http://elec3004.com

Introduction to Feedback Control

ELEC 3004: **Systems**: Signals & Controls

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Lecture 16

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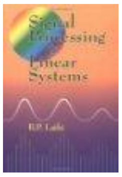
http://robotics.itee.uq.edu.au/~elec3004/

May 1, 2019

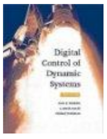
Lecture Schedule:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Systems as Maps & Signals as Vectors
	8-Mar	Systems: Linear Differential Systems
3	13-Mar	Sampling Theory & Data Acquisition
	15-Mar	Aliasing & Antialiasing
4	20-Mar	Discrete Time Analysis & Z-Transform
	22-Mar	Second Order LTID (& Convolution Review)
5	27-Mar	Frequency Response
	29-Mar	Filter Analysis
6	3-Apr	Digital Filters (IIR) & Filter Analysis
	5-Apr	PS 1: Q & A
7	10-Apr	Digital Windows
	12-Apr	Digital Filter (FIR)
8	17-Apr	Active Filters & Estimation
	19-Apr	
	24-Apr	**Holiday**
	26-Apr	
9	**1-May**	**Introduction to Feedback Control**
	3-May	Servoregulation & PID Control
10	8-May	Guest Lecture: FFT
	10-May	State-Space Control
11	15-May	Digital Control Design
	17-May	Stability
12	22-May	State Space Control System Design
	24-May	Shaping the Dynamic Response
13	29-May	System Identification & Information Theory
	31-May	Summary and Course Review

Follow Along Reading:



B. P. Lathi
Signal processing and linear systems
1998
TK5102.9.L38 1998



G. Franklin,
J. Powell,
M. Workman
Digital Control of Dynamic Systems
1990

TJ216.F72 1990
[Available as UQ Ebook]

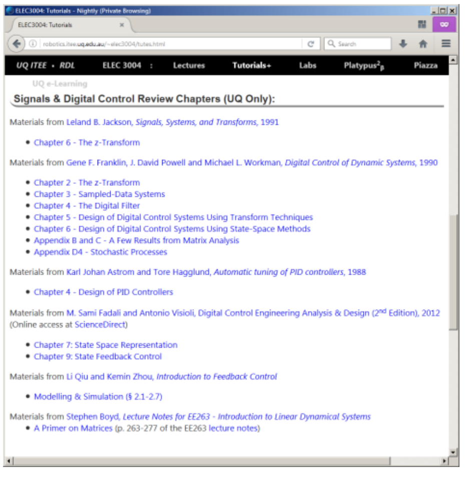
Today

- Chapter 6 (Continuous-Time System Analysis Using the Laplace Transform)
  - § 6.3 Solution of Differential and Integro-Differential Equations
  - § 6.5 Block Diagrams
  - ➔ § 6.4 KVL | KCL made easy!

- FPW
  - Chapter 3: Sampled-Data Systems
  - Chapter 2: Linear, Discrete, Dynamic-Systems Analysis

Next Time

Announcements: Controls Resources


ELEC3004: Tutorials - Nightly (Private Browsing)
ELEC3004: Tutorials
robotics.itee.uq.edu.au/~elec3004/tutes.html
UQ ITEE • RDL ELEC 3004 : Lectures Tutorials+ Labs Platypus$^2_\beta$ Piazza
UQ e-Learning

Signals & Digital Control Review Chapters (UQ Only):

Materials from Leland B. Jackson, *Signals, Systems, and Transforms*, 1991

- Chapter 6 - The z-Transform

Materials from Gene F. Franklin, J. David Powell and Michael L. Workman, *Digital Control of Dynamic Systems*, 1990

- Chapter 2 - The z-Transform
- Chapter 3 - Sampled-Data Systems
- Chapter 4 - The Digital Filter
- Chapter 5 - Design of Digital Control Systems Using Transform Techniques
- Chapter 6 - Design of Digital Control Systems Using State-Space Methods
- Appendix B and C - A Few Results from Matrix Analysis
- Appendix D4 - Stochastic Processes

Materials from Karl Johan Astrom and Tore Hagglund, *Automatic tuning of PID controllers*, 1988

- Chapter 4 - Design of PID Controllers

Materials from M. Sami Fadali and Antonio Visioli, Digital Control Engineering Analysis & Design (2nd Edition), 2012 (Online access at ScienceDirect)

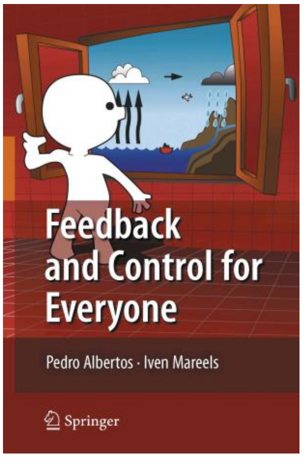
- Chapter 7: State Space Representation
- Chapter 9: State Feedback Control

Materials from Li Qiu and Kemin Zhou, *Introduction to Feedback Control*

- Modelling & Simulation (§ 2.1-2.7)

Materials from Stephen Boyd, *Lecture Notes for EE263 - Introduction to Linear Dynamical Systems*

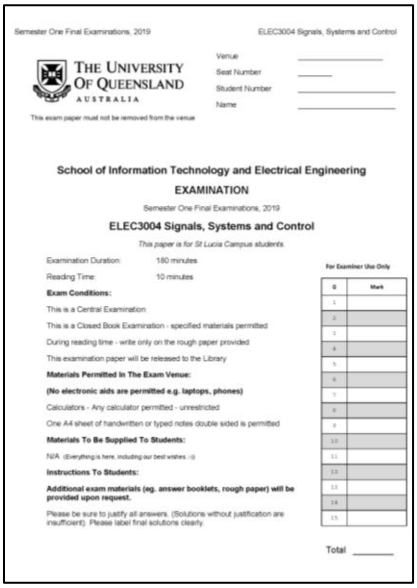
- A Primer on Matrices (p. 263-277 of the EE263 lecture notes)



- Free e-Book [via UQ]: https://library.uq.edu.au/record=b2508512~S7

Announcements: Final Exam Information

- Section 1:
  - Digital Linear Dynamical Systems
  - 4 Questions
  - 60 Points (33 %)
- Section 2:
  - Digital Processing / Filtering of Signals
  - 6 Questions
  - 60 Points (33 %)
- Section 3:
  - Digital & State-Space Control
  - 5 Questions
  - 60 Points (33 %)



Semester One Final Examinations, 2019 ELEC3004 Signals, Systems and Control

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

This exam paper must not be removed from the venue

Venue ________
Seat Number ____
Student Number ________
Name ________

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2019

ELEC3004 Signals, Systems and Control

This paper is for St Lucia Campus students

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

N/A (Everything is here, including our best wishes :-))

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

Please be sure to justify all answers. (Solutions without justification are insufficient). Please label final solutions clearly.

