http://elec3004.com

Sampling Theory & Data Acquisition

ELEC 3004: **Systems**: Signals & Controls
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Lecture 5

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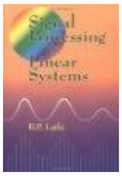
π-day (2017)

Lecture Schedule:

Week	Date	Lecture Title
1	28-Feb	Introduction
	2-Mar	Systems Overview
2	7-Mar	Systems as Maps & Signals as Vectors
	9-Mar	Systems: Linear Differential Systems
3	14-Mar	Sampling Theory & Data Acquisition
	16-Mar	Antialiasing Filters
4	21-Mar	Discrete System Analysis
	23-Mar	Convolution Review
5	28-Mar	Frequency Response
	30-Mar	Filter Analysis
5	4-Apr	Digital Filters (IIR)
	6-Apr	Digital Windows
6	11-Apr	Digital Filter (FIR)
	13-Apr	FFT
	18-Apr	Holiday
	20-Apr	
	25-Apr	
7	27-Apr	Active Filters & Estimation
8	2-May	Introduction to Feedback Control
	4-May	Servoregulation/PID
10	9-May	Introduction to (Digital) Control
	11-May	Digitial Control
11	16-May	Digital Control Design
	18-May	Stability
12	23-May	Digital Control Systems: Shaping the Dynamic Response
	25-May	Applications in Industry
13	30-May	System Identification & Information Theory
	1-Jun	Summary and Course Review

Follow Along Reading:



B. P. Lathi
Signal processing and linear systems
1998
[TK5102.9.L38 1998]

- Chapter 5:
  Sampling
  - § 5.1 **The Sampling Theorem**
  - § 5.2 Numerical Computation of Fourier Transform: The Discrete Fourier Transform (DFT)

Also:
- § 4.6 Signal Energy

Linear Differential Systems

(Recap)

Equivalence Across Domains

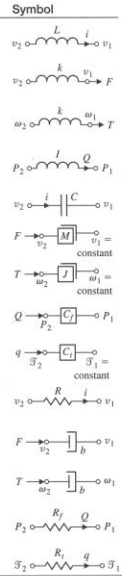
Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

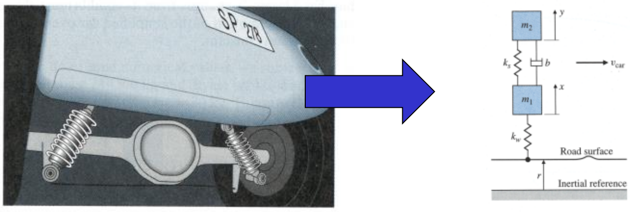
Table 2.2 Summary of Governing Differential Equations for Ideal Elements

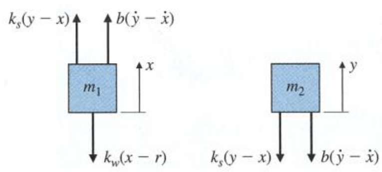
Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2^2$	
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2^2$	
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}^2$	
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}^2$	
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}^2$	
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	



Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 74

Example: Quarter-Car Model





Example: Quarter-Car Model (2)

$$\ddot{x} + \frac{b}{m_1}(\dot{x}-\dot{y}) + \frac{k_s}{m_1}(x-y) + \frac{k_w}{m_1}x = \frac{k_w}{m_1}r,$$

$$\ddot{y} + \frac{b}{m_2}(\dot{y}-\dot{x}) + \frac{k_s}{m_2}(y-x) = 0.$$

$$s^2X(s) + s\frac{b}{m_1}(X(s)-Y(s)) + \frac{k_s}{m_1}(X(s)-Y(s)) + \frac{k_w}{m_1}X(s) = \frac{k_w}{m_1}R(s),$$

$$s^2Y(s) + s\frac{b}{m_2}(Y(s)-X(s)) + \frac{k_s}{m_2}(Y(s)-X(s)) = 0,$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{k_w b}{m_1 m_2}\left(s + \frac{k_s}{b}\right)}{s^4 + \left(\frac{b}{m_1} + \frac{b}{m_2}\right)s^3 + \left(\frac{k_s}{m_1} + \frac{k_s}{m_2} + \frac{k_w}{m_1}\right)s^2 + \left(\frac{k_w b}{m_1 m_2}\right)s + \frac{k_w k_s}{m_1 m_2}}.$$

Economics: Cost of Production

Materials, parts, labour, etc. (***inputs***) are combined to make a number of products (***outputs***):

- x_j: price per unit of production input j
- a_{ij}: input j required to manufacture one unit of product i
- y_i: production cost per unit of product i
- For $y = Ax$:
  - i^{th} <u>row</u> of A is bill of materials for unit of product i
- Production inputs needed:
  - q_i is quantity of product i to be produced
  - r_j is total quantity of production input j needed

