

NAME: _____ Student ID: _____

Note: The purpose of this quiz is to review some of the mathematical functions/concepts needed for this course and hints at some of the initial topics that will be covered in the coming weeks.

① Complex Solutions to Real Problems

Can an ODE with only real constant coefficients have a complex solution?
(Briefly explain and, if so, please given an example.)

② Transfer Functions and the s-Domain

What is a transfer function? (Please briefly explain)

What are the final values (i.e. $t \rightarrow \infty$) for the systems governed by following transfer functions:

$$\bullet G_a(s) = \frac{3004}{s+4}$$

$$\bullet G_b(s) = \frac{3004}{s-4}$$

$$\bullet G_c(s) = \frac{3004}{s^2+4}$$

$$\bullet G_d(s) = \frac{3004}{s^4+4}$$

$$\bullet G_e(s) = \frac{3004}{s^2+4s}$$

$$\bullet G_f(s) = \frac{3004}{4}$$

What is an impulse response?

Extra Credit: Imagine a system, $W(t)$. If it has an normalized output to a unit, 1 Hz sinusoidal input given by $W_{out}(V_{[1\text{ Hz}]}(t)) = \sin(4\pi t)$, what is $W(t)$ xor $W(s)$ in general?

(Hint: What is $V_{[1\text{ Hz}]}(s)$? Recall that $\mathcal{L}[\delta(t)] = 1$ and $\mathcal{L}[\sin(at)] = \frac{a}{s^2+a^2}$)

③ Free Determination

True or False: $\det(\mathbf{A} + \mathbf{B}) = \det(\mathbf{A}) + \det(\mathbf{B})$?

True or False: $\det(\mathbf{AB}) = \det(\mathbf{A}) \cdot \det(\mathbf{B})$?

④ Process of Elimination

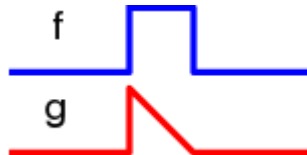
True or False: As the case $A = LU$ shows, factorization is basically elimination

True or False: If A is invertible, then the only solution to $Ax = 0$ is $x = 0$.

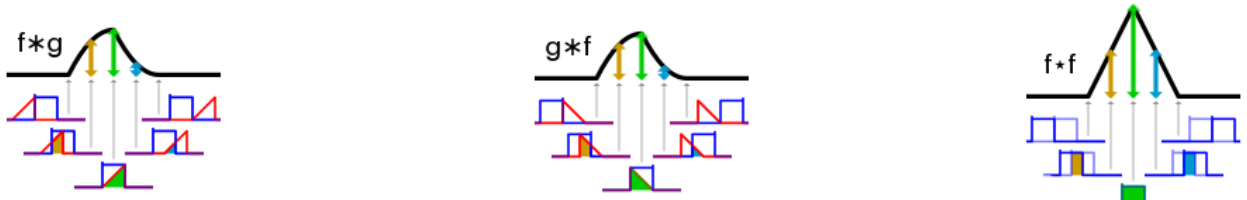
True or False: As Linear Equations ($Ax = b$) come from **steady-state problems**, then it can be said that eigenvalues ($Ax = \lambda x$) have importance in **dynamic problems**.

⑤ Convolution!

Given two functions, $f(t)$ and $g(t)$:



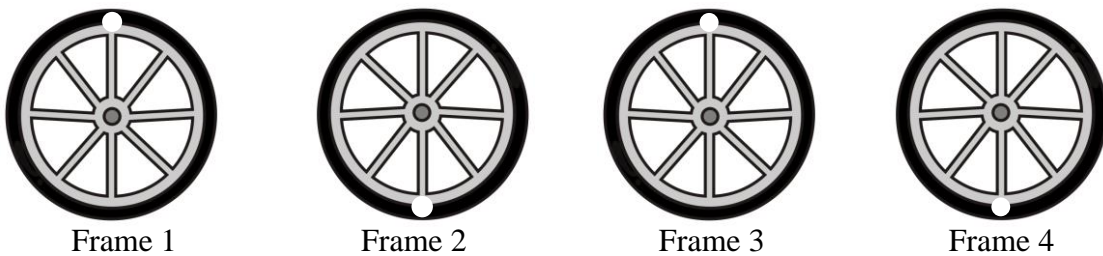
True or False: Are the following convolutions correct?



A) _____ B) _____ C) _____

⑥ A Signal Re-volution! (on Aliasing and Nyquist)

A (global-shutter) video camera captures video at 25 frames per second (fps). When pointed at the wheel (with 195/65R15 tyres) of 64 cm diameter with a dot logo marked on it, the following video frames are recorded:



Which way is the wheel rotating?
 What is its angular velocity? (If more than one solution, then provide one possible answer)

At what possible speeds (metres per second) could the wheel be travelling?
 When will this be the same as the speed of the car to which the said wheel is attached?