http://elec3004.com

# Servoregulation & PID Control

ELEC 3004: **Systems**: Signals & Controls
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Lecture 17

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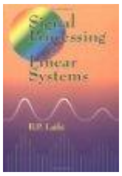
## Lecture Schedule:

| Week | Date | Lecture Title |
|---|---|---|
| 1 | 27-Feb | Introduction |
| | 1-Mar | Systems Overview |
| 2 | 6-Mar | Systems as Maps & Signals as Vectors |
| | 8-Mar | Systems: Linear Differential Systems |
| 3 | 13-Mar | Sampling Theory & Data Acquisition |
| | 15-Mar | Aliasing & Antialiasing |
| 4 | 20-Mar | Discrete Time Analysis & Z-Transform |
| | 22-Mar | Second Order LTID (& Convolution Review) |
| 5 | 27-Mar | Frequency Response |
| | 29-Mar | Filter Analysis |
| 6 | 3-Apr | Digital Filters (IIR) & Filter Analysis |
| | 5-Apr | PS 1: Q & A |
| 7 | 10-Apr | Digital Windows |
| | 12-Apr | Digital Filter (FIR) |
| 8 | 17-Apr | Active Filters & Estimation |
| | 19-Apr | Holiday |
| | 24-Apr | |
| | 26-Apr | |
| 9 | 1-May | Introduction to Feedback Control |
| | **3-May** | **Servoregulation & PID Control** |
| 10 | 8-May | Guest Lecture: FFT |
| | 10-May | State-Space Control |
| 11 | 15-May | Digital Control Design |
| | 17-May | Stability |
| 12 | 22-May | State Space Control System Design |
| | 24-May | Shaping the Dynamic Response |
| 13 | 29-May | System Identification & Information Theory |
| | 31-May | Summary and Course Review |

## Follow Along Reading:




**B. P. Lathi**
***Signal processing and linear systems***
**1998**
**TK5102.9.L38 1998**

**G. Franklin,**
**J. Powell,**
**M. Workman**
*Digital Control of Dynamic Systems*
1990

**TJ216.F72 1990**
[Available as UQ Ebook]

Today

**➔ P - I - D**

- FPW
  - Chapter 4:
    Discrete Equivalents to Continuous Transfer Functions: The Digital Filter

- FPW
  - Chapter 5: Design of Digital Control Systems Using Transform Techniques

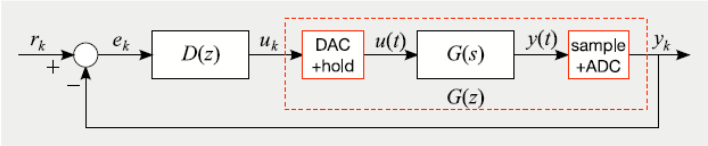
***Next Time***



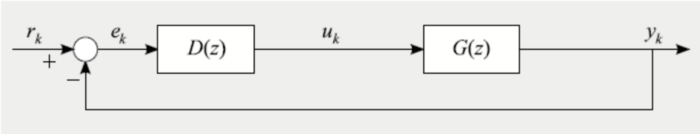
# Purely Discrete Design

## Designing in the Purely Discrete…

Analyse/design a discrete controller $D(z)$:




by considering the purely discrete time system:




Closed loop system tranfer function: $\frac{Y(z)}{R(z)} = \frac{G(z)D(z)}{1+G(z)D(z)}$

How do the closed loop poles relate to → stability?

→ performance?



## Now in discrete

- Naturally, there are discrete analogs for each of these controller types:

Lead/lag: $\frac{1-\alpha z^{-1}}{1-\beta z^{-1}}$

PID: $k\left(1+\frac{1}{\tau_i(1-z^{-1})}+\tau_d(1-z^{-1})\right)$

But, where do we get the control design parameters from?
The s-domain?