$\therefore \; r = A^T q$

& Total production cost is:

$$r^T x = (A^T q)^T x = q^T A x$$

Source: Boyd, EE263, Slide 2-18

Estimation (or inversion)

$$y = Ax$$

- y_i is i^{th} measurement or sensor reading (which we have)
- x_j is j^{th} parameter to be estimated or determined
- a_{ij} is sensitivity of i^{th} sensor to j^{th} parameter
- sample problems:
  - find x, given y
  - find all x's that result in y (i.e., all x's consistent with measurements)
  - if there is no x such that $y = Ax$, find x s.t. $y \approx Ax$
    (i.e., if the sensor readings are inconsistent, find x which is almost consistent)

Source: Boyd, EE263, Slide 2-26

Digital
Signals & Systems

Signal: A carrier of (desired) information [1]

- Need **NOT** be electrical:
  - Thermometer
  - Clock hands
  - Automobile speedometer

- Need **NOT** always being given
  - – "Abnormal" sounds/operations
  - – Ex: "pitch" or "engine hum" during machining as an indicator for feeds and speeds

Signal: A carrier of (desired) information [2]

- Electrical signals
  - Voltage
  - Current

- **Digital signals**
  - **Convert analog electrical signals to an appropriate digital electrical message**
  - **Processing by a microcontroller or microprocessor**

Transduction (sensor to an electrical signal)

- Sensor reacts to environment (physics)

- Turn this into an electrical signal:
  - V: voltage source
  - I: current source

- **Measure this signal**
  - Resistance
  - Capacitance
  - Inductance

Digital Signal

- Representation of a signal against a discrete set
- The set is fixed in by computing hardware

$$s \in \mathbb{Z}$$

- Can be scaled or normalized … but is limited

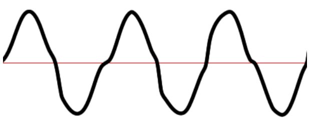
$$s \in \mathbb{Z}(0, \ldots, 2^{16})$$

- Time is also discretized

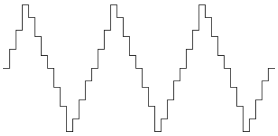
$$s' \in \frac{\mathbb{Z}(0,\ldots,2^{16})}{2^{16}}$$

Analog vs Digital

- *Analog Signal*: An analog or analogue signal is any variable signal **continuous** in both time and amplitude



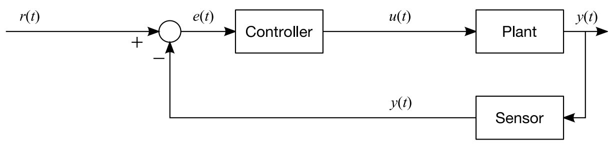
- *Digital Signal*: A digital signal is a signal that is both **discrete** and quantized



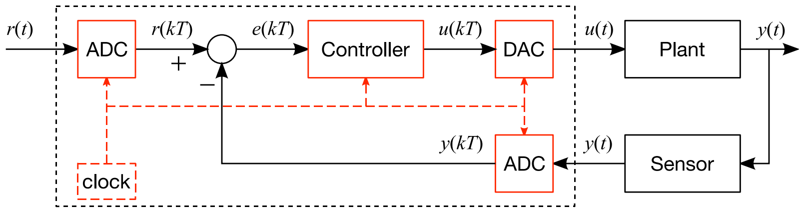
E.g. Music stored in a CD: 44,100 Samples per second and 16 bits to represent amplitude

Digital Systems

- Continuous:



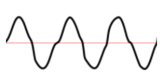
- Digital:



➔ Digital Systems ∵

Better SNR

- We trade-off "**certainty in time**" for "**signal noise/uncertainty**"
- Analog: ∞ time resolution
  - *Digital* has fixed time steps




- This avoids the noise and uncertainty in component values that affect analogue signal processing.

Better Processing

- **Digital microprocessors** are in a range of objects, from obvious (e.g. phone) to disposable (e.g. Go cards). (what doesn't have one?)

Compared to antilog computing (op-amp):

