http://elec3004.com

PS 1: Q&A

ELEC 3004: **Systems**: Signals & Controls
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Lecture 12

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Lecture Schedule:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Systems as Maps & Signals as Vectors
	8-Mar	Systems: Linear Differential Systems
3	13-Mar	Sampling Theory & Data Acquisition
	15-Mar	Aliasing & Antialiasing
4	20-Mar	Discrete Time Analysis & Z-Transform
	22-Mar	Second Order LTID (& Convolution Review)
5	27-Mar	Frequency Response
	29-Mar	Filter Analysis
6	3-Apr	Digital Filters (IIR) & Filter Analysis
	5-Apr	**PS 1: Q & A**
7	10-Apr	Digital Filter (FIR) & Digital Windows
	12-Apr	FFT
8	17-Apr	Active Filters & Estimation & Holiday
	19-Apr	Holiday
	24-Apr	
	26-Apr	
9	1-May	Introduction to Feedback Control
	3-May	Servoregulation/PID
10	8-May	PID & State-Space
	10-May	State-Space Control
11	15-May	Digital Control Design
	17-May	Stability
12	22-May	State Space Control System Design
	24-May	Shaping the Dynamic Response
13	29-May	System Identification & Information Theory
	31-May	Summary and Course Review

Question 1

Q1. Linearity: Starting Straight Away [10 points]
This question explores some of these interesting properties of Linear systems, notably superposition. This gives that if several inputs are acting on a linear system, then the total response of this system is the sum of the outputs from each input on its own.

Please **determine** and **justify** if these equations are linear

- $w(t) = 300t + 4$
- $y(t) = 300\frac{dx}{dt} + 4x(t)$
- $z(t) = 300 \cdot t \cdot \frac{dx}{dt} + 4t^2x(t)$

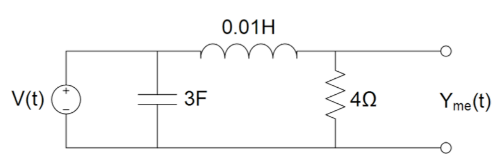
Please **determine** and then **generally prove** (or **disprove**) if these statements about linearity are true.

- Linearity and the Converse. Consider f(x)=[**A**]x.
  For this case, is matrix multiplication a linear operation?
  The Converse: Can **any** linear function f **always** be written as f(x)=[**A**]x?

- **Uniqueness**.
  For any linear function *f* there is **only** one matrix [A] for which f(x)=[**A**]x for all x.

Question 2

Let us consider the interplay between linearity, circuits and signals. A Magic Elf proposes the following following circuit consisting of ideal elements:



- Is the voltage output for this circuit (noted $Y_{me}(t)$) a linear system?

- If so, show that the output of the Magic Elf's circuit, $Y_{me}(t)$, satisfies the conditions of linearity with respect to the input, V(t) and the initial conditions.

- If not, what element(s) could be removed to make it linear?

Question 3

Pink has a new song, Noise, a highlight of which is a loud Mezzo-soprano A_5 note (880 Hz). This was recorded live at the recent concert at 1E6 Dreams† Stadium via a microphone connected to a preamp that approximates a consumer line level signal.

Upon inspection the signal recorded was found to be (in Volts):

$$V_{microphone}(t) = 0.42\cos(1760\pi t) + 0.314\cos(100\pi t) + 1$$

It appears that joint between the 3.5 mm connector and the unbalanced wire was not properly shielded and thus introducing a 50 Hz whine. To add insult to injury, the recording was rushed to get ahead of a demolition for refurbishment, so by accident it was sampled at 1,044 Hz (instead of the expected 44.1 KHz).

- Please plot the voltage signal from the microphone ($V_{microphone}(t)$) for t=0 to 1 second.
- Please plot the sampled, digitized signal captured on a basic audio card with simple line level (i.e., no negative voltage rail). Again, for t=0 to 1 second.
- It is proposed that all this can be solved "easily" by changing the anti-aliasing (or band-limiting) filter to add a high-pass filter with a cut-off of 100 Hz between the pre-amp and the line-level input on the audio card. Briefly discuss if this will work?

Question 4

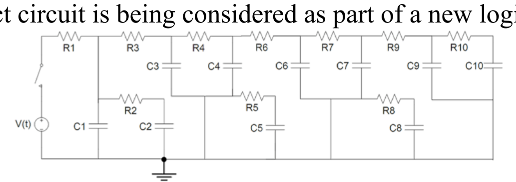
Let ***f(t)*** be a periodic continuous time signal with Period **P**. Then, let **f[k]** be the discrete time signal generated from ***f(t)*** with equally spaced samples of period **Q**; that is,

$$\boldsymbol{f}[\boldsymbol{k}] = \boldsymbol{f}(\boldsymbol{kQ})$$

- Show that the sequence **f[k]** will be periodic **if and only** if the ratio **P/Q** is itself rational.

Question 5

An interconnect circuit is being considered as part of a new logic architecture.



- Initially, assume unit circuit elements. That is, the capacitors $C_1,..,C_{10}$=1F, the resistors $R_1,..,R_{10}$=1Ω and voltage, V(t)=1 V. At t=0, the switch is closed.
- The voltages across the capacitors (C[i], here C_1 to C_{10}) are $\mathbf{x_i}$

Question 5 [continued]

- What are the steady-state values (i.e., the static gain matrix) of the voltages across the capacitors in this circuit. That is, what are the final values of $x_1,...,x_{10}$?
- Let's write this system as an LDS model. What are the [A] and [B] matrices? What are the eigenvalues and resolvent of [A]. And, importantly, what do the **eigenvalues**, **transfer matrix**, and **QR** [or Gram-Schmidt] **decomposition** indicate/signify about the system?
- Which of the ten voltages will reach within 99% of its steady-state values last? How long will it take for it (and thus the circuit) to reach within 99% of these final values?
- Please plot the step response of the system. That is, please plot the voltages (or states) $x_1,...,x_{10}$ as a function of time from 0 to the settling time found in (b)
- What is the effect of doubling the voltage on the overall settling time? Based on this, would it be possible to select a voltage such that the system will deliver 1 Volt across C10 in t=0.3004s (i.e., x_{10}(t=0.3004s) = 1 Volt).

Question 6

- Remember that a signal (vector) need not only be written in the standard basis (**S** consisting of basis vector $\mathbf{s}_1$,..., $\mathbf{s}_n$, where $\mathbf{s}_i$ are columns of an identity matrix (i.e., **S**=**I**))

- Consider x=[1,10,100,1000].

For this case, what are the standard basis vectors s1,..., s4?

Please determine the scalar coefficients c1,..., c4?

- Consider a small (4×4) wavelet basis given by:

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{w}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

- Using this basis, determine the basis matrix W and its inverse W^{-1}

- Now, please find the coefficients for the vectors:
  - x1=[1,10,100,1000]
  - x2=[1000,100,10,1]
  - x3=[3,5,3,2]
  - x4=[3,0,0,4]
  - x5=[3,1,4,1]
  - x6=[1,-1,0,1]

- For the case (b) above, it has been postulated that the coefficients, c, should always be given by $c = W^{-1}x$. Please prove or disprove this.

- For case (b), how does the coefficient c1 relate to the values x_1,...,x_4 ?