For Examiner Use Only

Q	Mark
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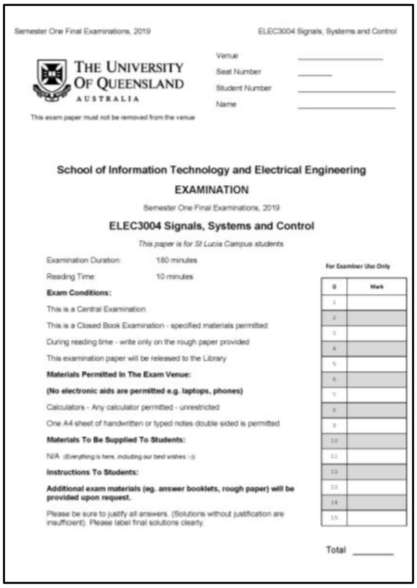
Total ______

Announcements: Final Exam Information

- **Tentative Date: June 10**
- **Tentative Time: 2:30 pm**
- Room: TBA
- Questions:
  - Some New
  - A couple based on previous exams (2012-2017)
- *"If there's no monster in the closet, then why am I scared?"*
- *Sometimes the backlash comes from people who are genuinely, if wrongly, fearful of change.*
  More often it's manufactured by the powerful and the privileged ... maintain the status-quo.
  -- Obama



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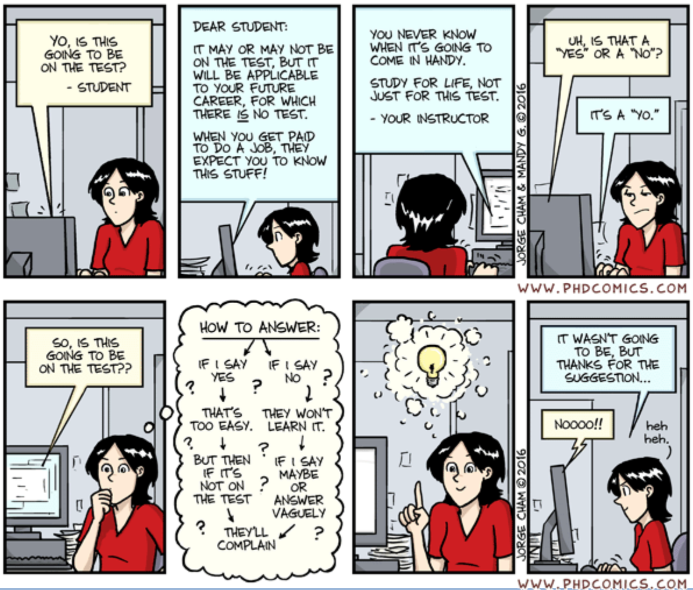
Total ______

Announcements: “Thanks for the Suggestion” ☺



Lab 3 – LeviLab I:

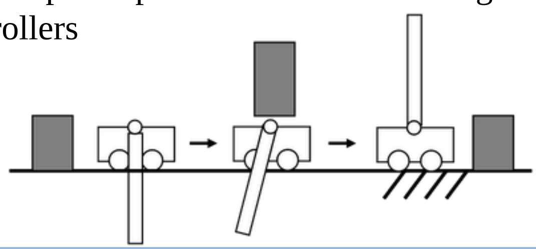
- *“not a long time ago in a lab down, down the way”*

Introducing Feedback

Control

Once upon a time…

- Electromechanical systems were controlled by electromechanical compensators
  - Mechanical flywheel governors, capacitors, inductors, resistors, relays, valves, solenoids (fun!)
  - But also complex and sensitive!
- Humans developed sophisticated tools for designing reliable analog controllers



Control

Once upon a time…

- Electromechanical systems were controlled by electromechanical compensators
  - Mechanical flywheel governors, capacitors, inductors, resistors, relays, valves, solenoids (fun!)
  - But also complex and sensitive!

➔Idea: Digital computers in real-time control

- Transform approach (classical control)
  - Root-locus methods (pretty much the same as METR 3200)
  - Bode's frequency response methods (these change compared to METR 3200)
- State-space approach (modern control)

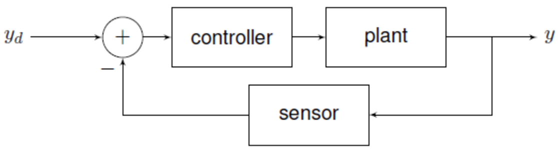
→ Model Making: Control of frequency response as well as Least Squares Parameter Estimation

Many advantages

- Practical improvement over analog control:
  - **Flexible**; reprogrammable to implement different control laws for different systems
  - **Adaptable**; control algorithms can be changed on-line, during operation
  - **Insensitive** to environmental conditions; (heat, EMI, vibration, etc.)
  - **Compact**; handful of components on a PCB
  - **Cheap**

Feedback Control

(Simple) control systems have three parts:

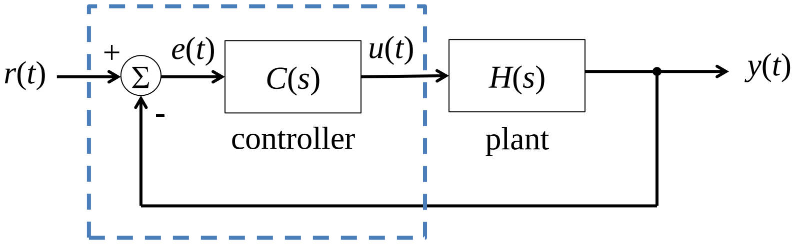


- The plant is the system to be controlled (e.g. the robot).
- The sensor measures the output of the plant.
- The controller sends an input command to the plant based on the difference from the actual output and the desired output.

