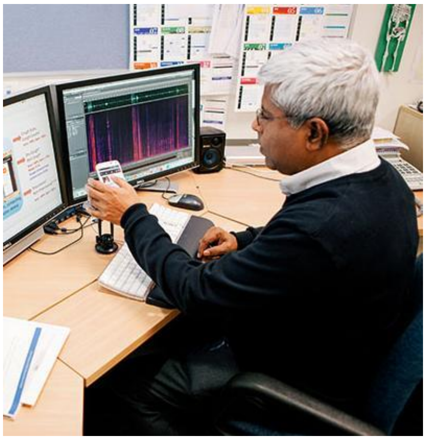
Computer Aided Surgery: "**Soft**" is "**Hard**"!

Research: Incorporating Stiffness (Haptics): Visual Deformable Object Analysis



Dansereau, Singh, Leitner, *ICRA 2016*

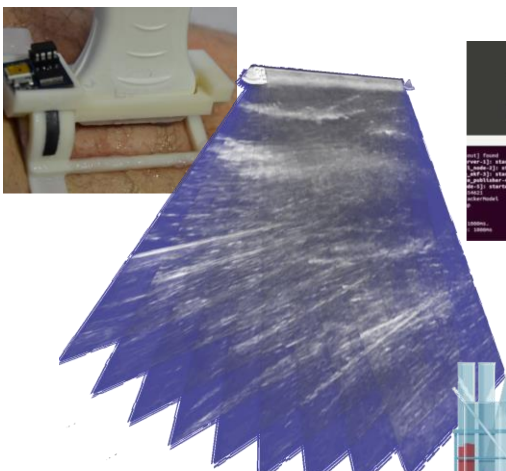
Iceberg to Titanic: Take Advantage of Information



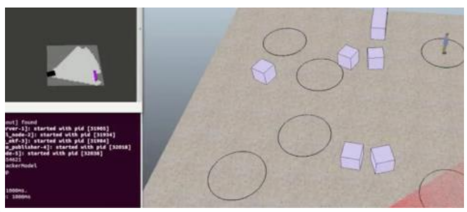
- 30 Min/Day Talking on Phone
  - 5.5 days/year of audio samples
  - Track this (notably the pauses) over time to detect onset of dimentia
- 150 Photos/Month

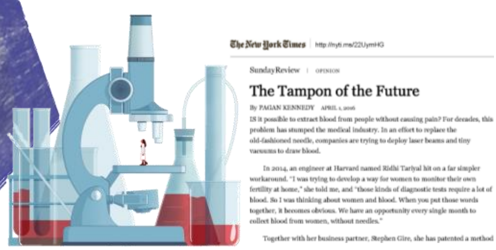
How?

- More Signals



- Stochastic Processing (Think **TAPIR!**)





The New York Times | http://nyti.ms/22UymHG

SundayReview | OPINION

The Tampon of the Future

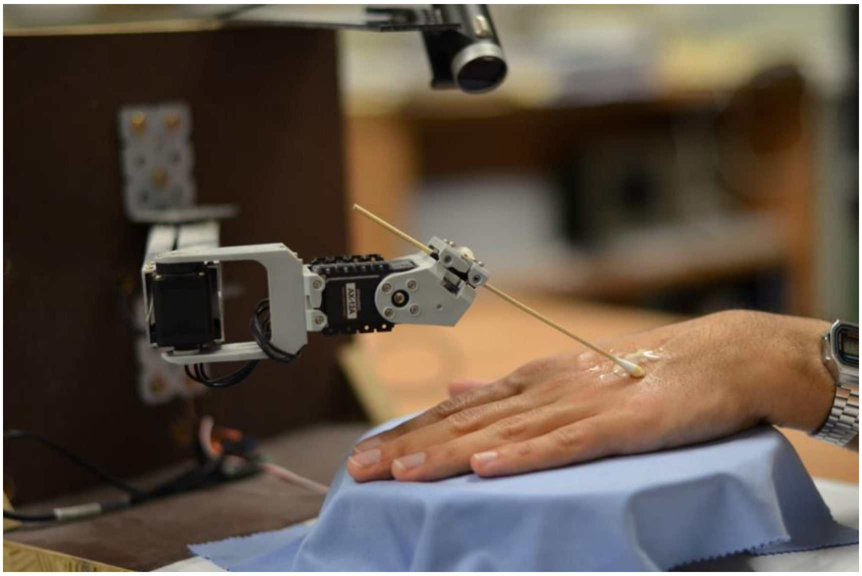
By PAGAN KENNEDY APRIL 1, 2016

IS it possible to extract blood from people without causing pain? For decades, this problem has stumped the medical industry. In an effort to replace the old-fashioned needle, companies are trying to deploy laser beams and tiny vacuums to draw blood.