## Sampling a continuous-time system

suppose $\dot{x} = Ax$

sample $x$ at times $t_1 \leq t_2 \leq \cdots$: define $z(k) = x(t_k)$

then $z(k+1) = e^{(t_{k+1}-t_k)A} z(k)$

for uniform sampling $t_{k+1} - t_k = h$, so

$$z(k+1) = e^{hA} z(k),$$

a discrete-time LDS (called *discretized version* of continuous-time system)

Source: Boyd, Lecture Notes for EE263, 10-22



## Piecewise constant system

consider *time-varying* LDS $\dot{x} = A(t)x$, with

$$A(t) = \begin{cases} A_0 & 0 \leq t < t_1 \\ A_1 & t_1 \leq t < t_2 \\ \vdots \end{cases}$$

where $0 < t_1 < t_2 < \cdots$ (sometimes called jump linear system)

for $t \in [t_i, t_{i+1}]$ we have

$$x(t) = e^{(t-t_i)A_i} \cdots e^{(t_3-t_2)A_2} e^{(t_2-t_1)A_1} e^{t_1 A_0} x(0)$$

(matrix on righthand side is called state transition matrix for system, and denoted $\Phi(t)$)

Source: Boyd, Lecture Notes for EE263, 10-23

## Qualitative behaviour of *x(t)*

suppose $\dot{x} = Ax$, $x(t) \in \mathbf{R}^n$

then $x(t) = e^{tA}x(0)$; $X(s) = (sI - A)^{-1}x(0)$

$i$th component $X_i(s)$ has form

$$X_i(s) = \frac{a_i(s)}{\mathcal{X}(s)}$$

where $a_i$ is a polynomial of degree $< n$

thus the poles of $X_i$ are all eigenvalues of $A$ (but not necessarily the other way around)

Source: Boyd, Lecture Notes for EE263, 10-24



## Qualitative behaviour of *x(t)* [2]

first assume eigenvalues $\lambda_i$ are distinct, so $X_i(s)$ cannot have repeated poles

then $x_i(t)$ has form

$$x_i(t) = \sum_{j=1}^{n} \beta_{ij} e^{\lambda_j t}$$

where $\beta_{ij}$ depend on $x(0)$ (linearly)

eigenvalues determine (possible) qualitative behavior of $x$:

- eigenvalues give exponents that can occur in exponentials
- real eigenvalue $\lambda$ corresponds to an exponentially decaying or growing term $e^{\lambda t}$ in solution
- complex eigenvalue $\lambda = \sigma + j\omega$ corresponds to decaying or growing sinusoidal term $e^{\sigma t}\cos(\omega t + \phi)$ in solution

Source: Boyd, Lecture Notes for EE263, 10-25

## Qualitative behaviour of *x(t)* [3]

first assume eigenvalues $\lambda_i$ are distinct, so $X_i(s)$ cannot have repeated poles

then $x_i(t)$ has form

$$x_i(t) = \sum_{j=1}^{n} \beta_{ij} e^{\lambda_j t}$$

where $\beta_{ij}$ depend on $x(0)$ (linearly)

eigenvalues determine (possible) qualitative behavior of $x$:

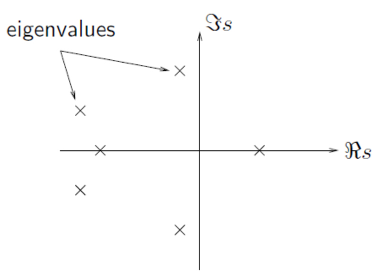
- eigenvalues give exponents that can occur in exponentials
- real eigenvalue $\lambda$ corresponds to an exponentially decaying or growing term $e^{\lambda t}$ in solution
- complex eigenvalue $\lambda = \sigma + j\omega$ corresponds to decaying or growing sinusoidal term $e^{\sigma t} \cos(\omega t + \phi)$ in solution

Source: Boyd, Lecture Notes for EE263, 10-26



## Qualitative behaviour of *x(t)* [4]

- $\Re\lambda_j$ gives exponential growth rate (if $> 0$), or exponential decay rate (if $< 0$) of term
- $\Im\lambda_j$ gives frequency of oscillatory term (if $\neq 0$)




Source: Boyd, Lecture Notes for EE263, 10-27

## Qualitative behaviour of *x(t)* [5]

now suppose $A$ has repeated eigenvalues, so $X_i$ can have repeated poles

express eigenvalues as $\lambda_1, \ldots, \lambda_r$ (distinct) with multiplicities $n_1, \ldots, n_r$, respectively ($n_1 + \cdots + n_r = n$)

then $x_i(t)$ has form

$$x_i(t) = \sum_{j=1}^{r} p_{ij}(t) e^{\lambda_j t}$$

where $p_{ij}(t)$ is a polynomial of degree $< n_j$ (that depends linearly on $x(0)$)

