ELEC 3004 / 7312 – Systems: Signals & Controls **2019**

Problem Set 3: Digital Feedback Control

Total marks: 100 **Due Date:** Monday, **June 3, 2019** at 23:59 AEST [*end of week 13*]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and **submitted directly in Platypus** (preferred) or as **PDF**. Note that Microsoft **Word** documents and scanned images of **handwritten pages** are specifically **disallowed**. The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Please double-check that your name is not in the solution directly or via the associated metadata.

Also, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

Finally, a note of remembrance for our colleagues and compatriots in Sri Lanka.
A disturbance, no matter how extreme, may always be rejected via an integral effort.

Thank you. **:-)**

For Questions Q1 to Q3: Please answer **2 out of the 3** questions (your choice).
If all three questions are submitted, *the tutors will only mark two chosen at random*.

For Questions Q6 and Q7: These questions are optional and for extra credit only.
There is no cap on the total extra credit for the problem set.

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. The Stately State Transition Matrix, $\mathbf{\Phi}$ **[20 points]**

Consider a state transition matrix, $\mathbf{\Phi}$, for a LTI system **A**[1].

$$\mathbf{A} = \begin{bmatrix} 16 & 2 & 3 & 13 \\ 5 & 11 & 10 & 8 \\ 9 & 7 & 6 & 12 \\ 4 & 14 & 15 & 1 \end{bmatrix}$$

$$\mathbf{\Gamma} = \begin{bmatrix} 3 & 1 & 4 & 1 \\ 1 & 3 & 1 & 4 \\ 4 & 1 & 3 & 1 \\ 1 & 4 & 1 & 3 \end{bmatrix}$$

Please determine and justify :

(a) $\mathbf{\Phi}(t)$ for this system **A**

(b) $\mathbf{\Phi}(s)$ for this system **A**

(c) System **A**'s characteristic polynomial

(d) $\widehat{\Phi}(z)$ for this system **A** via Tustin's method (i.e. trapazoid-rule)

(e) A difference equation assuming: (1) a step input at the first step ($u(k)$), followed by (2) ZOH sampling, with (3) **H**=**I**, and $\mathbf{\Gamma}$ as given.
[**Hint:** What is z-transform of the ZOH, i.e., $G(z)$ for a system $G(s)$?]

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[1] **Note:** In Matlab, **A**=`magic(4);` $\mathbf{\Gamma}$=`toeplitz([3 1 4 1]);`

Q2. PID: Possibly Insufficient Design **[20 points]**

The PID control architecture might be popular, but that does not mean it can be used for everything. This question explores when the PID controller is (in)sufficient: when it is too much, when it is "goldilocks", and when it is not enough.

Consider a plant given by $P(s) = \frac{1}{(s+a)^n}$. As n increases, so does the order of the system. Let's consider three orders and how PID controller design works for them (or does not).

(a) **First Order (** $a = 1, n = 1$ **)**: It has been said that PI control is adequate for processes where the dynamics is (essentially) first order. **Please explain why this is the case.**

(b) **Simple Second Order (** $a = 0, n = 2$ **)**: It has been said that a double integrator **can not** be controlled by a PI controller. (1) **Please explain why this is the case.** (2) Also, could a PID controller be sufficient for servoregulation in this case?
[**Possible Hint**: For part (1), if it helps, think about what might happen if $a = 1, n = 2$].

(c) **Third Order (** $a = 1, n = 3$ **)**: It has been said that when a process is more than second order, that a PID might be workable, but insufficient. **Please explain why this might be the case**.

(d) **Delay-Dominated First Order**: Control of systems with large time delays can be difficult.Consider a variation on P with large delay (T_D , $T_D > a$): $Q(s) = \frac{1}{1+as} e^{-T_D s}$. It is has been said that for such systems that adding derivative action does not help much. **Please explain why this might be the case**.

Q3. LeviLab: Floating the (Magnetic) Data **[20 points]**

Laboratory 3 and 4 involve involve modelling and control of a levitating magnetic mass. Based on your analysis and laboratory experiments, please answer the following:

(a) **Did it levitate?** ***If so*, ** please provide:
  a. The final control law (equation)
  b. A picture of it levitating

If it did not levitate, Explain why and show a picture including how far you got. Please give the system model**.** That is, the function / equation describing the system's behavior. [**Note**: a set of differential equations or a state-space form is acceptable.]

(b) **Tuning the system**
Discuss how you went about tuning the system. What gains did you select in the end? Please provide any evidence possible to support this (e.g. sensitivity plots, pole placements, random luck, Taguchi method, Ziegler Nichols process calculations, etc.).