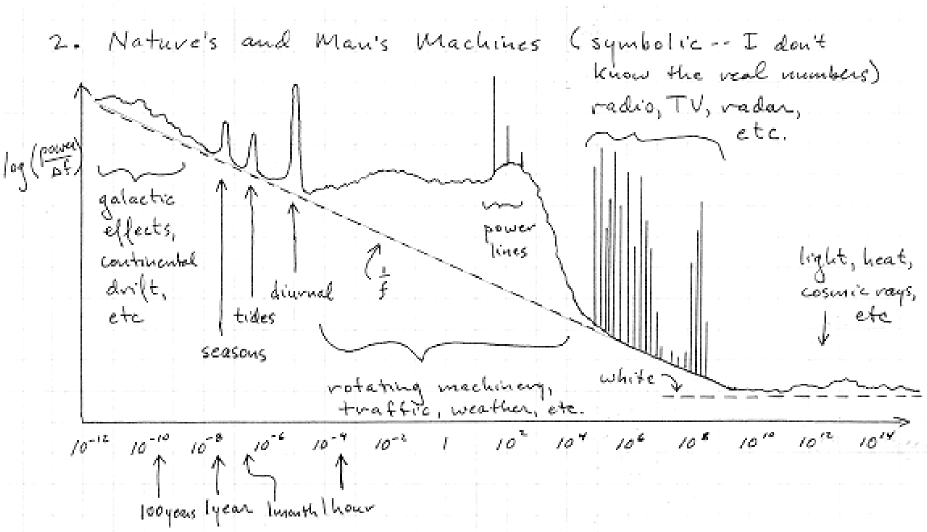
- **Accuracy**: digital signals are usually represented using 12 bits or more.
- **Reliability**: The ALU is stable over time.
- **Flexibility**: limited only programming ability!
- **Cost**: advances in technology make microcontrollers economical even for small, low cost applications. (Raspberry Pi 3: US$35)

Digital
Signals & Systems

WHY?

Noise!

BUT there is Noise …



Note: this picture illustrates the concepts but it is not quantitatively precise

Source: Prof. M. Siegel, CMU

Noise: "Unwanted" Signals Carrying Errant Information

- Cross-coupled measurements
- Cross-talk (at a restaurant or even a lecture)
- A bright sunny day obstructing picture subject
- Strong radio station near weak one
- observation-to-observation variation
  - Measurement fluctuates (ex: student)
  - Instrument fluctuates (ex: quiz !)
- Unanticipated effects / variation (Temperature)
- **One man's noise might be another man's signal**

Noise: Fundamental Natural Sources

- Voltage (EMF) – Capacitive & Inductive Pickup

- Johnson Noise – thermal / Brownian

- 1/f ($V_j = \sqrt{4 k_b T R}$

- Shot noise (interval-to-interval statistical count)

$$V_f = \sqrt{\frac{\alpha V_R^2}{N f}}$$

SNR : Signal to Noise Ratio

$$V = V_s + V_n$$

$$\text{Magnitude: } \bar{V}^2 = \bar{V}_s^2 + \bar{V}_n^2 + V_s \bar{V}_n$$

$$\frac{S}{N} = \frac{V_s^2}{V_n^2}$$

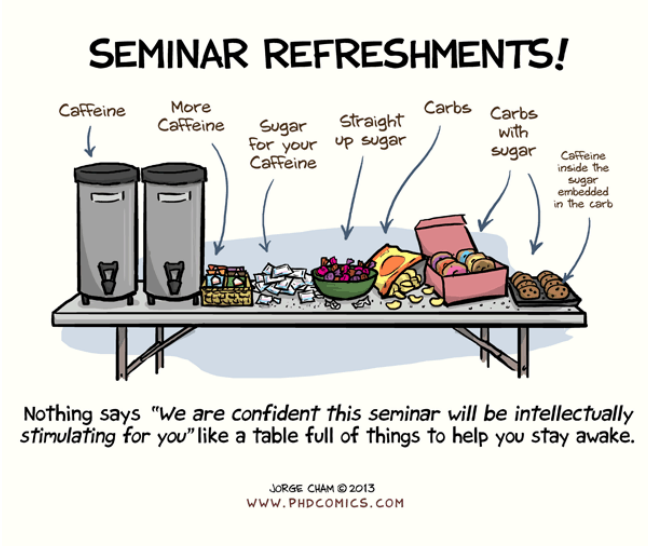
$$\text{in dB: } 10 \log \left(\frac{\bar{V_s}^2}{\bar{V_n}^2} \right) = 20 \log \left(\frac{V_s^{rms}}{V_n^{rms}} \right)$$

BREAK

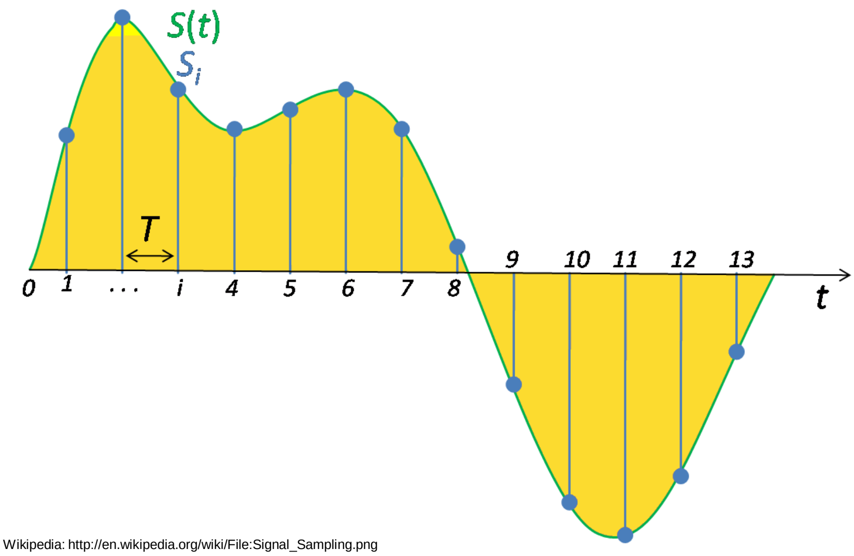
A theory for all this …

Sampling!