Source: Boyd, Lecture Notes for EE263, 10-28



## Emulation vs Discrete Design

- Remember: polynomial algebra is the same, whatever symbol you are manipulating:

  eg. $s^2 + 2s + 1 = (s+1)^2$

  $z^2 + 2z + 1 = (z+1)^2$

  Root loci behave the same on both planes!

- Therefore, we have two choices:
  - Design in the s-domain and digitise (emulation)
  - Design only in the z-domain (discrete design)

# Lead/Lag



## Some standard approaches

- Control engineers have developed time-tested strategies for building compensators
- Three classical control structures:
  - Lead
  - Lag
  - Proportional-Integral-Derivative (PID)
    (and its variations: P, I, PI, PD)

How do they work?

## Lead/lag compensation

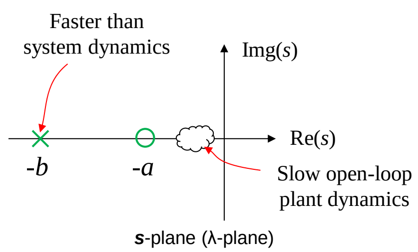
- Serve different purposes, but have a similar dynamic structure:

$$D(s) = \frac{s+a}{s+b}$$

Note:
Lead-lag compensators come from the days when control engineers cared about constructing controllers from networks of op amps using frequency-phase methods. These days pretty much everybody uses PID, but you should at least know what the heck they are in case someone asks.

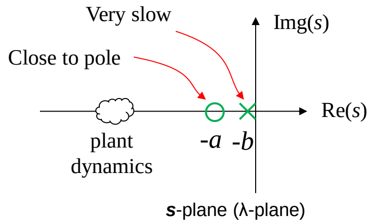


## Lead compensation: *a* < *b*




- Acts to decrease rise-time and overshoot
  - Zero draws poles to the left; adds phase-lead
  - Pole decreases noise
- Set $a$ near desired $\omega_n$; set $b$ at ~3 to 20x $a$

## Lag compensation: $a > b$




- Improves steady-state tracking
  - Near pole-zero cancellation; adds phase-lag
  - Doesn't break dynamic response (too much)
- Set $b$ near origin; set $a$ at ~3 to 10x $b$



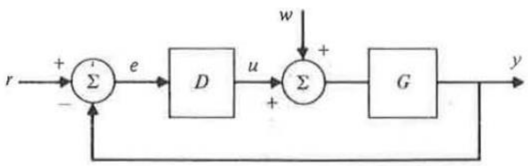
BREAK

PID
(Intro)



## PID

- Three basic types of control:
  - Proportional
  - Integral, and
  - Derivative
- The next step up from lead compensation
  - Essentially a combination of proportional and derivative control



## Proportional Control

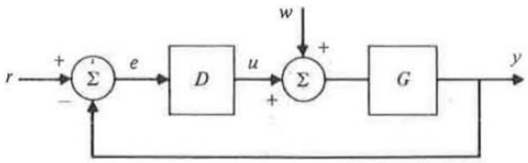
A discrete implementation of proportional control is identical to continuous; that is, where the continuous is

$$u(t) = K_p e(t) \quad \Rightarrow \quad D(s) = K_p,$$

the discrete is

$$u(k) = K_p e(k) \quad \Rightarrow \quad \boxed{D(z) = K_p}$$

where $e(t)$ is the error signal as shown in Fig 5.2.