Digital Control

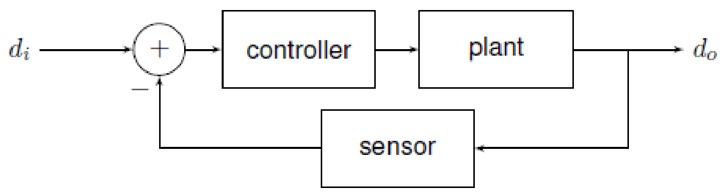
Archetypical control system

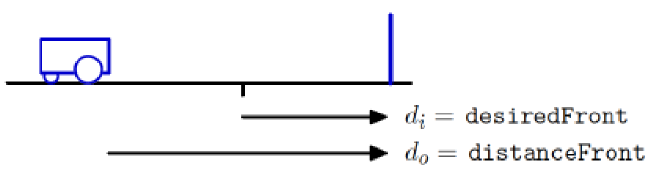
- Consider a continuous control system:



- The functions of the controller can be entirely represented by a discretised computer system

Simple Controller Goes Digital



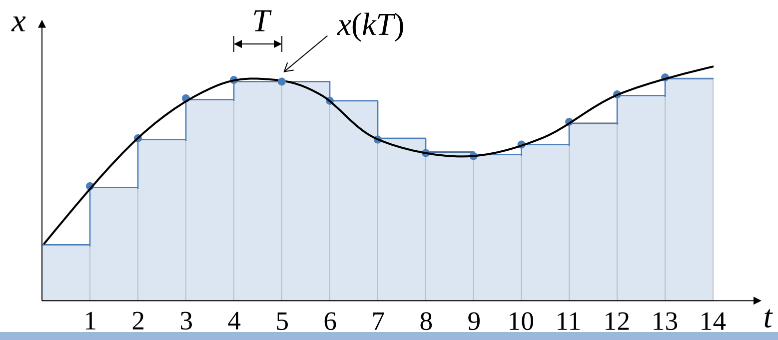


$$\begin{aligned} \text{plant:} \quad & y[n] = y[n-1] - Tu[n-1] \\ \text{sensor:} \quad & y[n] = u[n-1] \\ \text{controller:} \quad & y[n] = Ku[n] \end{aligned}$$

Complex system behaviors, depending on K

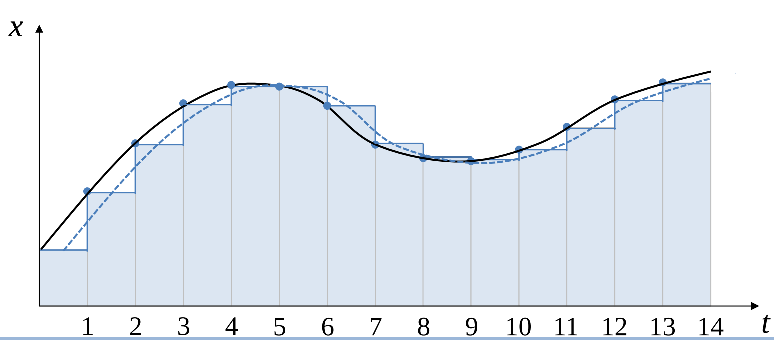
Return to the discrete domain

- Recall that continuous signals can be represented by a series of samples with period T



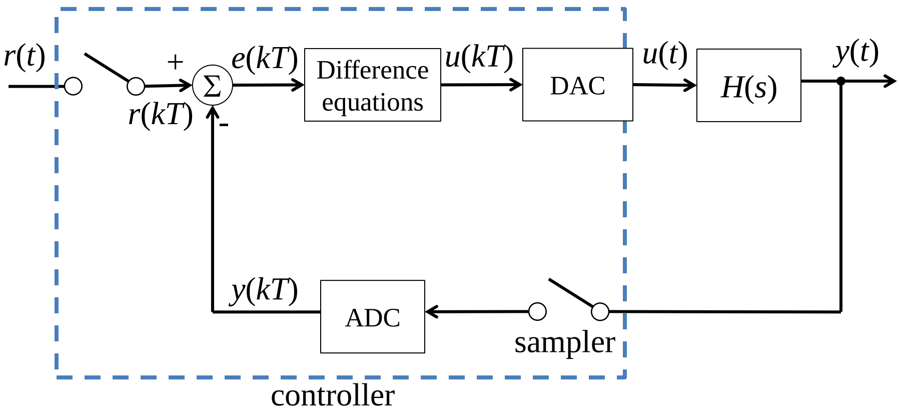
Zero Order Hold

- An output value of a synthesised signal is held constant until the next value is ready
  - This introduces an effective delay of $T/2$



Digitisation

- Continuous signals sampled with period T
- kth control value computed at $t_k = kT$



Digitisation

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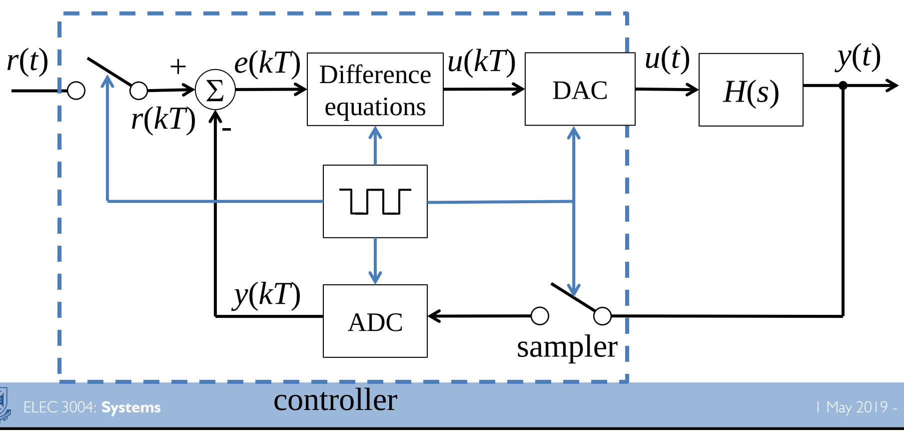


Difference equations

- How to represent differential equations in a computer? Difference equations!
- The output of a difference equation system is a function of current and previous values of the input and output:

$$y(t_k) = D(x(t_k), x(t_{k-1}), \dots, x(t_{k-n}), y(t_{k-1}), \dots, y(t_{k-n}))$$

  – We can think of x and y as parameterised in k

  Useful shorthand: $x(t_{k+i}) \equiv x(k+i)$

➔ Discrete-time transfer function

take $\mathcal{Z}$-transform of system equations

$$x(t+1) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) + Du(t)$$

yields

$$zX(z) - zx(0) = AX(z) + BU(z), \qquad Y(z) = CX(z) + DU(z)$$

solve for $X(z)$ to get

$$X(z) = (zI - A)^{-1}zx(0) + (zI - A)^{-1}BU(z)$$

(note extra z in first term!)