In 2014, an engineer at Harvard named Ridhi Tariyal hit on a far simpler workaround. "I was trying to develop a way for women to monitor their own fertility at home," she told me, and "those kinds of diagnostic tests require a lot of blood. So I was thinking about women and blood. When you put those words together, it becomes obvious. We have an opportunity every single month to collect blood from women, without needles."

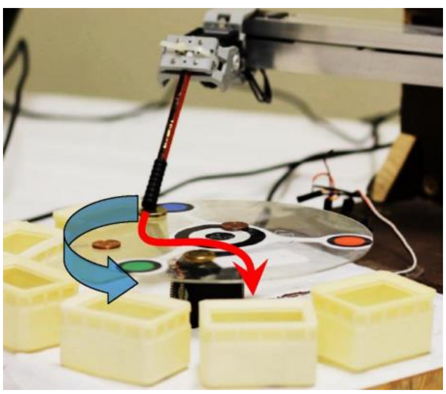
Together with her business partner, Stephen Gire, she has patented a method

Robotics & Health: A Friendly Touch!

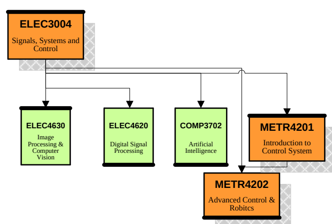


What's Next?

- METR 4202:



- ELEC/METR Programs

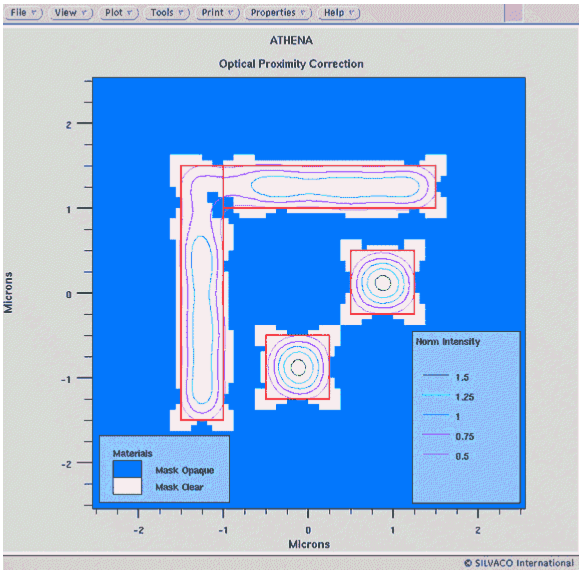


Is it Useful?

Yes.

(For example … Next Year – ELEC/METR 4810)

Advanced Application: Optical Proximity Correction



It Can Rock Your Boat Gently Down The Stream: IMU Deaduced Reckoning (Navigation)

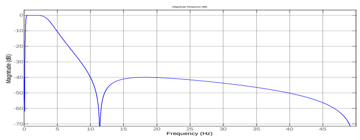
Idea: Integrate your motion (twice for $\ddot{x} \rightarrow x$ and once for $\dot{\theta} \rightarrow \theta$)

Problem:

- (DC) bias in accelerometer → <u>drift</u>

Solution:

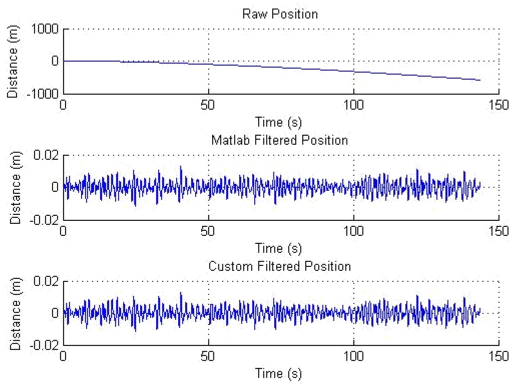
- IIR Bandpass filter (0.1-10 Hz)



```
A = [1.0000   -5.8235   14.1332  -18.2966   13.3259   -5.1772    0.8382];
B = [0.9155   -5.4933   13.7331  -18.3109   13.7331   -5.4933    0.9155];
```


It Can Rock Your Boat Gently Down The Stream: IMU Deaduced Reckoning (Navigation) [2]

Solution:



SECATs:
Let's look back at the topic list from Lecture 1

The course is has a huge mandate:

- It is really 3 · ½ courses in one !
  - Linear Systems
  - Signal Processing
  - Controls & Digital Controls

- ∴ It is **b r o a d !!**

- There is a logic to it
  - They share the same mathematical nature (poles & zeros)
  - The math is common to more than just circuits!