Not this type of sampling … ☺



This type of sampling…



Source: Wikipedia: http://en.wikipedia.org/wiki/File:Signal_Sampling.png

Sampling Theorem

- The Nyquist criterion states:

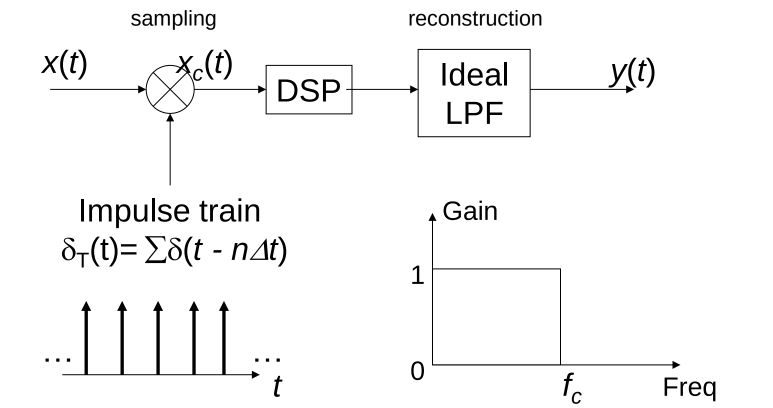
To prevent aliasing, a ***bandlimited*** *signal of bandwidth* w_B *rad/s must be sampled at a rate greater than* $2w_B$ *rad/s*

$$w_s > 2w_B$$

Note: this is a > sign not a ≥

Also note: Most real world signals require band-limiting with a lowpass (anti-aliasing) filter

Mathematics of Sampling and Reconstruction



Sampling frequency $f_s = 1/\Delta t$

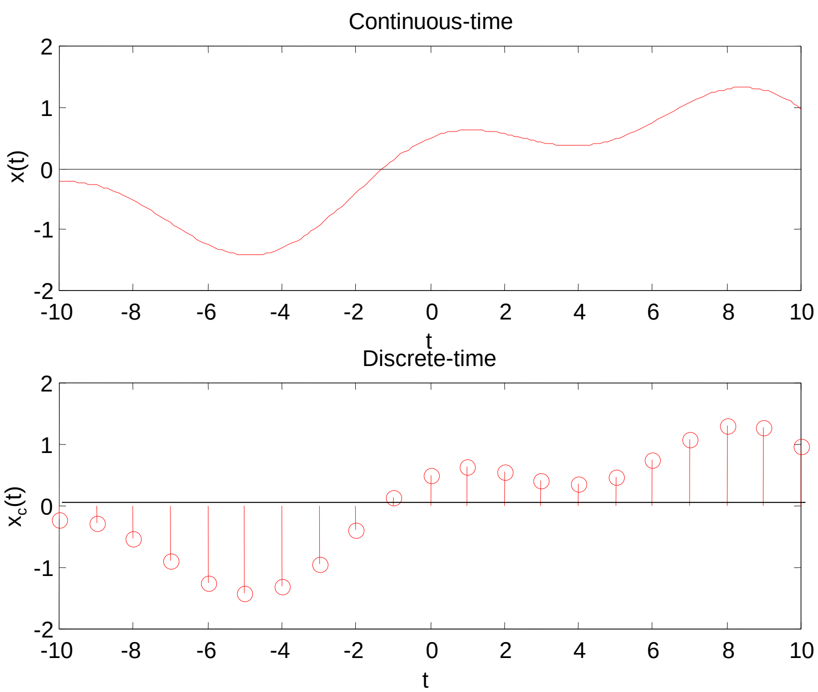
Cut-off frequency = f_c

Mathematical Model of Sampling

- x(t) multiplied by impulse train δT(t)