hence

$$Y(z) = H(z)U(z) + C(zI - A)^{-1}zx(0)$$

where $H(z) = C(zI - A)^{-1}B + D$ is the *discrete-time transfer function*

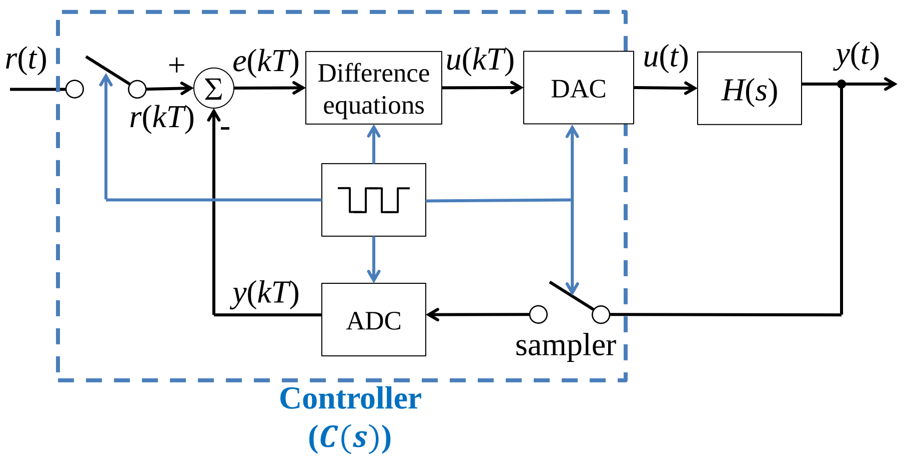
note power series expansion of resolvent:

$$(zI - A)^{-1} = z^{-1}I + z^{-2}A + z^{-3}A^2 + \cdots$$

Source: Boyd, Lecture Notes for EE263, 13-39

Modelling

(Digital) **Feedback Control**



- Continuous signals sampled with period T
- kth control value computed at $t_k = kT$

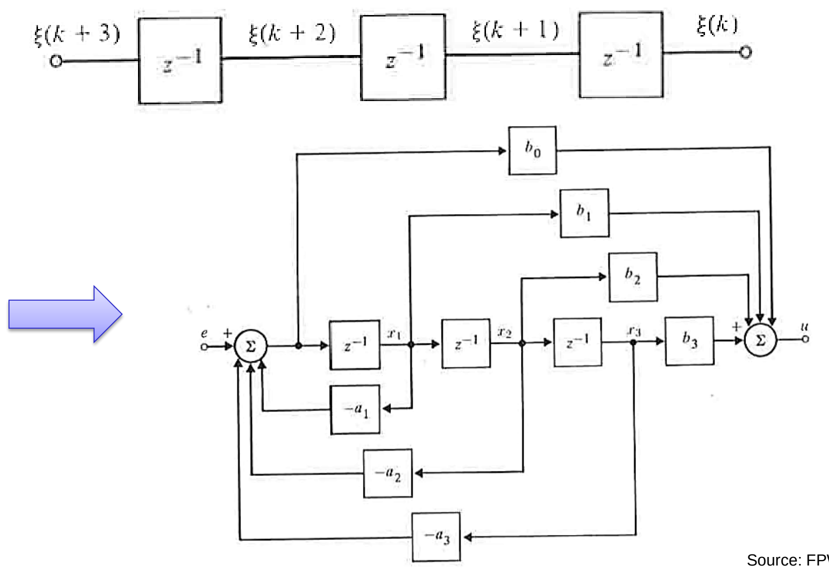
C(s): PID = Control for the PID-dly minded

- Proportional-Integral-Derivative control is the control engineer's hammer*
  - For P,PI,PD, etc. just remove one or more terms

$$C(s) = k\left(\underbrace{1}_{\text{Proportional}} + \underbrace{\frac{1}{\tau_i s}}_{\text{Integral}} + \underbrace{\tau_d s}_{\text{Derivative}}\right)$$

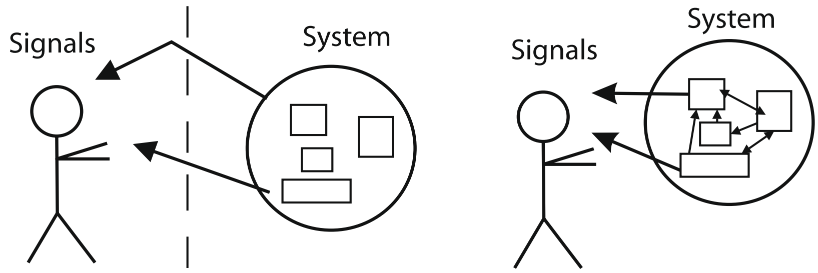
*Everything is a nail. That's why it's called "Bang-Bang" Control ☺

Feedback Control: Tuning Nightmare!



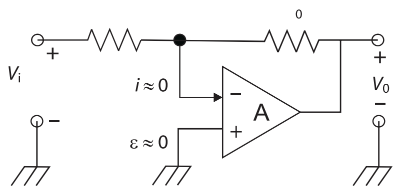
Source: FPW, Fig. 2.8

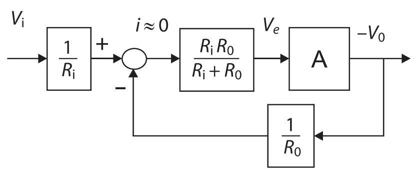
Signals and Systems: Modelling Tools!



- Signals: Often came from a system
- Now we want feedback tools so as to understand the structure of the systems and how they interact so as to get desired signals

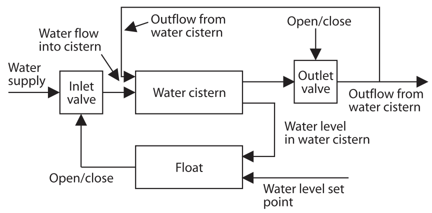
Feedback Control + Models: Everywhere

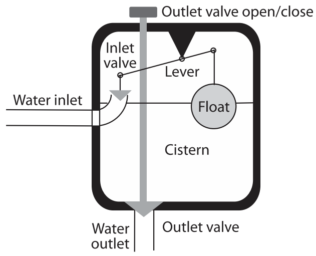


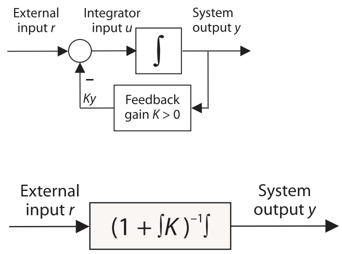


- $\frac{V_0}{V_1} = -\frac{R_0}{R_1} \frac{1}{1+\frac{1}{A}\left(1+\frac{R_0}{R_1}\right)}$