## Integral: P Control only

- Consider a first order system with a constant load disturbance, $w$; (recall as $t \to \infty$, $s \to 0$)

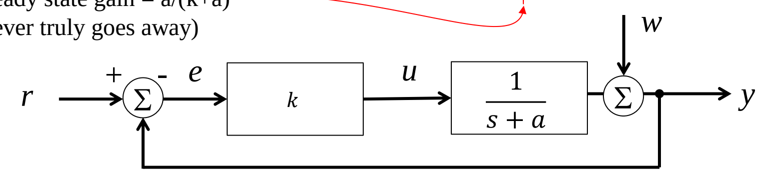
$$y = k\frac{1}{s+a}(r-y) + w$$

$$(s+a)y = k\,(r-y) + (s+a)w$$

$$(s+k+a)y = kr + (s+a)w$$

$$y = \frac{k}{s+k+a}r + \frac{(s+a)}{s+k+a}w$$

Steady state gain = a/(k+a)
(never truly goes away)



## Integral

- Integral applies control action based on accumulated output error
  - Almost always found with P control
- Increase dynamic order of signal tracking
  - Step disturbance steady-state error goes to zero
  - Ramp disturbance steady-state error goes to a constant offset

Let's try it!



## Integral Control

For continuous systems, we integrate the error to arrive at the control,

$$u(t) = \frac{K_p}{T_I}\int_{t_o}^{t} e(t)dt \;\Rightarrow\; D(s) = \frac{K_p}{T_I s},$$

where $T_I$ is called the *integral*, or *reset time*. The discrete equivalent is to sum all previous errors, yielding

$$u(k) = u(k-1) + \frac{K_p T}{T_I}e(k) \;\Rightarrow\; \boxed{D(z) = \frac{K_p T}{T_I(1-z^{-1})} = \frac{K_p T z}{T_I(z-1)}.} \quad (5.60)$$

Just as for continuous systems, the primary reason for integral control is to reduce or eliminate steady-state errors, but this typically occurs at the cost of reduced stability.

## Now with added integral action

$$y = k\left(1 + \frac{1}{\tau_i s}\right)\frac{1}{s+a}(r - y) + w$$

$$y = k\frac{s + \tau_i^{-1}}{s}\frac{1}{s+a}(r - y) + w$$

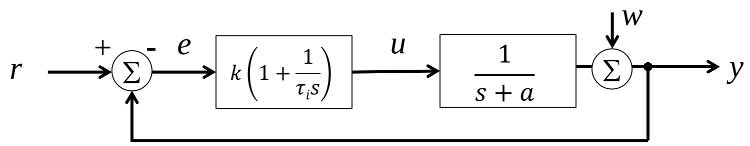
Same dynamics

$$s(s+a)y = k(s + \tau_i^{-1})(r - y) + s(s+a)w$$

$$(s^2 + (k+a)s + \tau_i^{-1})y = k(s + \tau_i^{-1})r + s(s+a)w$$

$$y = \frac{k(s + \tau_i^{-1})}{(s^2 + (k+a)s + \tau_i^{-1})}r + \frac{s(s+a)}{k(s + \tau_i^{-1})}w$$

Must go to zero for constant *w*!






## Derivative Control

For continuous systems, derivative or rate control has the form

$$u(t) = K_p T_D \dot{e}(t) \quad \Rightarrow \quad D(s) = K_p T_D s$$

where $T_D$ is called the *derivative time*. Differentiation can be approximated in the discrete domain as the first difference, that is,

$$u(k) = K_p T_D \frac{(e(k) - e(k-1))}{T} \quad \Rightarrow \quad \boxed{D(z) = K_p T_D \frac{1 - z^{-1}}{T} = K_p T_D \frac{z-1}{Tz}.}$$

In many designs, the compensation is a sum of proportional and derivative control (or PD control). In this case, we have

$$D(z) = K_p\left(1 + \frac{T_D(z-1)}{Tz}\right).$$

or, equivalently,

$$\boxed{D(z) = K\frac{z - \alpha}{z}}$$

## Derivative Control [2]

- Similar to the lead compensators
  - The difference is that the pole is at $z = 0$

  [Whereas the pole has been placed at various locations along the z-plane real axis for the previous designs. ]

- In the continuous case:
  - pure derivative control represents the ideal situation in that there is no destabilizing phase lag from the differentiation
  - the pole is at $s = -\infty$

- In the discrete case:
  - $z=0$
  - However this has phase lag because of the necessity to wait for one cycle in order to compute the first difference



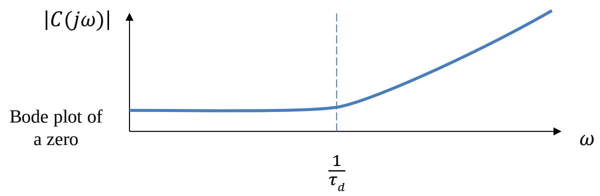
## Derivative

- Derivative uses the rate of change of the error signal to anticipate control action
  - Increases system damping (when done right)
  - Can be thought of as ‘leading’ the output error, applying correction predictively
  - Almost always found with P control*

**What kind of system do you have if you use D, but don’t care about position?* Is it the same as P control in velocity space?