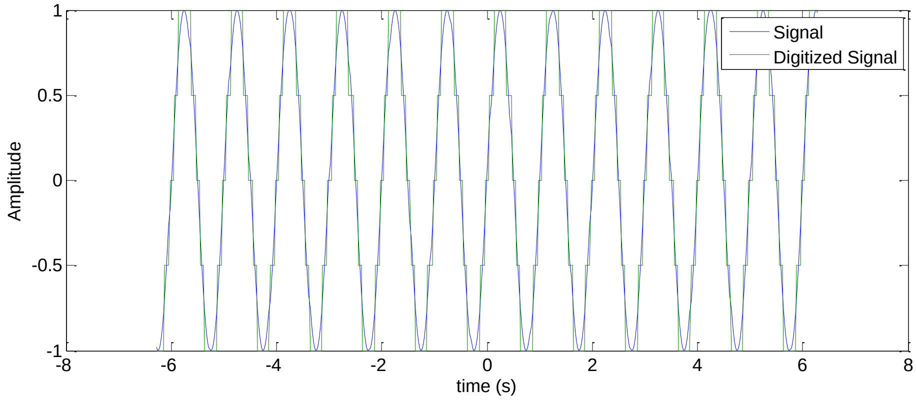
$$
\begin{aligned}
x_c(t) &= x(t)\delta_T(t) \\
&= x(t)\left[\delta(t)+\delta(t-\Delta t)+\delta(t-2\Delta t)+\cdots\right] \\
&= \sum_n x(n\Delta t)\delta(t-n\Delta t)
\end{aligned}
$$

- $x_c(t)$ is a train of impulses of height $x(t)|_{t=n\Delta t}$



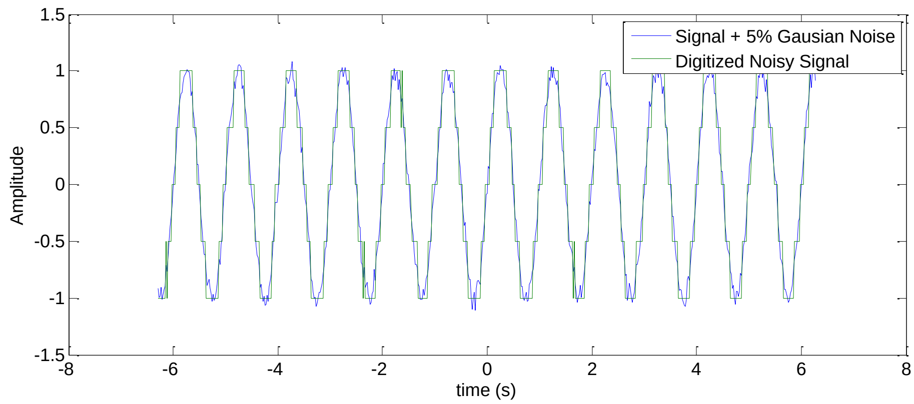
Discrete Time Signal

- Image a signal…



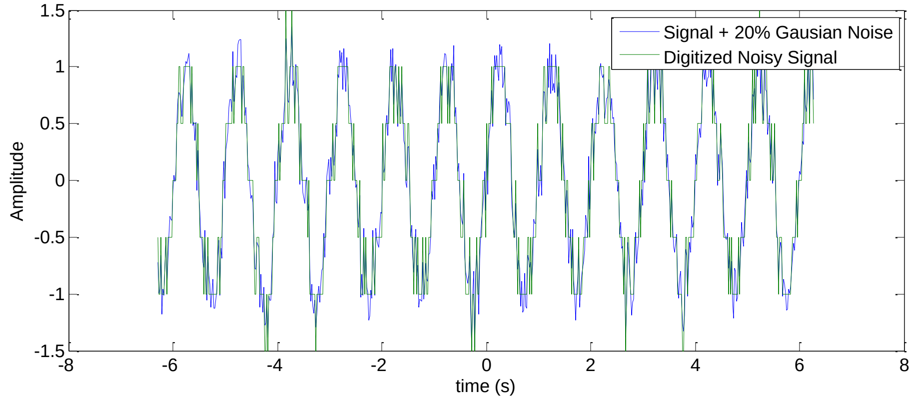
Discrete Time Signals

- Digitization helps beat the Noise!



Discrete Time Signals

- But only so much…



Signal Manipulations

- Shifting

$$y(n) = x(n - n_0)$$

- Reversal

$$y(n) = x(-n)$$

- Time Scaling
  (Down Sampling)

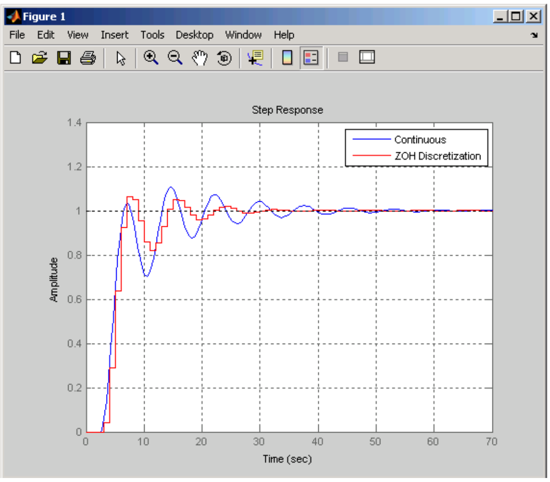
$$y(M) = x(Mn)$$

  (Up Sampling)