Feedback Control + Models: Everywhere





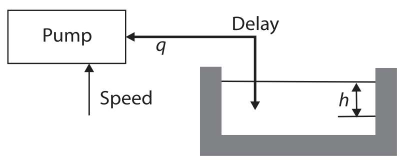


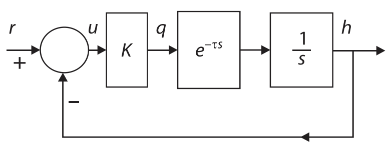
- Integrator:

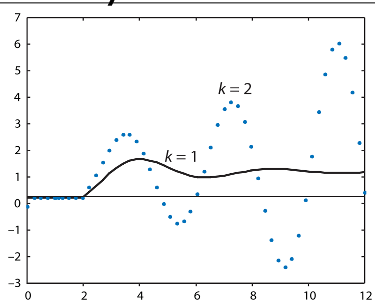
$$y(t) = y_0 + \int_0^t u(\tau)d\tau$$

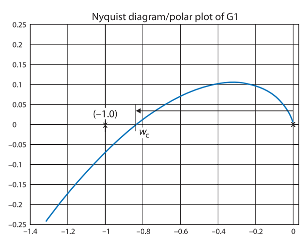
$$y(t) = y_0 + \frac{A\sin(\omega t)}{\omega}$$

Feedback Control + Models: Everywhere





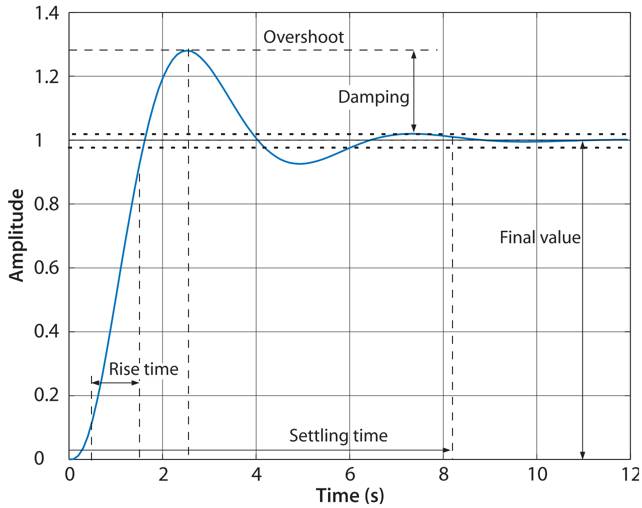




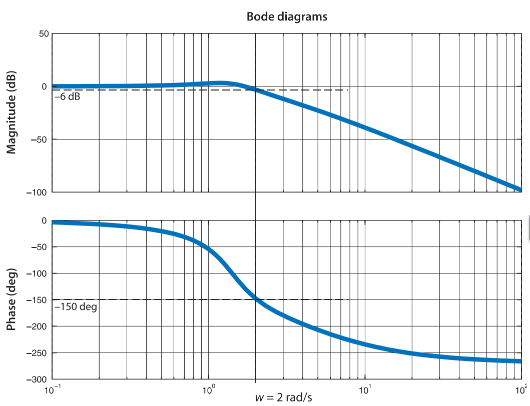
BREAK

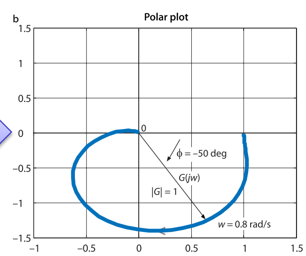
Feedback as a Filter

Time Response



Frequency Domain Analysis

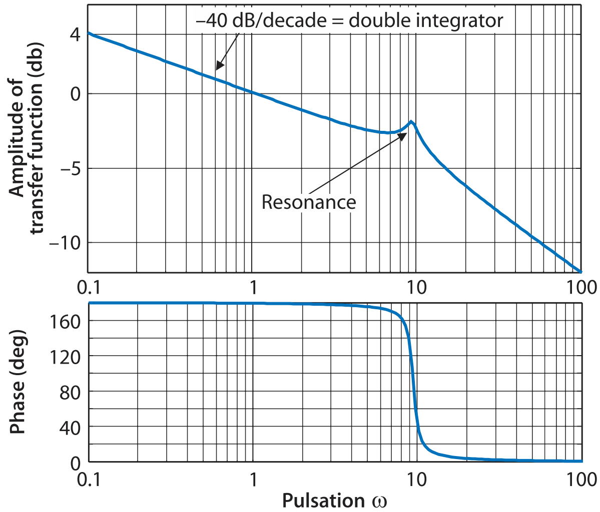




- Bode (Magnitude + Phase Plots)
- Nyquist Plot (Polar)

In This Way Feedback May Be Seen as a Filter

- Ex: Lightly Damped Robot Arm



How to Design?
Back to Analog !

Two cases for control design

The system…

- Isn't fast enough
- Isn't damped enough
- Overshoots too much
- Requires too much control action

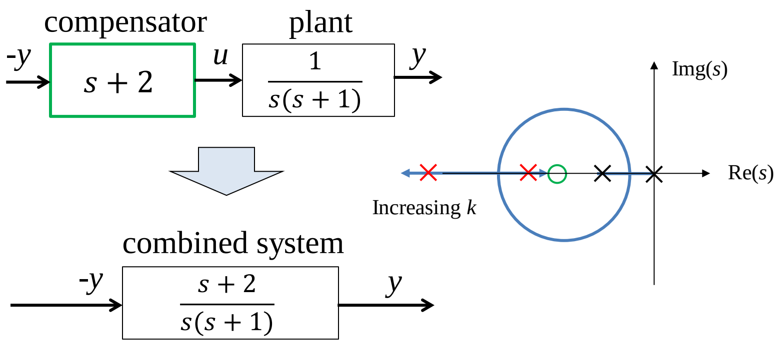
("Performance")

- Attempts to spontaneously disassemble itself

("Stability")

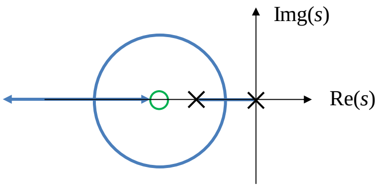
Dynamic compensation

- We can do more than just apply gain!
  - We can add dynamics into the controller that alter the open-loop response



But what dynamics to add?

