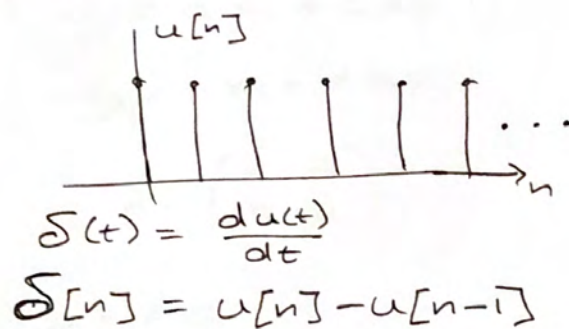


Question 1

A.)



B.)
$$S[n] = \sum_{k=0}^{\infty} h[n] \cdot u[n-k]$$

C.)
$$h[n] = S[n] - S[n-1]$$

Question 2

$$y[n] = 3\delta[n-4] - \delta[n] - \delta[n+7] + 4\delta[n+2]$$

A.)
$$Y(z) = 3z^{-4} - 1 - z^7 + 4z^2$$

B.)
$$z = e^{j\omega n}, \quad \omega = \frac{2\pi k}{N}$$

$$Y(\omega) = 3e^{-j4\omega} - 1 - e^{j7\omega} + 4e^{j2\omega}$$

Euler $\Rightarrow e^{-j\omega} = \cos\omega - j\sin\omega$

$$\therefore Y(\omega) = 3\cos(4\omega) - j3\sin(4\omega) - 1 - \cos(7\omega) - j\sin(7\omega) + 4\cos(2\omega) + j4\sin(2\omega)$$

Question 3

$$A.) \quad y_1 = ax_1 + cx_2$$

$$y_2 = bx_1 + dx_2$$

$$\therefore A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\underline{y} = A \underline{x}$$

$$B.) \quad A = \begin{bmatrix} a & c \\ 0 & d \end{bmatrix} \therefore y_2 \text{ only depends on } x_2.$$



6.)

$$y(t) = 3 \cos(2\pi t) + \cos(3\pi(t+0.5)) + 4 \sin(5\pi t)$$

$$\omega_s = n\pi$$

Assuming $n \in \mathbb{Z}$ ω_n = Nyquist frequency• Nyquist frequency = Maximum frequency = $5\pi \text{ rad.s}^{-1}$

$$\therefore \omega_s > \omega_n \times 2 = 10\pi \text{ rad.s}^{-1}$$

$$\therefore \omega_s \approx 11\pi \text{ rad.s}^{-1} \text{ (could be anything more than } 10\pi)$$

$$\Rightarrow f_s = \frac{11}{2} \text{ Hz} \quad T_s = \frac{2}{11} \text{ s}$$

$$\text{A.) } \omega_s = \underline{\underline{11\pi}}$$

To get the full signal:

- Capture at least one full period
- $\frac{P}{Q}$ should be a whole number
 - P is the period of the signal
 - Q is the sampling period = $\frac{2}{11} \text{ s}$

$$\frac{P}{\frac{2}{11}} = P \cdot \frac{11}{2} \quad \therefore P = 2 \text{ s}$$

Signal's periodicity is the lowest number with the common factors of signal's periods:

$$\omega_1 = 2\pi \quad T_1 = 1 \text{ s}$$

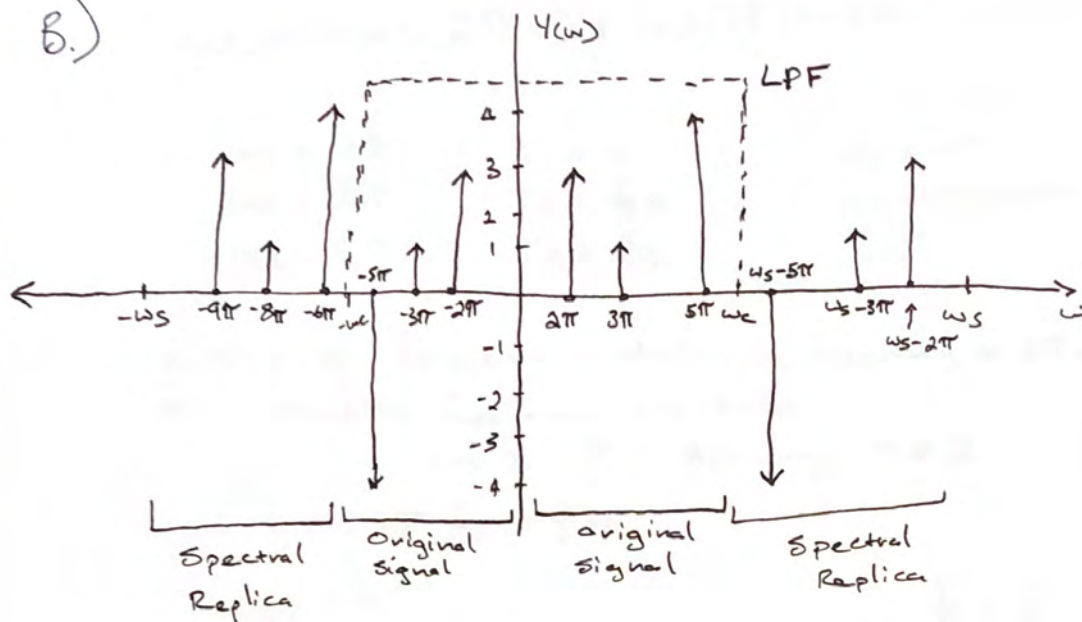
$$\omega_2 = 3\pi \quad T_2 = \frac{2}{3} \text{ s}$$