$$y(n) = x\left(\frac{n}{N}\right)$$

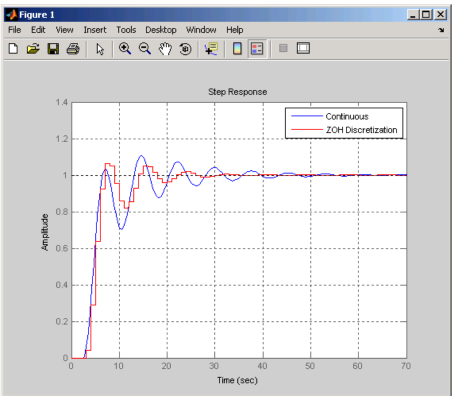
Discrete Time Signals

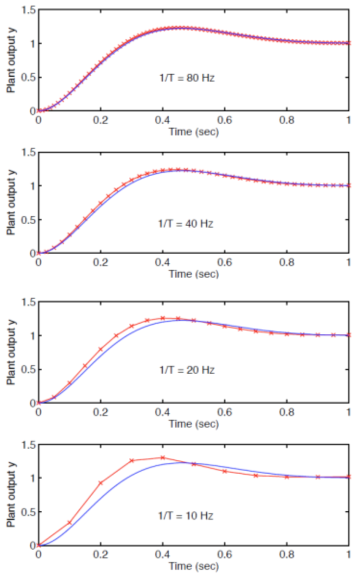
- Can make control tricky!



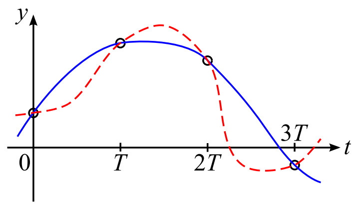
Discrete Time Signals

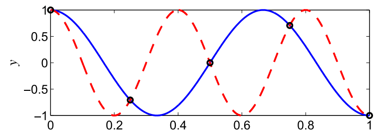
- Can make control tricky!





Nyquist Sampling Theorem and Aliasing





- A signal $y(t)$ is uniquely defined by its samples $y(kT)$ if the sampling frequency is more than twice the bandwidth of $y(t)$.

Sampling Theorem

- The Nyquist criterion states:

To prevent aliasing, a ***bandlimited*** *signal of bandwidth w_B rad/s must be sampled at a rate greater than $2w_B$ rad/s*