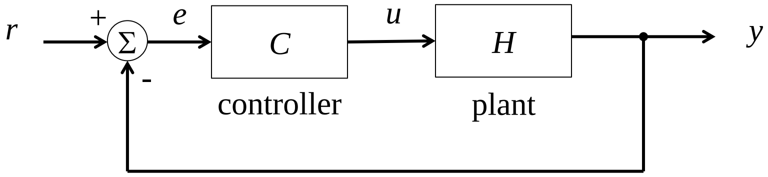
- Recognise the following:
  - A root locus starts at poles, terminates at zeros
  - "Holes eat poles"
  - Closely matched pole and zero dynamics cancel
  - The locus is on the real axis to the left of an odd number of poles (treat zeros as 'negative' poles)



The Root Locus (Quickly)

- The transfer function for a closed-loop system can be easily calculated:

$$y = CH(r - y)$$
$$y + CHy = CHr$$
$$\therefore \frac{y}{r} = \frac{CH}{1 + CH}$$



The Root Locus (Quickly)

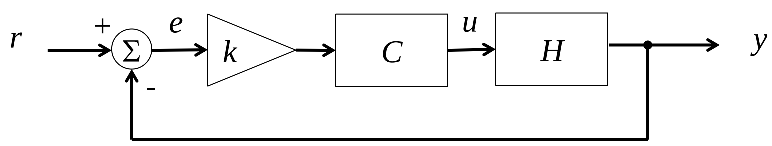
- We often care about the effect of increasing gain of a control compensator design:

$$\frac{y}{r} = \frac{kCH}{1 + kCH}$$

Multiplying by denominator:

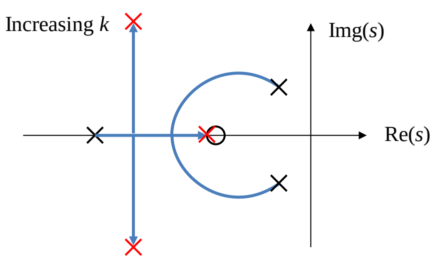
characteristic polynomial

$$\frac{y}{r} = \frac{kC_n H_n}{\boxed{C_d H_d + kCnHn}}$$



The Root Locus (Quickly)

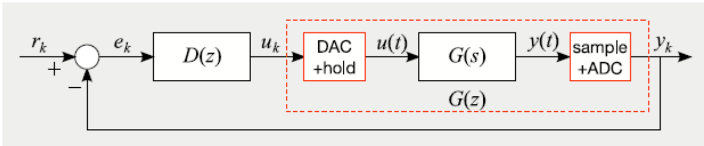
- Pole positions change with increasing gain
  - The trajectory of poles on the pole-zero plot with changing *k* is called the “root locus”
  - This is sometimes quite complex



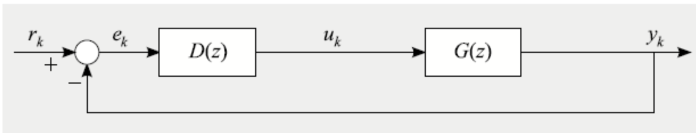
(In practice you’d plot these with computers)

Designing in the Purely Discrete…

Analyse/design a discrete controller $D(z)$:



by considering the purely discrete time system:



Closed loop system tranfer function: $\frac{Y(z)}{R(z)} = \frac{G(z)D(z)}{1+G(z)D(z)}$

How do the closed loop poles relate to $\rightarrow$ stability?

$\rightarrow$ performance?

Now in discrete

- Naturally, there are discrete analogs for each of these controller types:

Lead/lag: $\frac{1-\alpha z^{-1}}{1-\beta z^{-1}}$

PID: $k\left(1+\frac{1}{\tau_i(1-z^{-1})}+\tau_d(1-z^{-1})\right)$

But, where do we get the control design parameters from?
The s-domain?

Sampling a continuous-time system

suppose $\dot{x} = Ax$

sample x at times $t_1 \leq t_2 \leq \cdots$: define $z(k) = x(t_k)$

then $z(k+1) = e^{(t_{k+1}-t_k)A} z(k)$

for uniform sampling $t_{k+1} - t_k = h$, so

$$z(k+1) = e^{hA} z(k),$$

a discrete-time LDS (called *discretized version* of continuous-time system)

Source: Boyd, Lecture Notes for EE263, 10-22

Piecewise constant system

consider *time-varying* LDS $\dot{x} = A(t)x$, with

$$A(t) = \begin{cases} A_0 & 0 \leq t < t_1 \\ A_1 & t_1 \leq t < t_2 \\ \vdots \end{cases}$$

where $0 < t_1 < t_2 < \cdots$ (sometimes called jump linear system)

for $t \in [t_i, t_{i+1}]$ we have

$$x(t) = e^{(t-t_i)A_i} \cdots e^{(t_3-t_2)A_2} e^{(t_2-t_1)A_1} e^{t_1 A_0} x(0)$$

(matrix on righthand side is called state transition matrix for system, and denoted $\Phi(t)$)

Source: Boyd, Lecture Notes for EE263, 10-23

Qualitative behaviour of ***x(t)***

suppose $\dot{x} = Ax$, $x(t) \in \mathbf{R}^n$

then $x(t) = e^{tA}x(0)$; $X(s) = (sI - A)^{-1}x(0)$

ith component $X_i(s)$ has form

$$X_i(s) = \frac{a_i(s)}{\mathcal{X}(s)}$$

where a_i is a polynomial of degree $< n$

thus the poles of X_i are all eigenvalues of A (but not necessarily the other way around)

Source: Boyd, Lecture Notes for EE263, 10-24

Qualitative behaviour of ***x(t)*** [2]

first assume eigenvalues λ_i are distinct, so $X_i(s)$ cannot have repeated poles

then $x_i(t)$ has form

$$x_i(t) = \sum_{j=1}^{n} \beta_{ij} e^{\lambda_j t}$$

where β_{ij} depend on $x(0)$ (linearly)

eigenvalues determine (possible) qualitative behavior of x:

- eigenvalues give exponents that can occur in exponentials
- real eigenvalue λ corresponds to an exponentially decaying or growing term $e^{\lambda t}$ in solution
- complex eigenvalue $\lambda = \sigma + j\omega$ corresponds to decaying or growing sinusoidal term $e^{\sigma t}\cos(\omega t + \phi)$ in solution

Source: Boyd, Lecture Notes for EE263, 10-25

Qualitative behaviour of *x(t)* [3]

first assume eigenvalues λ_i are distinct, so $X_i(s)$ cannot have repeated poles

then $x_i(t)$ has form

$$x_i(t) = \sum_{j=1}^{n} \beta_{ij} e^{\lambda_j t}$$

where β_{ij} depend on $x(0)$ (linearly)