$$\omega_3 = 5\pi \quad T_3 = \frac{2}{5} \text{ s}$$

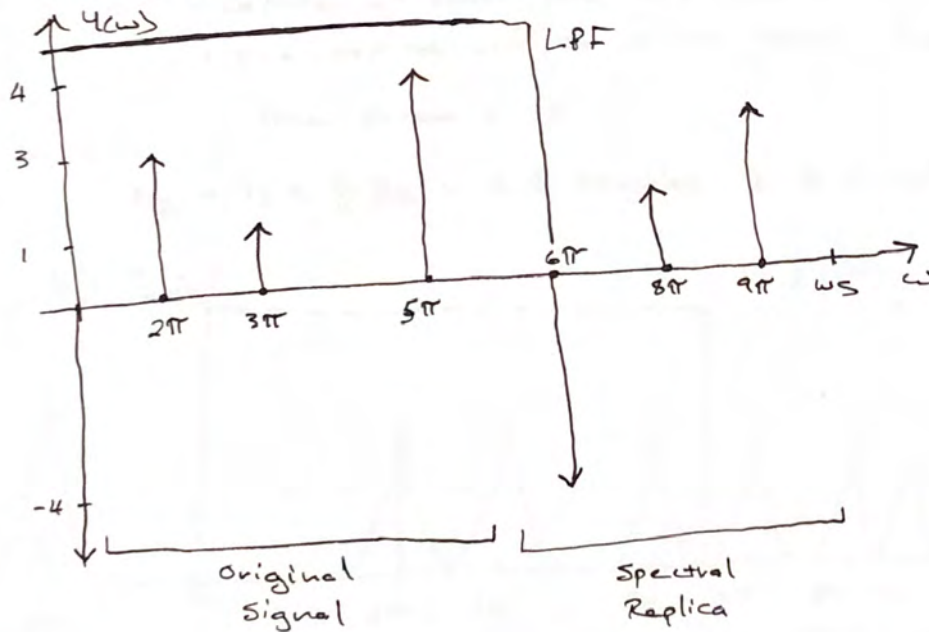
2 is the lowest number with common factors of $1, \frac{2}{3}, \frac{2}{5}$. $\therefore 2 \text{ s}$ will capture at least 1 period of the signal. [*Actually exactly one period]

$$\therefore N_s = 2 \text{ s} \times \frac{11}{2} \text{ Hz} = 11 \text{ samples}$$

B.)



Signal is mirrored across $\omega = 0$.
Taking just the +ve side: $\omega \geq 0$



The cutoff must be higher than $5\pi \text{ rad.s}^{-1}$ to include the signal but it must exclude the spectral replica.

$$\therefore 5\pi < \omega_c \leq 6\pi$$

Since the LPF is ideal at $\omega_c = 6\pi \text{ rad.s}^{-1}$ it will cutoff all frequencies from and including $6\pi \text{ rad.s}^{-1}$.

Question 7

$$|H_n(\omega)| = \frac{1}{\sqrt{1 + \omega^{2n}}}$$

for LPF: $\omega = \frac{\omega_0}{\omega_c}$ HPF: $\omega = \frac{\omega_c}{\omega_0}$

- n is the order
- ω_0 is the frequency of the signal
- ω_c is the cut off frequency

$$\omega_c = 2\pi \cdot 6\text{kHz} = 12\pi \text{ k rad.s}^{-1}$$

$$n = 2 \quad (\text{2nd order})$$

- Transfer function can be found by taking the square of the magnitude:

$$|H(\omega)|^2 = \frac{1}{1 + \omega^{2n}} = H(\omega)$$

- for HPF:

$$|H(\omega)|^2 = \frac{1}{1 + \left(\frac{\omega_c}{\omega_0}\right)^{2n}} = H(\omega)$$

