Worked example
Impulse Response

ELEC3004 / ELEC7312

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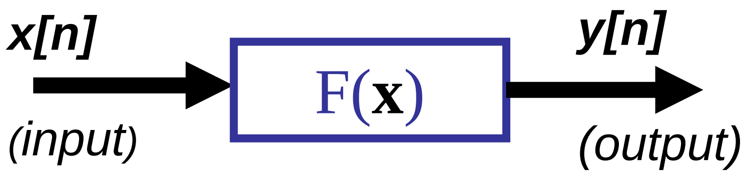
Create change

Blackbox System

A **signal** can be observed output from an unknown linear time-invariant **system**



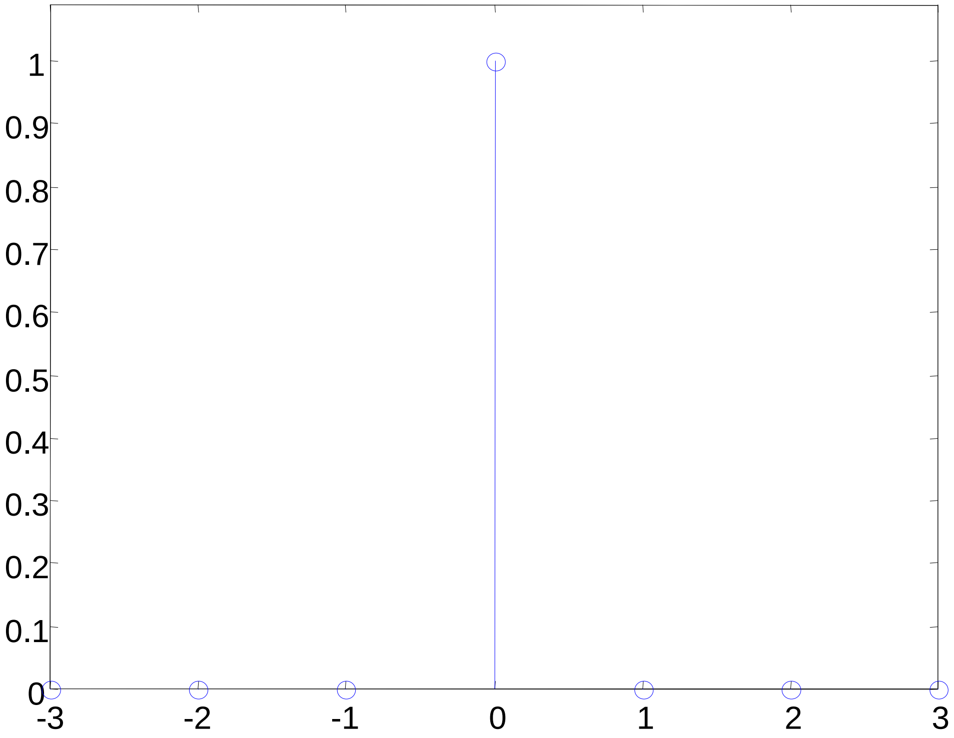
How can we characterise it?

Impulse Functions

Discrete-time

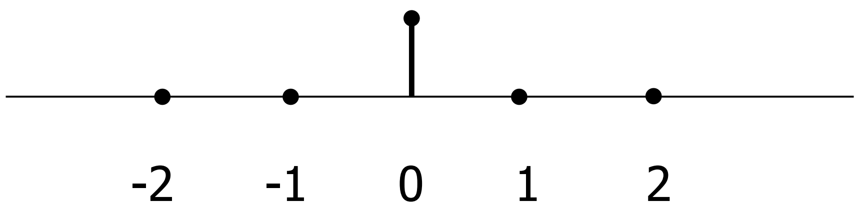
- unit sample, or unit impulse
- $\delta[n]$
- unit area