(c) LabView
Please provide a "selfie" with of you (and/or team) **and also** the tutor with the levitating magnet (if relevant) in the background. Thus, there should be two photographs submitted. [**Please** keep them reasonably sized -- <2 MB is fine.]

Q4. A Sound Strategy? **[30 points]**

As part of its "Think. Difference. Function." student strategy, SNaF[2] University wants to add a live in-house band to accompany lectures and play musical interludes. Its President, an award-winning media personality, points out that this will "be distinctive," has "co-creation in partnership," and that this "collaborative, connected and active" learning will lead to to a more "sound education." (Also, "jazzing it up" helps justify higher fees. :-))

To support this "blended learning," a microphone and speaker are added to every classroom. Consider the situation where a speaker outputs amplified signals from a microphone, but where the microphone itself senses some of the sound coming from said speaker (shown above). The attenuating properties of the air decrease the strength of the speaker's signal picked up by the microphone. This increases with greater distance between the microphone and speaker. In addition, there is a time delay between the signal produced amplifier and then sent to the speaker and that sensed by the microphone.

Initially assume the amplifier is $K_G(s)$, the speaker is $K_S(s)$, the attenuation the due to the air gap is $K_A(s)$, the delay due to the air gap is T_A (in sec.), the microphone is given by $K_M(s)$, and the sampled signal is ZOH with a sampling period of T_S (in sec.).

(a) Please draw the full block diagram from audio input to speaker output for the system assuming it is a fully **digital** (i.e. the amplifier is digital) an in a negative feedback configuration.

(b) What is the closed loop transfer function in both the z domain? (i.e. $TF(z)$)?

(c) If $K_G(s) = k_1$, $K_S(s) = s^2 + k_2 s + k_3$, $K_A(s) =- k_4 e^{-sT_A}$, $K_M(s) = s^2 + k_5 s + k_6$, what is the order of the overall closed loop transfer function in the **z-domain**?

(d) If the terms $k_1, ..., k_6$ inside K_G, K_S, K_A, K_M, T_A and T_S are constants (and time-invariant), what can we say about the system's stability?

(e) Pick a series of stable, initial system gains (K_G, K_S, K_M, etc.). (1) Plot the overall system on the z-plane (γ-plane). (2) What happens as the air gap in increased?
[**Hint**: Consider what happens to k_4 and T_A as the gap is increased?]

(f) Prof. Ring-Out, ever-tired of the President's superficial nonsense, is asked to consult. She hypothesizes that a notch filter filter tuned to the resonant modes of system's closed loop transfer function could be added to the amplifier to remove "the feedback". Using the your design from part (e) above and assuming the room is large (and thus has a negligible effect and echo), **please determine these resonant modes**. Then please expand on Prof. Ring-Out's strategy — (1) Which type (IIR or FIR) would you recommend? (2) What general filter order is suggested (i.e. log(filter order))? **Please explain.**
[**Hint**: What are the tradeoffs inherent with a filter in a feedback loop?]

[2] *Scientia Nummus ac Fortuna* (Know Money and Luck) - University

Q5. For Whom the Camera Clicks **[30 points]**

It has been suggested that colour photography, chiefly its colour calibration, might exhibit a racial bias[3]. Perhaps linear systems theory might be able to help explore this[4].

Colour images are typically captured using a Bayer color filter array with R, G and B responses for "red" (long), "green" (medium), and "blue" (short) wavelengths. For simplicity, divide the visible spectrum into D bands, and model the response as follows:

$$R_{pixel} = \sum_{i=1}^{D} r_i p_i \qquad G_{pixel} = \sum_{i=1}^{D} g_i p_i \qquad B_{pixel} = \sum_{i=1}^{D} b_i p_i$$

where p_i is the incident power in the i^{th} wavelength band, and r_i, g_i, and b_i are non-negative constants that describe the spectral response of the different pixel colors. The sensed colour values (R_{pixel}, G_{pixel}, B_{pixel}) are a vector function of the three pixel responses that are then sampled and digitized with Q-bits (where Q is typically 8, and helps establish the sensor's (but not necessarily the final image's) dynamic range).