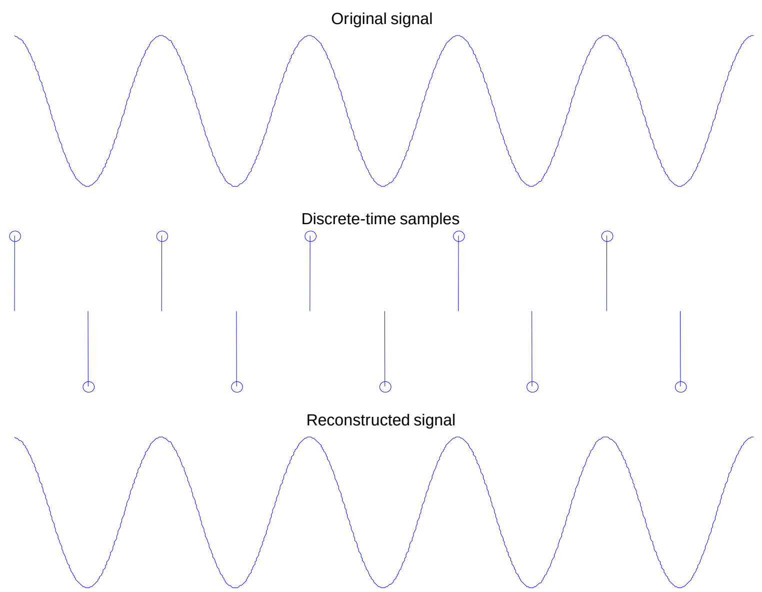
$$-w_s > 2w_B$$

Note: this is a > sign not a ≥

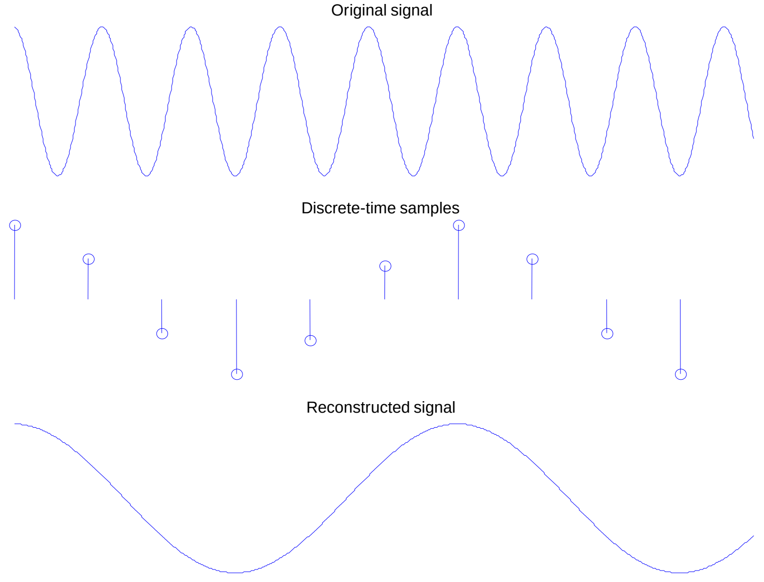
Also note: Most real world signals require band-limiting with a lowpass (anti-aliasing) filter

Time Domain Analysis of Sampling

- Frequency domain analysis of sampling is very useful to understand
  - sampling ($X(w)*\sum \delta(w - 2\pi n/\Delta t)$)
  - reconstruction (lowpass filter removes replicas)
  - aliasing (if $w_s \leq 2w_B$)
- Time domain analysis can also illustrate the concepts
  - sampling a sinewave of increasing frequency
  - sampling images of a rotating wheel

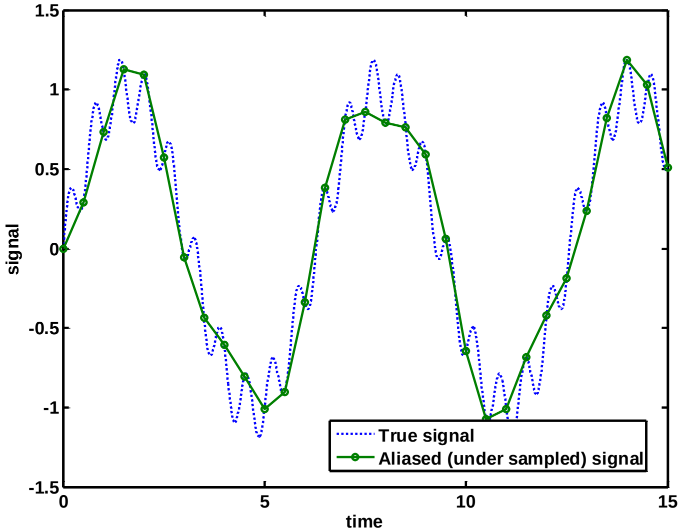


A signal of the original frequency is reconstructed

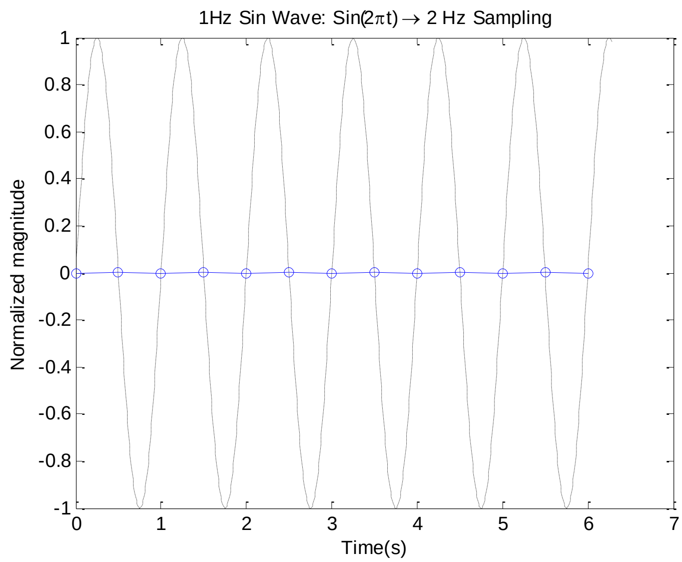


A signal with a reduced frequency is recovered, i.e., the signal is aliased to a lower frequency (we recover a replica)