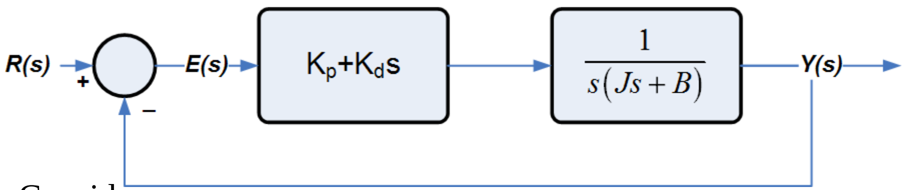
## Derivative

- It is easy to see that PD control simply adds a zero at $s = -\frac{1}{\tau_d}$ with expected results
  - Decreases dynamic order of the system by 1
  - Absorbs a pole as $k \to \infty$
- Not all roses, though: derivative operators are sensitive to high-frequency noise






## PD for 2nd Order Systems




- Consider:

$$\frac{Y(s)}{R(s)} = \frac{(K_P + K_D s)}{Js^2 + (B + K_D)s + (1 + K_P)}$$

- Steady-state error: $e_{ss} = \frac{1}{(1+K_P)}$
- Characteristic equation: $Js^2 + (B + K_D)s + (1 + K_P) = 0$
- Damping Ratio: $\zeta = \frac{B+K_D}{2\sqrt{(1+K_P)J}}$

➔ It is possible to make $e_{ss}$ and overshoot small (↓) by making B small (↓), $K_P$ large ↑, $K_D$ such that ζ:between [0.4 – 0.7]

## PID – Control for the PID-dly minded

- Proportional-Integral-Derivative control is the control engineer's hammer*
  – For P,PI,PD, etc. just remove one or more terms

$$C(s) = k\left(1 + \frac{1}{\tau i s} + \tau d s\right)$$

Proportional
Integral
Derivative

*Everything is a nail. That's why it's called "Bang-Bang" Control ☺



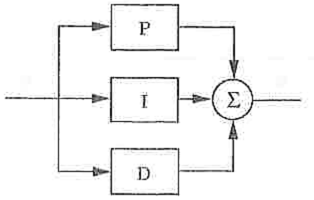
## PID

- Collectively, PID provides two zeros plus a pole at the origin
  – Zeros provide phase lead
  – Pole provides steady-state tracking
  – Easy to implement in microprocessors

- Many tools exist for optimally tuning PID
  – Zeigler-Nichols
  – Cohen-Coon
  – Automatic software processes