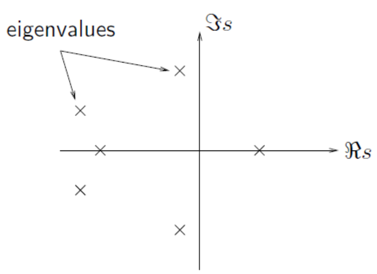
eigenvalues determine (possible) qualitative behavior of x:

- eigenvalues give exponents that can occur in exponentials
- real eigenvalue λ corresponds to an exponentially decaying or growing term $e^{\lambda t}$ in solution
- complex eigenvalue $\lambda = \sigma + j\omega$ corresponds to decaying or growing sinusoidal term $e^{\sigma t}\cos(\omega t + \phi)$ in solution

Source: Boyd, Lecture Notes for EE263, 10-26

Qualitative behaviour of *x(t)* [4]

- $\Re\lambda_j$ gives exponential growth rate (if > 0), or exponential decay rate (if < 0) of term
- $\Im\lambda_j$ gives frequency of oscillatory term (if $\neq 0$)



Source: Boyd, Lecture Notes for EE263, 10-27

Qualitative behaviour of ***x(t)*** [5]

now suppose A has repeated eigenvalues, so X_i can have repeated poles

express eigenvalues as $\lambda_1, \ldots, \lambda_r$ (distinct) with multiplicities $n_1, \ldots, n_r$, respectively ($n_1 + \cdots + n_r = n$)

then $x_i(t)$ has form

$$x_i(t) = \sum_{j=1}^{r} p_{ij}(t) e^{\lambda_j t}$$

where $p_{ij}(t)$ is a polynomial of degree $< n_j$ (that depends linearly on $x(0)$)

Source: Boyd, Lecture Notes for EE263, 10-28

Emulation vs Discrete Design

- Remember: polynomial algebra is the same, whatever symbol you are manipulating:

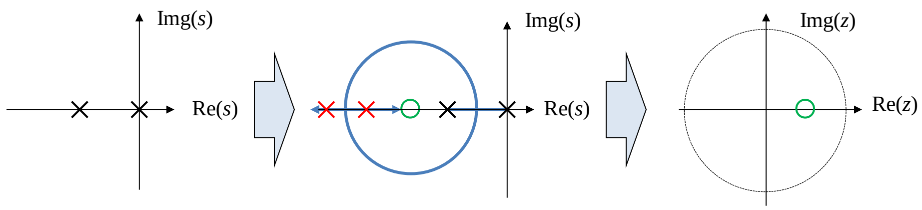
  eg. $s^2 + 2s + 1 = (s + 1)^2$

  $z^2 + 2z + 1 = (z + 1)^2$

  Root loci behave the same on both planes!
- Therefore, we have two choices:
  - Design in the s-domain and digitise (emulation)
  - Design only in the z-domain (discrete design)

Emulation design process

1. Derive the dynamic system model ODE
2. Convert it to a continuous transfer function
3. Design a continuous controller
4. Convert the controller to the z-domain
5. Implement difference equations in software



Emulation design process

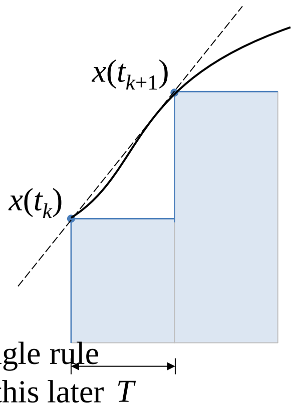
- Handy rules of thumb:
  - Use a sampling period of 20 to 30 times faster than the closed-loop system bandwidth
  - Remember that the sampling ZOH induces an effective $T/2$ delay
  - There are several approximation techniques:
    - Euler's method
    - Tustin's method
    - Matched pole-zero
    - Modified matched pole-zero

Euler's method*

- Dynamic systems can be approximated† by recognising that:

$$\dot{x} \cong \frac{x(k+1) - x(k)}{T}$$

- As $T \rightarrow 0$, approximation error approaches 0



*Also known as the forward rectangle rule
†Just an approximation – more on this later

Back to the future

A quick note on causality:

- Calculating the "(k+1)th" value of a signal using

$$y(k+1) = \underbrace{x(k+1)}_{\text{future value}} + \underbrace{Ax(k) - By(k)}_{\text{current values}}$$

relies on also knowing the next (future) value of $x(t)$.

(this requires very advanced technology!)

- Real systems always run with a delay:

$$y(k) = x(k) + Ax(k-1) - By(k-1)$$

Back to the example!

```
T = 0.02;  //period of 50 Hz, a number pulled from thin air
A = 2*T-1; //pre-calculated control constants
B = T-1;

...

while(1)
   {
      if(interrupt_flag)           //this triggers every 20 ms
      {
         x0 = x;                   //save previous values
         y0 = y;
         x = update_input();       //get latest x value
         y = x + A*x0 - B*y0;      //do the difference equations
         update_output(y);         //write out current value
      }
   }
```

(The actual calculation)

Tustin's method

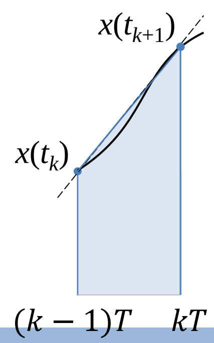
- Tustin uses a trapezoidal integration approximation (compare Euler's rectangles)
- Integral between two samples treated as a straight line:

$$u(kT) = \frac{T}{2}\left[x(k-1) + x(k)\right]$$

Taking the derivative, then z-transform yields:

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

which can be substituted into continuous models



Matched pole-zero

- If $z = e^{sT}$, why can't we just make a direct substitution and go home?

$$\frac{Y(s)}{X(s)} = \frac{s+a}{s+b} \Rightarrow \frac{Y(z)}{X(z)} = \frac{z-e^{-aT}}{z-e^{-bT}}$$

- Kind of!
  - Still an approximation
  - Produces quasi-causal system (hard to compute)
  - Fortunately, also very easy to calculate.