$$\delta[n] = \begin{cases} 0, & n \neq 0; \\ 1, & n = 0. \end{cases}$$



Example: Discrete Sequence

$\{x[n]\}_{-2}^{2} = [0 \quad 0 \quad 1 \quad 0 \quad 0]$

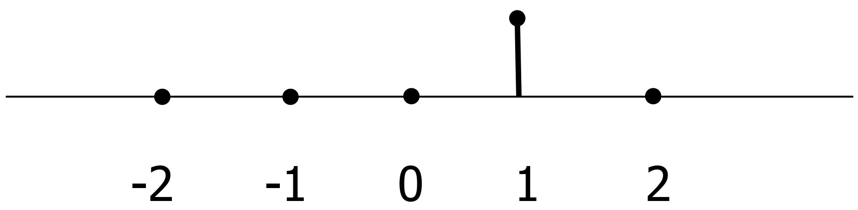


$x[n] = x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2]$

$x[n] = \delta[n]$ <- Impulse signal

Example: Discrete Sequence

$\{x[n]\}_{-2}^{2} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$

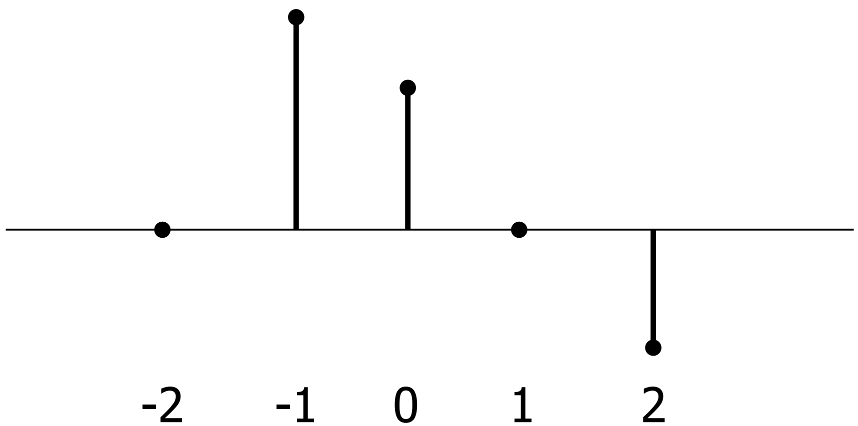


$x[n] = x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2]$