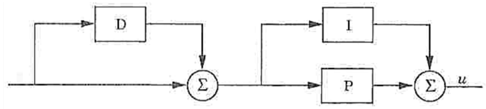
## PID Implementation

- Non-Interacting




$$C(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right)$$

- Interacting Form



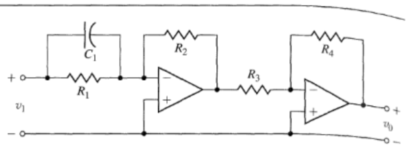

$$C'(s) = K\left(1 + \frac{1}{sT_i}\right)(1 + sT_d)$$

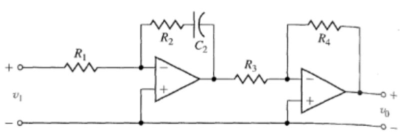
- Note: Different $K$,$T_i$ and $T_d$

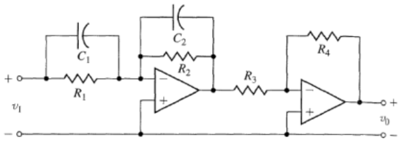


## Operational Amplifier Circuits for Compensators

| Type of Controller | $G_c(s) = \frac{V_0(s)}{V_1(s)}$ |
|---|---|
| PD | $G_c = \frac{R_4R_2}{R_3R_1}(R_1C_1s + 1)$ |
| PI | $G_c = \frac{R_4R_2(R_2C_2s + 1)}{R_3R_1(R_2C_2s)}$ |
| Lead or lag<br>Lead if $R_1C_1 > R_2C_2$<br>Lag if $R_1C_1 < R_2C_2$ | $G_c = \frac{R_4R_2(R_1C_1s + 1)}{R_3R_1(R_2C_2s + 1)}$ |




- (Yet Another Way to See PID)

**Source**: Dorf & Bishop, *Modern Control Systems*, p. 828

## PID Control

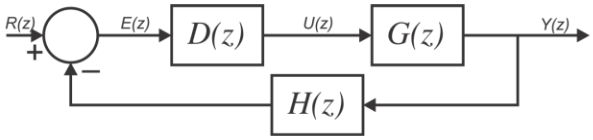
$$D(z) = K_p\left(1 + \frac{Tz}{T_I(z-1)} + \frac{T_D(z-1)}{Tz}\right).$$

The user simply has to determine the best values of

- $K_p$
- $T_D$ and
- $T_I$



## PID as Difference Equation




$$\frac{U(z)}{E(z)} = D(z) = K_p + K_i\left(\frac{Tz}{z-1}\right) + K_d\left(\frac{z-1}{Tz}\right)$$

$$u(k) = \left[K_p + K_i T + \left(\frac{K_d}{T}\right)\right] \cdot e(k) - [K_d T] \cdot e(k-1) + [K_i] \cdot u(k-1)$$

## PID Algorithm (in various domains):

FPW § 5.8.4 [p.224]

- PID Algorithm (in Z-Domain):

$$D(z) = K_p\left(1 + \frac{Tz}{T_I(z-1)} + \frac{T_D(z-1)}{Tz}\right)$$

- As Difference equation:

$$u(t_k) = u(t_{k-1}) + K_p\left[\left(1 + \frac{\Delta t}{T_i} + \frac{T_d}{\Delta t}\right)e(t_k) + \left(-1 - \frac{2T_d}{\Delta t}\right)e(t_{k-1}) + \frac{T_d}{\Delta t}e(t_{k-2})\right]$$

- Pseudocode [Source: Wikipedia]:

```
previous_error = 0, integral = 0
start:
  error = setpoint - measured_value
  integral = integral + error*dt
  derivative = (error - previous_error)/dt
  output = Kp*error + Ki*integral + Kd*derivative
  previous_error = error
  wait(dt)
  goto start
```



# PID Intuition

## Another way to see P I|D

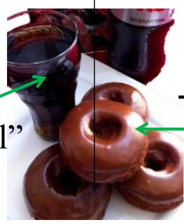
- Derivative

  D provides:
  - High sensitivity
  - Responds to change
  - Adds "damping" & ∴ permits larger $K_P$
  - Noise sensitive
  - Not used alone (∵ its on rate change of error – by itself it wouldn't get there)

→ "**D**iet Coke of control"

- Integral
  - Eliminates offsets (makes regulation ☺)
  - Leads to Oscillatory behaviour
  - Adds an "order" but instability (Makes a 2nd order system 3rd order)

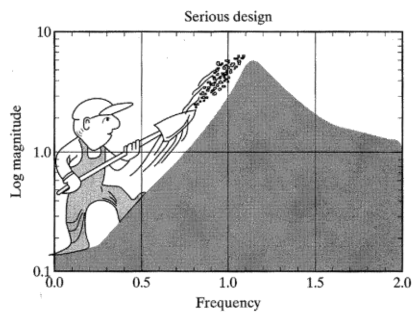
→ "**I**nteresting cake of control"





## Seeing PID – No Free Lunch

- The energy (and sensitivity) moves around (in this case in "frequency")




- Sensitivity reduction at low frequency unavoidably leads to sensitivity increase at higher frequencies.