Matched pole-zero

The process:

1. Replace continuous poles and zeros with discrete equivalents:

$$(s+a) \Rightarrow (z-e^{-aT})$$

2. Scale the discrete system DC gain to match the continuous system DC gain

3. If the order of the denominator is higher than the enumerator, multiply the numerator by $(z+1)$ until they are of equal order*

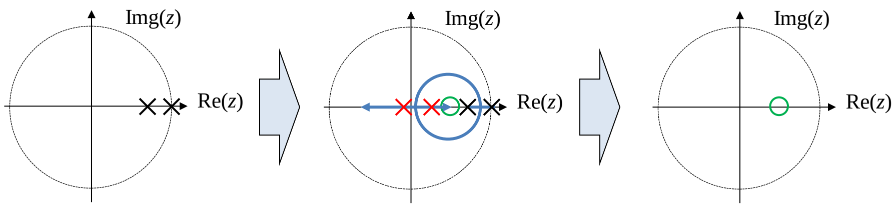
* This introduces an averaging effect like Tustin's method

Modified matched pole-zero

- We're prefer it if we didn't require instant calculations to produce timely outputs
- Modify step 2 to leave the dynamic order of the numerator one less than the denominator
  - Can work with slower sample times, and at higher frequencies

Discrete design process

1. Derive the dynamic system model ODE
2. Convert it to a discrete transfer function
3. Design a digital compensator
4. Implement difference equations in software
5. Platypus Is Divine!



Discrete design process

- Handy rules of thumb:
  - Sample rates can be as low as twice the system bandwidth
    - but 5 to 10× for "stability"
    - 20 to 30 × for better performance
  - A zero at $z = -1$ makes the discrete root locus pole behaviour more closely match the s-plane
  - Beware "dirty derivatives"
    - dy/dt terms derived from sequential digital values are called 'dirty derivatives' – these are especially sensitive to noise!
    - Employ actual velocity measurements when possible

Lead/Lag

Some standard approaches

- Control engineers have developed time-tested strategies for building compensators
- Three classical control structures:
  - Lead
  - Lag
  - Proportional-Integral-Derivative (PID)
    (and its variations: P, I, PI, PD)

How do they work?

Lead/lag compensation

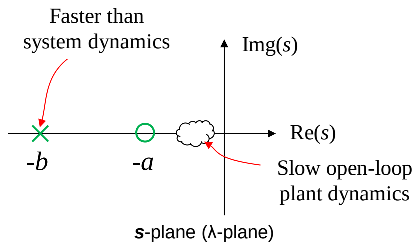
- Serve different purposes, but have a similar dynamic structure:

$$D(s) = \frac{s+a}{s+b}$$

Note:

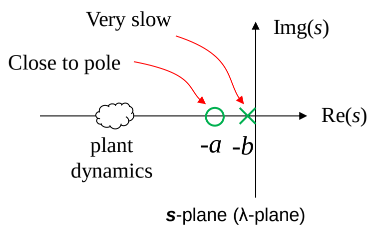
Lead-lag compensators come from the days when control engineers cared about constructing controllers from networks of op amps using frequency-phase methods. These days pretty much everybody uses PID, but you should at least know what the heck they are in case someone asks.

Lead compensation: $a < b$



- Acts to decrease rise-time and overshoot
  - Zero draws poles to the left; adds phase-lead
  - Pole decreases noise
- Set a near desired ω_n; set b at ~3 to 20x a

Lag compensation: $a > b$



- Improves steady-state tracking
  - Near pole-zero cancellation; adds phase-lag
  - Doesn’t break dynamic response (too much)
- Set b near origin; set a at ~3 to 10x b

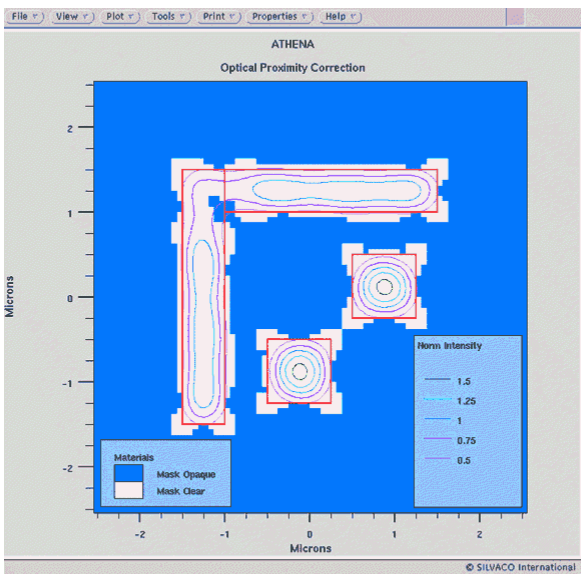
Next Time…

- **Digital Feedback Control**

- Review:
  - Chapter 2 of FPW

- More Pondering??

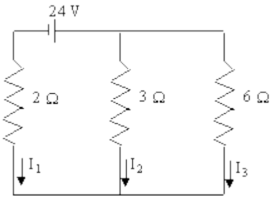
Advanced Application: Optical Proximity Correction



Fun Application: Linear Algebra & KVL!

We can write this as:

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 3 & 0 \\ 0 & -3 & 6 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 24 \\ 0 \end{pmatrix}$$



So we have:

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 3 & 0 \\ 0 & -3 & 6 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 24 \\ 0 \end{pmatrix}$$

Using a computer algebra system to perform the inverse and multiply by the constant matrix, we get:

$$I_1 = -6 \text{ A}$$

$$I_2 = 4 \text{ A}$$

$$I_3 = 2 \text{ A}$$

We observe that I_1 is negative, as expected from the circuit diagram.

Source: http://www.intmath.com/matrices-determinants/6-matrices-linear-equations.php

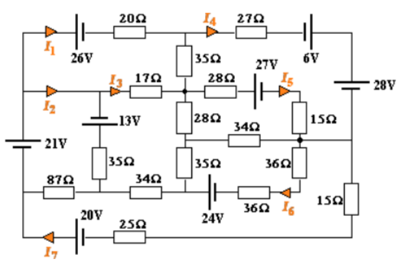
Fun Application: Linear Algebra & KCL!

We solve this using a computer as follows. We just write the coefficient matrix on the left, find the inverse (raise the matrix to the power -1) and multiply the result by the constant matrix.

You can use Matlab, Mathcad or similar math software to do this. Wolfram|Alpha is a free alternative.

$$X = \begin{bmatrix} 72 & 0 & -17 & -35 & 0 & 0 & 0 \\ 0 & 122 & -35 & 0 & 0 & 0 & -87 \\ 0 & -87 & -34 & 0 & 0 & -72 & 233 \\ -17 & -35 & 149 & 0 & -28 & -35 & -34 \\ 0 & 0 & -28 & -43 & 105 & -34 & 0 \\ 0 & 0 & -35 & 0 & -34 & 141 & -72 \\ -35 & 0 & 0 & 105 & -43 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -26 \\ 34 \\ -4 \\ -13 \\ -27 \\ 24 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -0.46801 \\ 0.42932 \\ 5.193 \times 10^{-3} \\ -0.22243 \\ -0.27848 \\ 0.21115 \\ 0.20914 \end{bmatrix}$$



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