$x[n] = \delta[n-1]$ <- Signal will arrive at n=1, so it is a delay

Example: Discrete Sequence

$\{x[n]\}_{-2}^{2} = [0 \quad 3 \quad 2 \quad 0 \quad -1]$

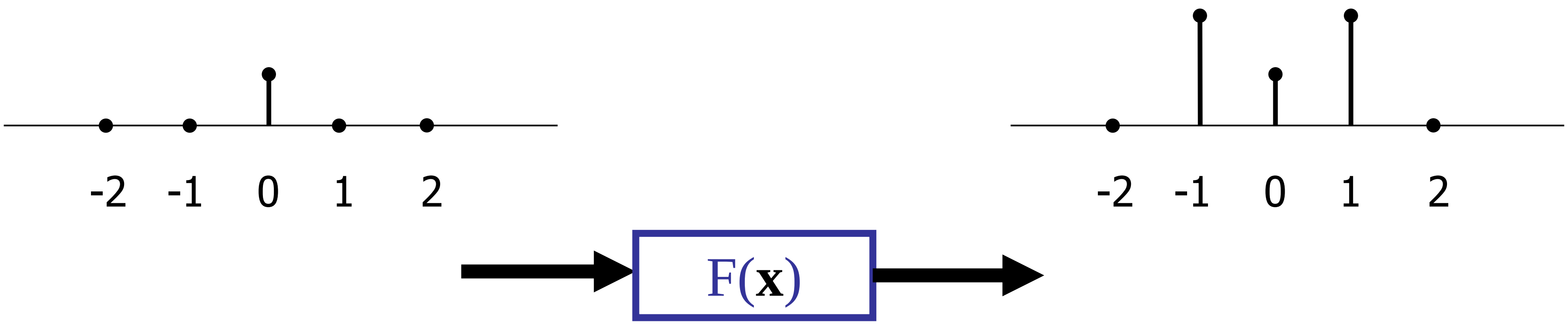


$x[n] = x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2]$

$x[n] = 3\delta[n+1] + 2\delta[n] - \delta[n-2]$

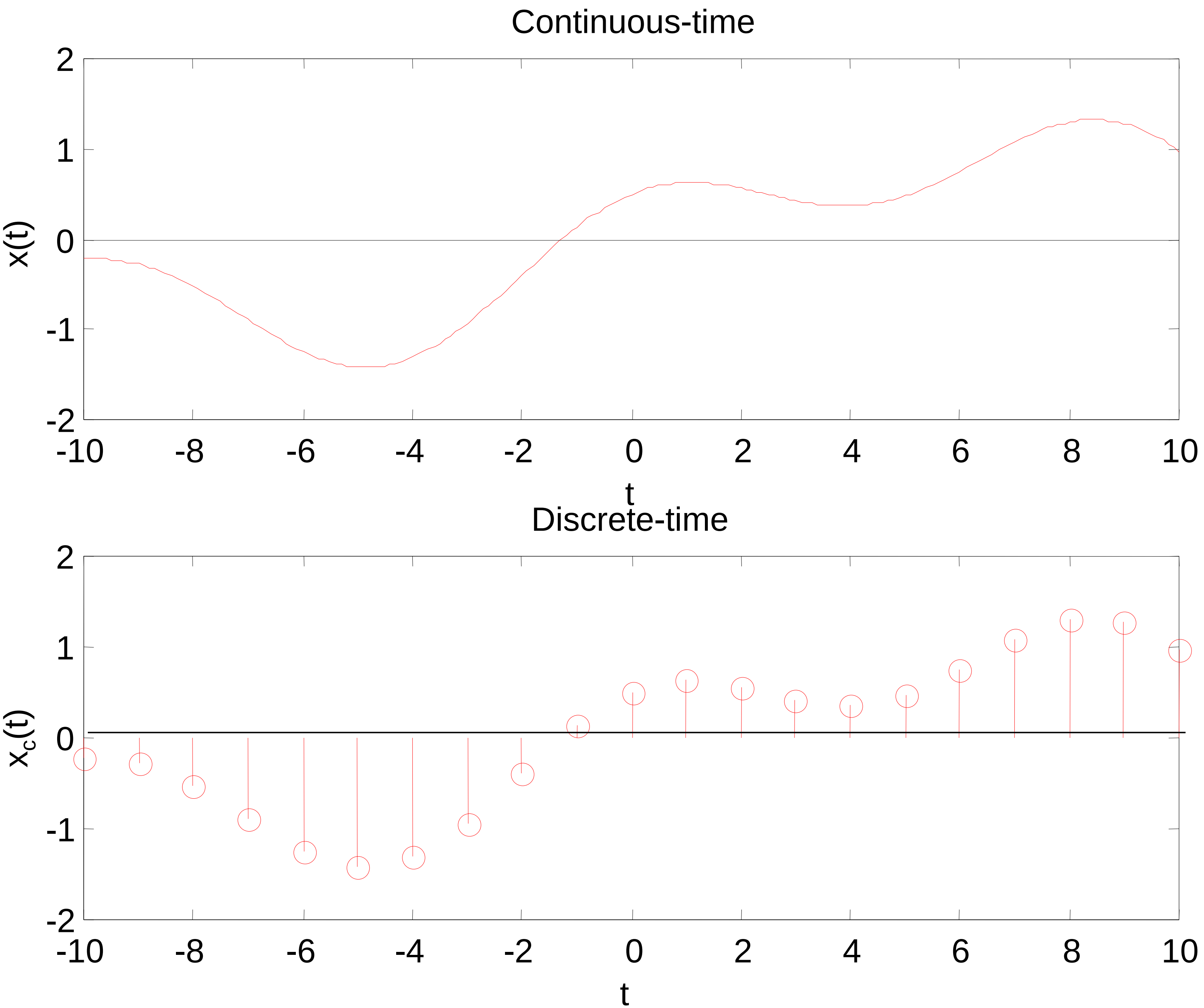
Example: Impulse Response

$\{x[n]\}_{-2}^{2} = [0 \quad 0 \quad 1 \quad 0 \quad 0]$



$x[n] = \delta[n]$

$y[n] = h[n]$
$\quad = 2*\delta[n+1] + \delta[n] + 2*\delta[n-1]$



Any signal is just a superposition of weighted impulses