**Source**: Gunter Stein's interpretation of the water bed effect – G. Stein, *IEEE Control Systems Magazine*, 2003.

## PID Intuition & Tuning

- Tuning – How to get the "magic" values:
  - Dominant Pole Design
  - Ziegler Nichols Methods
  - Pole Placement
  - Auto Tuning

- Although PID is common it is often poorly tuned
  - The derivative action is frequently switched off!
  (Why ∵ it's sensitive to noise)
  - Also lots of "I" will make the system more transitory & leads to integrator wind-up.



## PID Intuition

| **Effects of increasing a parameter independently** | | | | | |
|---|---|---|---|---|---|
| **Parameter** | Rise time | Overshoot | Settling time | Steady-state error | Stability |
| $K_p$ | ↓ | ⇑ | Minimal change | ↓ | ↓ |
| $K_I$ | ↓ | ⇑ | ⇑ | Eliminate | ↓ |
| $K_D$ | Minor change | ↓ | ↓ | No effect / minimal change | Improve (if $K_D$ small) |

## PID Intuition

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int e(s)\,ds + T_d\frac{de(t)}{dt}\right]$$

- P:
  - Control action is proportional to control error
  - It is necessary to have an error to have a non-zero control signal
- I:
  - The main function of the integral action is to make sure that the process output agrees with the set point in steady state

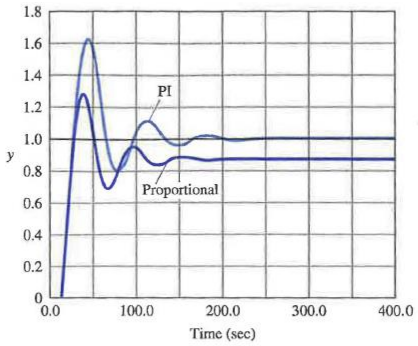


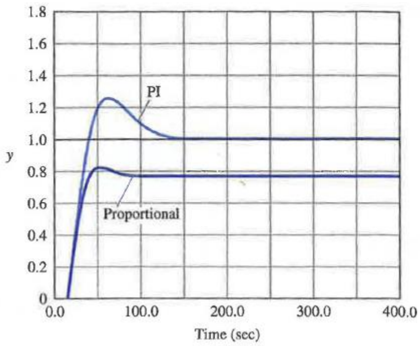
## PID Intuition

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int e(s)\,ds + T_d\frac{de(t)}{dt}\right]$$

- P:
- I:
- D:
  - The purpose of the derivative action is to improve the closed loop stability.
  - The instability "mechanism" "controlled" here is that because of the process dynamics it will take some time before a change in the control variable is noticeable in the process output.
  - The action of a controller with proportional and derivative action may e interpreted as if the control is made proportional to the *predicted* process output, where the prediction is made by extrapolating the error by the tangent to the error curve.

## PID Intuition: P and PI

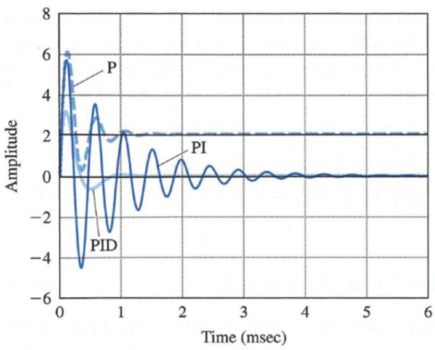


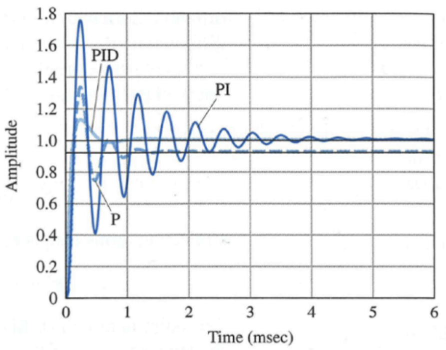






## PID Intuition: P and PI and PID

- Responses of P, PI, and PID control to




(a) step disturbance input (b) step reference input

# PID
(Tuning)



## Ziegler-Nichols Tuning – Reaction Rate

FPW § 5.8.5 [p.224]