- Substitute in:

$$H(\omega) = \frac{1}{1 + \left(\frac{12\pi \text{ k}}{\omega_0}\right)^4}$$

- Substitute $s = j\omega \Rightarrow \omega = \frac{s}{j}$

$$H(s) = \frac{1}{1 + \left(\frac{j12\pi \text{ k}}{s}\right)^4}$$

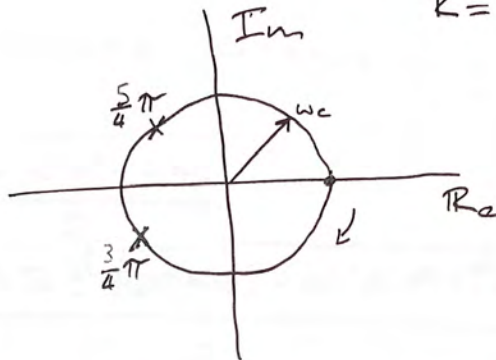
- Now can find the poles

$$H(s) = \frac{1}{1 + \left(\frac{j12\pi k}{s}\right)^4}$$

Poles $\Rightarrow \left(-\frac{j12\pi k}{s_k}\right)^4 = e^{j\pi(2k+n-1)}$ * (Lathi: pg 507)

$$s_k = 12\pi k e^{-j\frac{\pi}{4}(2k+n-1)}$$

↑ circle with magnitude ω_c
 $k = 1, \dots, n; n = 1, 2$



$k=1$

$$s_k = \omega_c e^{-j\frac{\pi}{4}(2+2-1)} = \omega_c e^{-j\frac{3}{4}\pi}$$

$$= \omega_c \left(\cos \frac{3}{4}\pi - j \sin \frac{3}{4}\pi\right)$$

$$\cong -8.5\pi k - j 8.5\pi k$$

$k=2$

$$s_k = \omega_c e^{-j\frac{\pi}{4}(2 \cdot 2 + 2 - 1)} = \omega_c e^{-j\frac{5}{4}\pi}$$

$$= \omega_c \left(\cos \frac{5}{4}\pi - j \sin \frac{5}{4}\pi\right)$$

$$\cong -8.5\pi k + j 8.5\pi k$$

$\therefore$ The transfer function is:

A.)
$$H(s) = \frac{1}{(s - (-8.5\pi k - j 8.5\pi k))(s - (-8.5\pi k + j 8.5\pi k))}$$

- Otherwise just do this: ^{*}[From Elec2004]

$$H(s) = \frac{1}{s^2 + \omega_c \sqrt{2} s + \omega_c^2}$$

$$H(s) = \frac{1}{s^2 + 12\pi k \sqrt{2} s + (12\pi k)^2}$$

- Proof: Use quadratic on this 

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

$$s = \frac{-12\pi k \sqrt{2} \pm \sqrt{(12\pi k \sqrt{2})^2 - 4(1)(12\pi k)^2}}{2}$$

$$\therefore s_1 = -26657.3 + j26657.3 (\div \pi)$$

$$s_1 = -8.49k\pi + j8.49k\pi$$

$$s_1 \cong -8.5k\pi + j8.5k\pi$$

$$s_2 = -26657.3 - j26657.3 (\div \pi)$$

$$s_2 \cong -8.5k\pi - j8.5k\pi$$



B.) Use the magnitude:

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega_0}\right)^4}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{12\pi k}{\omega_0}\right)^4}}$$

$$\text{Gain} = 20 \log(|H(\omega)|) \quad \text{Use } \omega = \frac{\omega_c}{\omega_0}$$

$$-3 = 20 \log\left(\frac{1}{\sqrt{1 + \omega^4}}\right)$$

$$-3 = -20 \log(\sqrt{1 + \omega^4})$$

$$-3 = -10 \log(1 + \omega^4)$$

$$+0.3 = \log(1 + \omega^4)$$

$$10^{+0.3} = 1 + \omega^4$$

$$1.995 = 1 + \omega^4$$

$$\omega^4 = 0.995$$

$$\omega = 0.999$$

$$\frac{12\pi k}{\omega_0} = 0.999$$

$$\omega = 12012\pi \text{ rad. s}^{-1}$$

- -3dB is typically the gain at ω_c

$$-30 \text{ dB} = -10 \log(1 + \omega^4)$$

$$3 = \log(1 + \omega^4)$$

$$10^3 = 1 + \omega^4$$

$$\omega^4 = 999$$

$$\omega = 5.622$$

$$\frac{12\pi K}{\omega_0} = 5.622$$

$$\omega_0 = 2.134\pi K \text{ rad.s}^{-1}$$