Lots of Stuff To Cover…

- Systems
- Signal Abstractions
- Signals as Vectors / Systems as Maps

- Linear Systems and Their Properties
- LTI Systems
- Autonomous Linear Dynamical Systems

- Convolution
- FIR & IIR Systems
- Frequency domain
- Fourier Transform (CT)
- Fast Fourier Transform (DT)

- Even and Odd Signals
- Likelihood
- Causality

- Impulse Response
- Root Locus
- Bode Functions

- Left-hand Plane

- Frequency Response

- Discrete Time
- Continuous Time

- Laplace Transformation
- Feedback and Control
- Additional Applications

- Linear Functions
- Linear Algebra Review
- Least Squares
- Least Squares Problems
- Least Squares Applications
- Matrix Decomposition and Linear Algebra
- Regularized Least Squares

- Least-squares
- Least-squares applications
- Orthonormal sets of vectors
- Eigenvectors and diagonalization
- Linear dynamical systems with inputs and outputs
- Symmetric matrices, quadratic forms, matrix norm, and SVD

- Controllability and state transfer
- Observability and state estimation

- And that, of course,
  Linear Systems are Cool! ☺

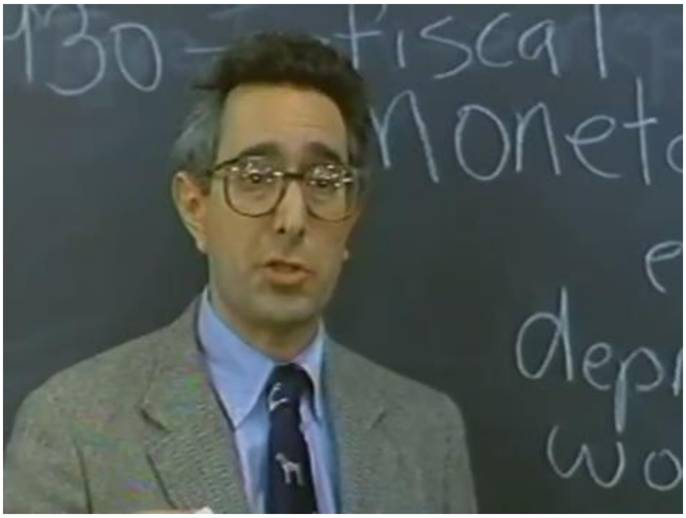
Yes, this is a Theoretical Approach! Why?

- Theory wins because the importance of any one application seems limited
- Breath
  - Books, books, everywhere, yet we're all on Wikipedia!!
- Assumptions:
  - Numerous conditions that need to be remembered
- Tacit Details:

  →The need for examples (but these are few and always seem the same)
- Time consuming

"4" Is Average

- What is a 3?