Principle of Superposition

Combination of

- Scaling and
- **Additivity**

e.g., 2 inputs & 2 constants (*a* & *b*)

$$H\{ax_1[n]+bx_2[n]\}=aH\{x_1[n]\}+bH\{x_2[n]\}$$

Linear systems obey the principle of superposition
i.e., they obey associative and distributive rules

Time Invariance

Conceptually,

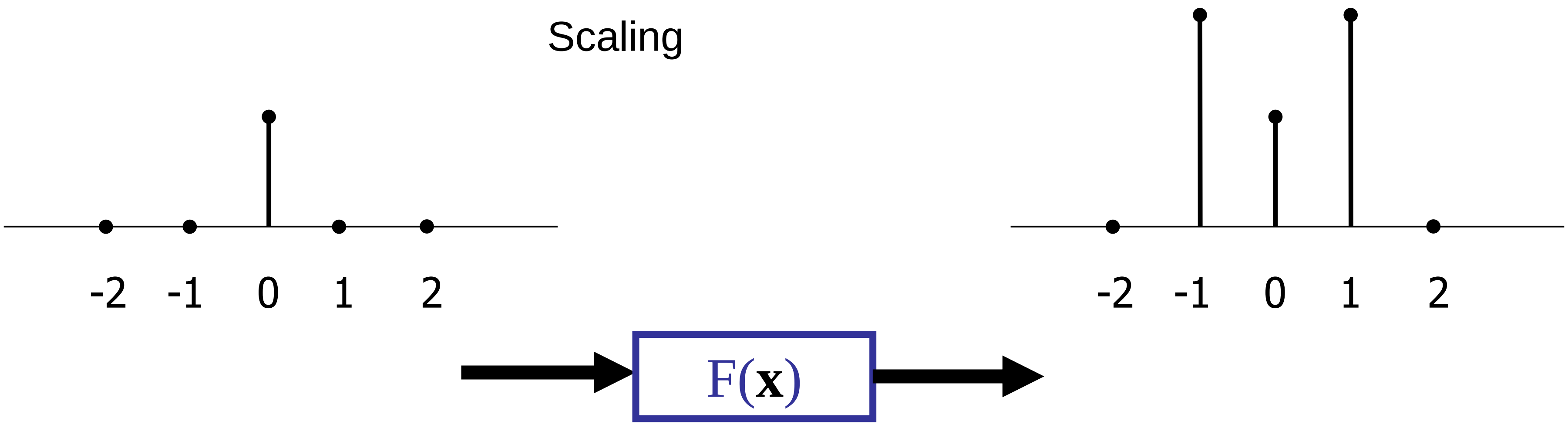
- System does not change over time
- e.g., Circuit R & C fixed

Formally,

- **Time shift in the input causes an**
- **Identical time shift in the output** (only)
- $y[n - n_o] = H\{x[n - n_o]\}$