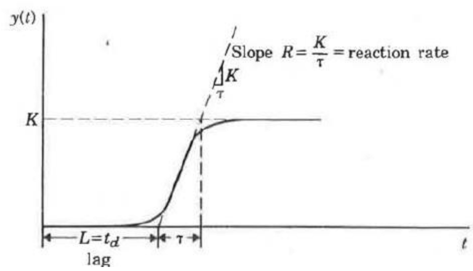
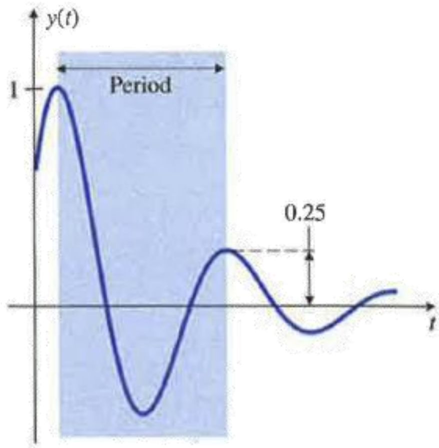



**Table 5.2** Ziegler-Nichols tuning parameters using transient response.

| | $K_p$ | $T_I$ | $T_D$ |
|---|---|---|---|
| $P$ | $1/RL$ | | |
| $PI$ | $0.9/RL$ | $3L$ | |
| $PID$ | $1.2/RL$ | $2L$ | $0.5L$ |

## Quarter decay ratio






## Ziegler-Nichols Tuning – Stability Limit Method

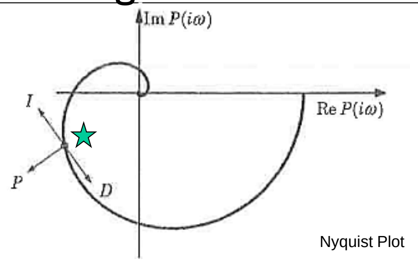
FPW § 5.8.5 [p.226]

- Increase $K_P$ until the system has continuous oscillations
  - ≡ $K_U$ : Oscillation Gain for "Ultimate stability"
  - ≡ $P_U$ : Oscillation Period for "Ultimate stability"

**Table 5.3** Ziegler-Nichols tuning parameters using stability limit.

| | $K_p$ | $T_I$ | $T_D$ |
|---|---|---|---|
| $P$ | $0.5K_u$ | | |
| $PI$ | $0.45K_u$ | $P_u/1.2$ | |
| $PID$ | $0.6K_u$ | $P_u/2$ | $P_u/8$ |

## Ziegler-Nichols Tuning / Intuition



Nyquist Plot

$$C(i\omega_u) = K\left(1 + i\left(\omega_u T_d - \frac{1}{\omega_u T_i}\right)\right) \approx 0.6K_u(1 + 0.467i)$$

- For a Given Point (★), the effect of increasing P,I and D in the "s-plane" are shown by the arrows above Nyquist plot

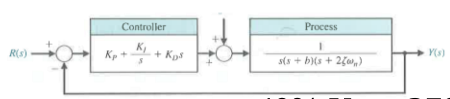


## PID Example

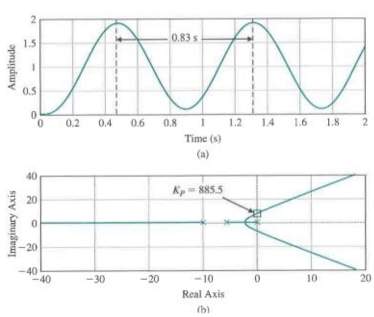
- A 3rd order plant: b=10, $\zeta$=0.707, $\omega_n$=4

$$G(s) = \frac{1}{s(s+b)(s+2\zeta\omega_n)}$$

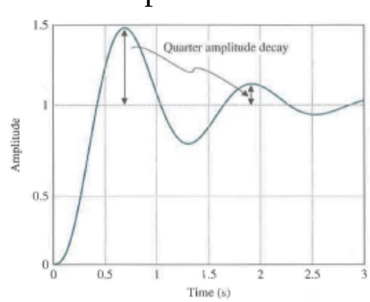
- PID:




- Kp=855:




- 40% Kp = 370



## Next Time…

- **Digital Feedback Control**

- Review:
  – Chapter 2 of FPW

- More Pondering??