SECaTs: Some Lessons in the Works for Next Year

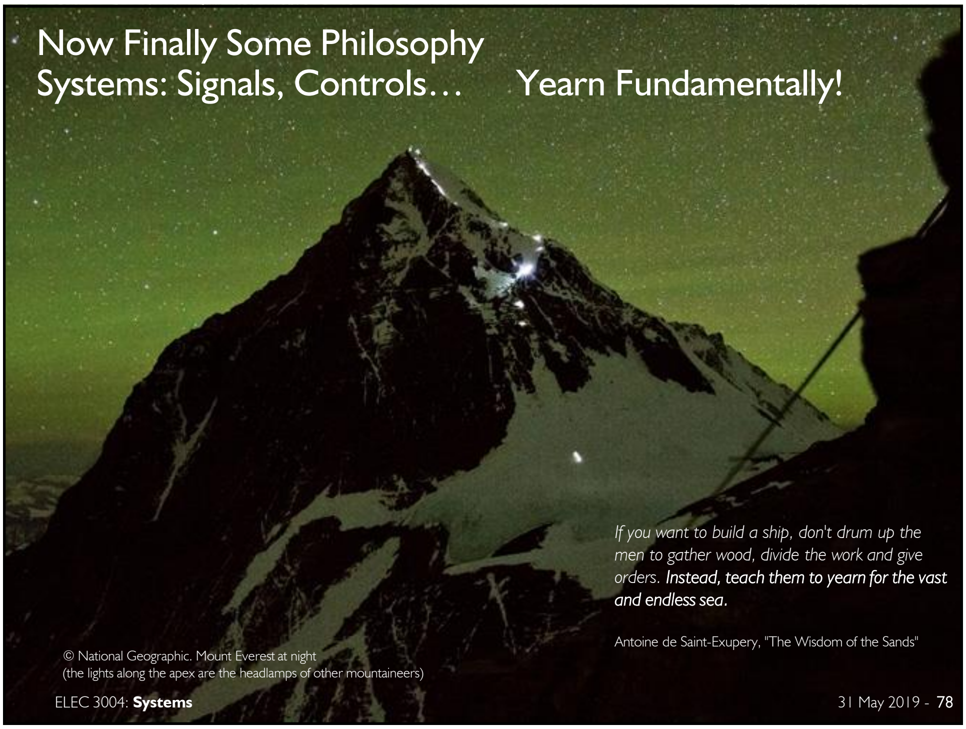
- I shall only use my own slides

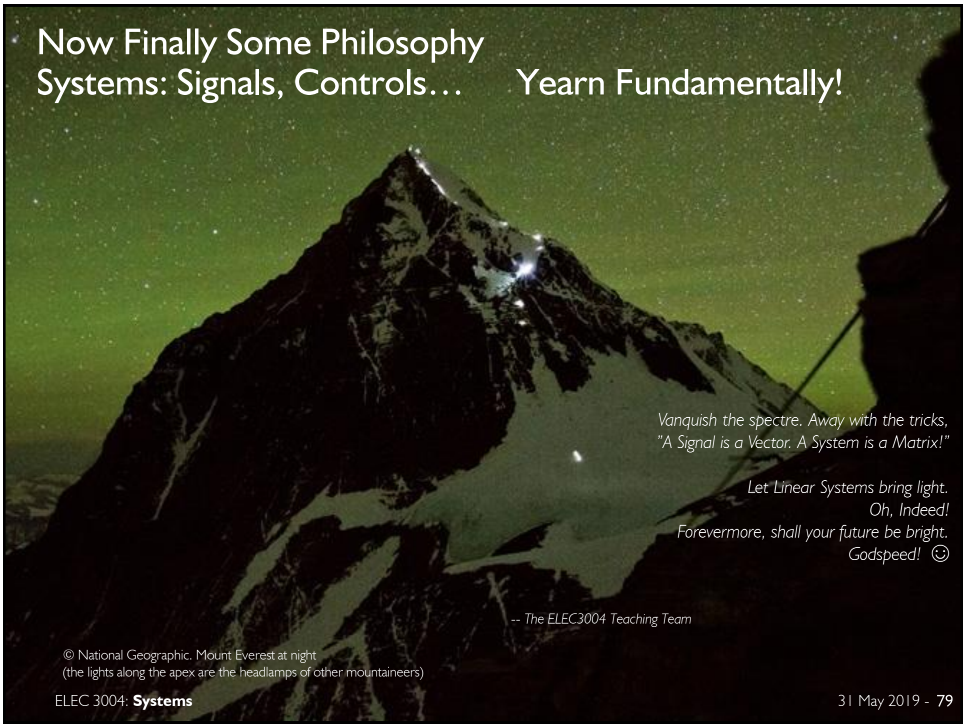
- Less is more!
  - Smaller assignments
  - More time for Examples

- Better organization
  - Better tutorials
  - More examples!!
  - I get that. But, we’ve come a long way

➔ To make this happen I need your support!

ELEC 3004: **Systems** 31 May 2019 - 77





Next Time…

- **There is no next time! ☺**

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Systems as Maps & Signals as Vectors
	8-Mar	Systems: Linear Differential Systems
3	13-Mar	Sampling Theory & Data Acquisition
	15-Mar	Aliasing & Antialiasing
4	20-Mar	Discrete Time Analysis & Z-Transform
	22-Mar	Second Order LTID (& Convolution Review)
5	27-Mar	Frequency Response
	29-Mar	Filter Analysis
6	3-Apr	Digital Filters (IIR) & Filter Analysis
	5-Apr	PS 1: Q & A
7	10-Apr	Digital Windows
	12-Apr	Digital Filter (FIR)
8	17-Apr	Active Filters & Estimation
	19-Apr	**Holiday**
	24-Apr	
	26-Apr	
9	1-May	Introduction to Feedback Control
	3-May	Servoregulation & PID Control
10	8-May	State-Space Control
	10-May	Guest Lecture: FFT
11	15-May	Advanced PID & & FFT Processes
	17-May	State Space Control System Design
12	22-May	Shaping the Dynamic Response
	24-May	Stability and Examples
13	29-May	System Identification & Information Theory & Information Space
	31-May	Summary and Course Review

- We're at the End. It's (the) final!
- Thank you folks!