Example: Impulse Response

$$\{x[n]\}_{-2}^{2} = [0 \quad 0 \quad 2 \quad 0 \quad 0]$$



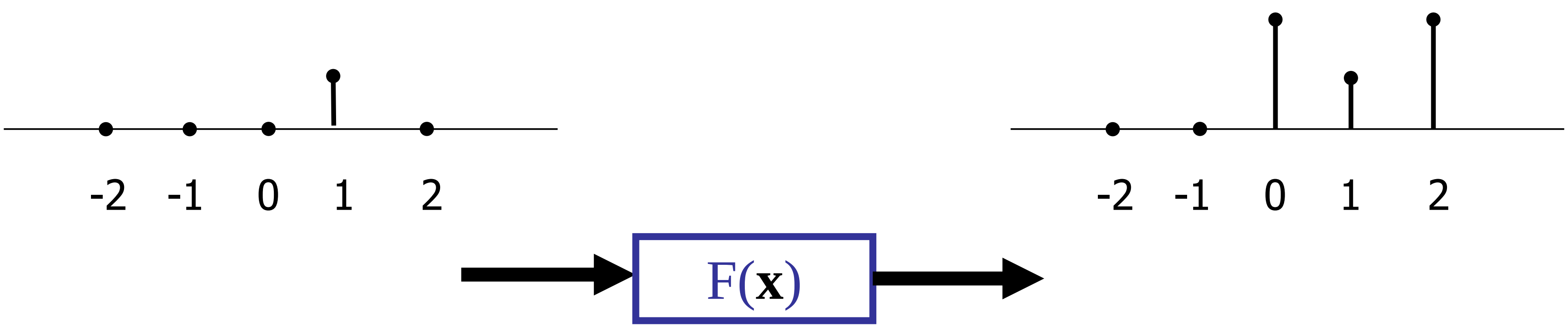
$x[n] = 2*\delta[n]$

$y[n] = h[n]$
$\quad = 4*\delta[n+1] + 2\delta[n] + 4*\delta[n-1]$

Example: Impulse Response

$$\{x[n]\}_{-2}^{2} = [0 \quad 0 \quad 2 \quad 0 \quad 0]$$

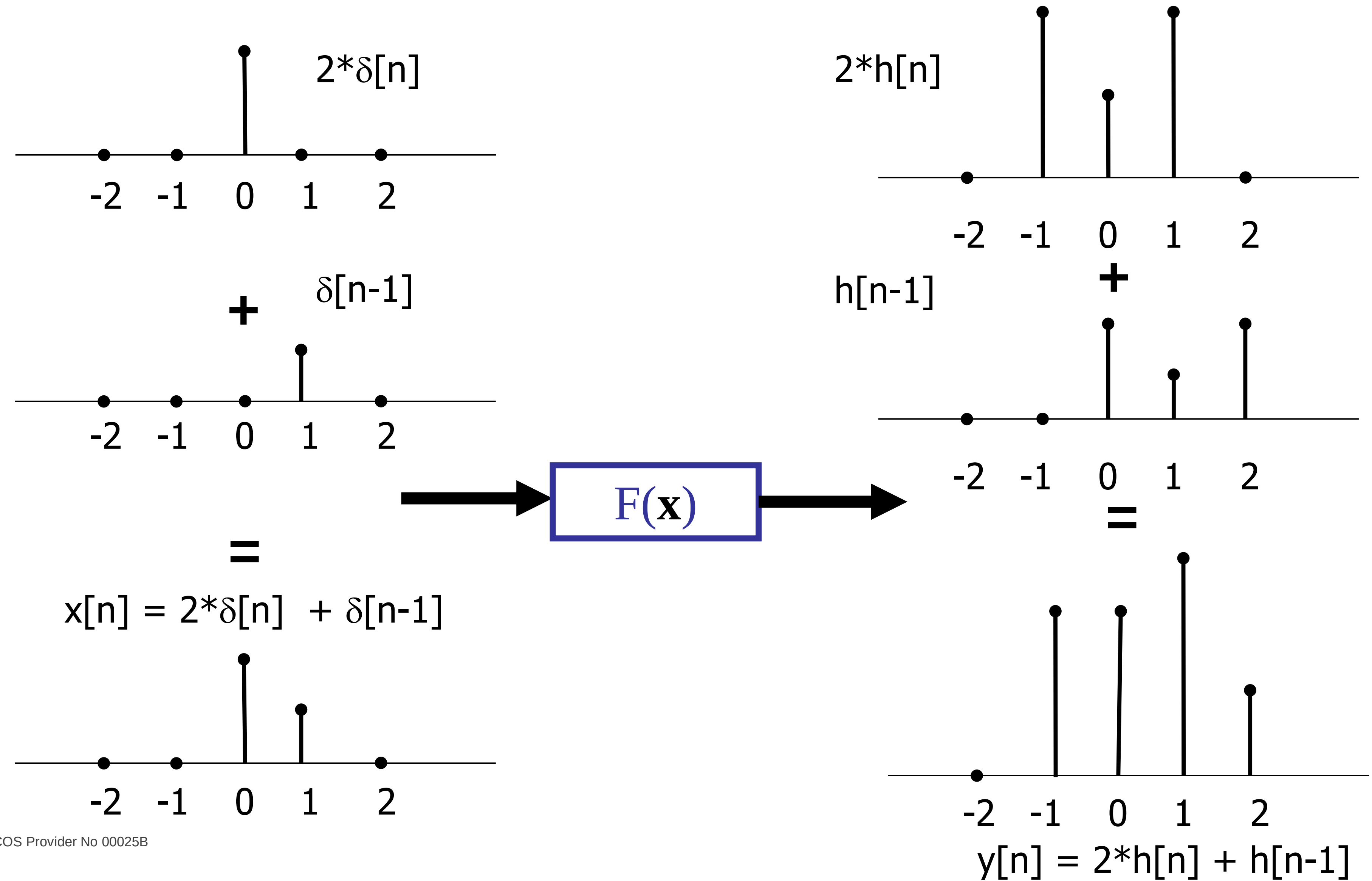
Time Shift



$x[n] = \delta[n-1]$

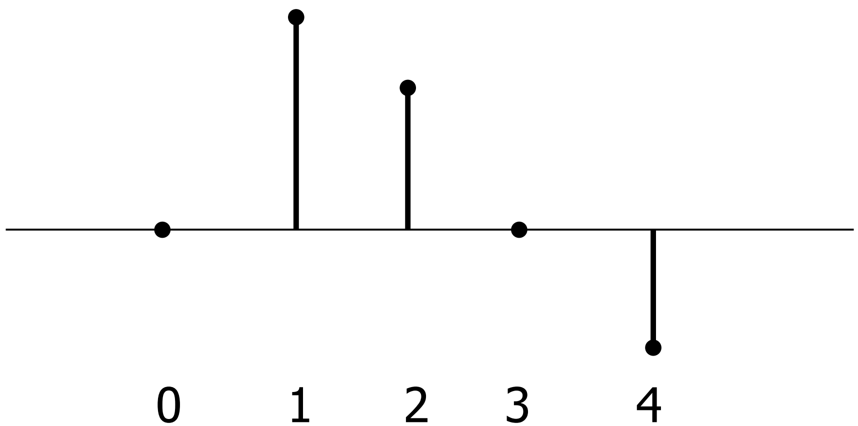
$y[n] = h[n]$
$= 2*\delta[n] + \delta[n-1] + 2*\delta[n-2]$

Example: Impulse Response



Example: Impulse Response

$\{x[n]\}_0^4 = [0 \quad 3 \quad 2 \quad 0 \quad -1]$



x[n] = x[0]δ[n-0] + x[1]δ[n-1] + x[2]δ[n-2] + x[3]δ[n-3] + x[4]δ[n-4]
= 0*δ[n-0] + 3*δ[n-1] + 2*δ[n-2] + 0*δ[n-3] + -1*δ[n-4]

y[n] = 0*h[n-0] + 3*h[n-1] + 2*h[n-2] + 0*h[n-3] + -1*h[n-4]
= x[0]h[n-0] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3] + x[4]h[n-4]

$$y[n] = \sum_{m=0}^{\infty} h[n-m]x[m] = h[n] * x[n]$$