(a) Determine (non-trivially) when two light spectra, p and $\widehat{p}$ are visually indistinguishable? (This may be a specific case or, preferably, a general rule)
[**Note**: Visually identical cases with different spectral power compositions are called *metamers*]

(b) In a colour matching operation, a camera is calibrated against a test light where one changes the intensities of three primary lights until the sum of the primary lights looks the same. (n.b. monitor/TV calibration does something similar, but in reverse). That is, to find a spectrum of the form $p_{match} = a_1\lambda_u + a_2\lambda_v + a_3\lambda_w$, where λ_u, λ_v, λ_w are the spectra of the primary lights, and a_i are the intensities that, if/when found, are indistinguishable from a test light spectrum p_{test}. Can this always be done? How does this vary depending on the number of D bands and Q bits? **Please discuss.**

(c) *Is it just the camera?* An object's surface can be characterized by its reflectance (i.e. the fraction of light it reflects) for each band of wavelengths (λ). Now consider two objects illuminated (at different times) by two different light sources (e.g., LED lights [color temperature: 6500K] and candle light [1850K]). **Engineer A** argues that if the two objects look identical when illuminated by LED lights, they will look identical when illuminated by candlelight. **Engineer B** claims that two objects can appear identical when illuminated by LED lights, but look different when lit by candlelight. **Who is right?** Please discuss.

(d) *Calibrate to what?* **Engineer C** of the *Compassionate Optical Instruments Laboratory* is asked to design a set of spectral curves r_i, g_i, and b_i for their new camera, *The Amaterasu*. How should Engineer C design these spectral curves? Please discuss.[5]
[**Hint**: How might the number of D bands and Q bits affect this? Are there computational photography methods that might inform this design decision?]

[3] Sarah Lewis, "The Racial Bias Built Into Photography", *The New York Times*, April 25, 2019. Online at: https://nyti.ms/2GwtSzj. (UQ Cached copy)

[4] For more information see also S. Boyd, *Lecture Notes for EE263*, 2012.

[5] Feel free to use your knowledge of Linear Systems theory, but also Economics, etc. (e.g., ENGG4900). Would it be advisable to have different curves for different regions (Asia, Oceania, etc.)?

Q6. Lofty Estimation Goals **[+5 Points Extra Credit]**

Biogen Aeronautic Defence and Space Systems makes Biofuel-powered jet airplanes. For their new model, the *Bein' Green-17*, they placed a greener jet forward of the wing; but, as its nacelle (or inlet cowl) generates lift at high angles of attack (such as those during take-off), it is necessary to compensate for the positive feedback mode this causes in the plane's pitch. [Note, this is particularly the case when the air is a little thinner, such as at "hot or high" airports. Also, other design changes (a new inlet, larger landing gear, etc.) would be less fuel efficient and more costly.]

In response, a control program is added to automatically pitch the nose down just before a stall (i.e. a condition in which lift is lost due to airflow separation at large angles of attack). This, in turn, requires an estimate of the airplane's angle of attack. Four engineers are debating what strategy to take to observe this value.

- **Engineer W** suggests that median value from three of the same sensors is taken.
- **Engineer X** suggests that only two of the most accurate and robust sensors are needed. Engineer X recommends a sensor that has an MTBF (mean time between failures) of 20,000 hours (2.2 years continuous operation) and to take the maximum value of the two sensors.
- **Engineer Y** suggests state estimation of the value from different two different types of sensors (of which, at least one of each is needed, though more is nice). Engineer Y's initial suggestion is an Extended Kalman Filter, to get the average value (μ) and its variance (σ).
- **Engineer Z** suggests an integrated Bayesian estimation and control strategy that considers the value,its likelihood, and the next controlled action, States would be conditioned on the sequence of flight motions, state of the overall flight (e.g., taxi, take-off, etc.), etc. Engineer Z adds that this approach is more robust and even allows for active-sensing (e.g. self-diagnosis and calibrating the sensor when on (presumably level) ground, etc.).

Who (if any, some, or all) **is right? Please discuss.**

Q7. Teleoperation Without Haste **[+15 Points Extra Credit]**

Teleoperation involves robots over a communication channel with the operator handling one manipulator (the master) to operate on another manipulator at the remote end (the slave). This can be modeled using an extension of the state-space ideas seen in this class, chiefly a two-port model. This is introduced and analyzed in the seminal paper, B. Hannaford, "A Design Framework for Teleoperators with Kinesthetic Feedback", *IEEE Transactions on Robotics and Automation*, **5**(4):426-434, 1989. (UQ cached copy)

Review this paper and using the methods from this course, discuss how this approach allows for modelling teleoperation such that explicit models of the human and environment are not needed to analyze aspects of system performance and stability. Additionally, compare and contrast this teleoperation and delay compensation strategy to a PID control strategy and to a Smith Predictor.