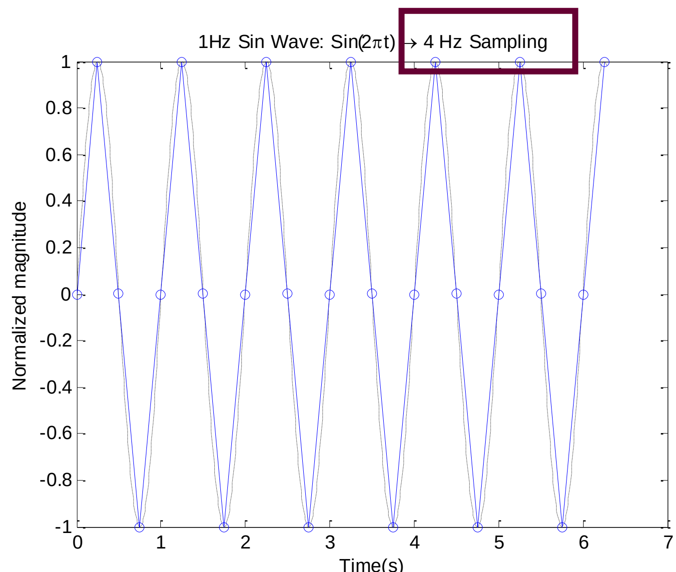
Sampling < Nyquist → Aliasing



Nyquist is not enough …



A little more than Nyquist is not enough …



Frequency Domain Analysis of Sampling

- Consider the case where the DSP performs no filtering operations
  - i.e., only passes xc(t) to the reconstruction filter
- To understand we need to look at the frequency domain
- Sampling: we know
  - multiplication in time ≡ convolution in frequency
  - F{x(t)} = X(w)
  - F{δT(t)} = ∑δ(w - 2πn/Δt),
  - i.e., an impulse train in the frequency domain

Frequency Domain Analysis of Sampling

- In the frequency domain we have

$$X_c(w) = \frac{1}{2\pi}\left(X(w) * \frac{2\pi}{\Delta t}\sum_n \delta\left(w - \frac{2\pi n}{\Delta t}\right)\right)$$
$$= \frac{1}{\Delta t}\sum_n X\left(w - \frac{2\pi n}{\Delta t}\right)$$

Remember convolution with an impulse? Same idea for an impulse train

- Let's look at an example
  - where X(w) is triangular function
  - with maximum frequency w_m rad/s
  - being sampled by an impulse train, of frequency w_s rad/s

Sampling Frequency

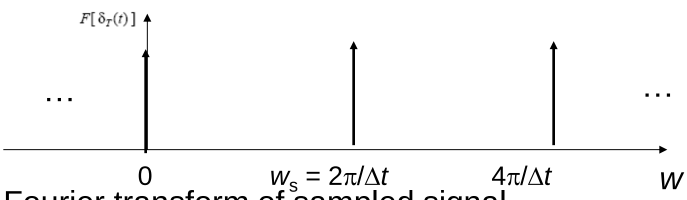
- In this example it was possible to recover the original signal from the discrete-time samples

- But is this always the case?

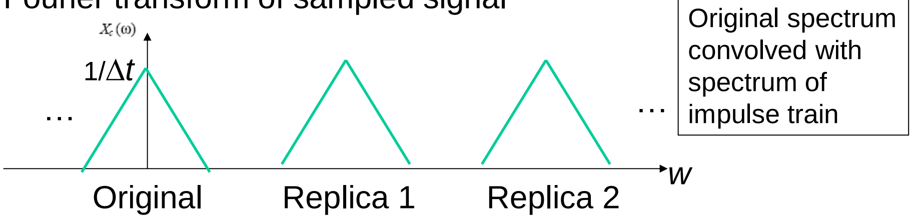
- Consider an example where the sampling frequency w_s is reduced
  - i.e., Δt is increased

Fourier transform of original signal X(ω) (signal spectrum)

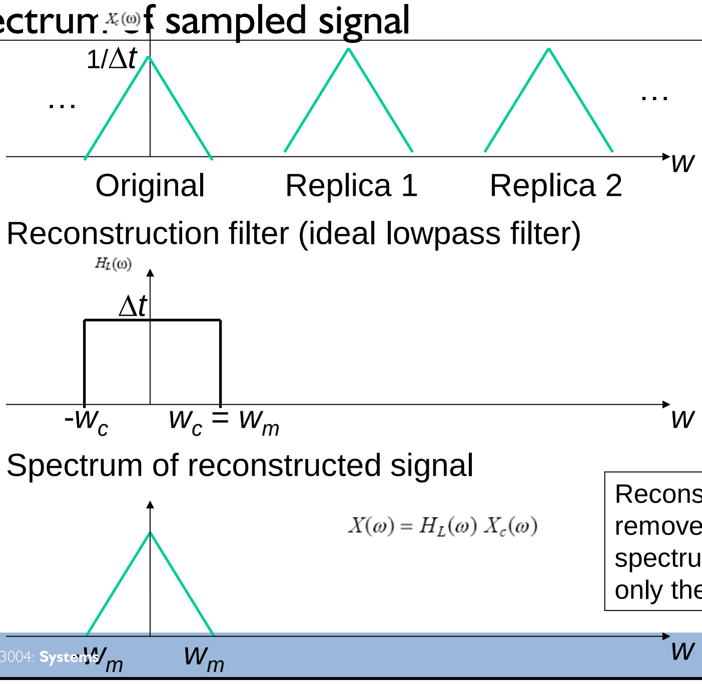
Fourier transform of impulse train $\delta_T(\omega/2\pi)$ (sampling signal)



Fourier transform of sampled signal

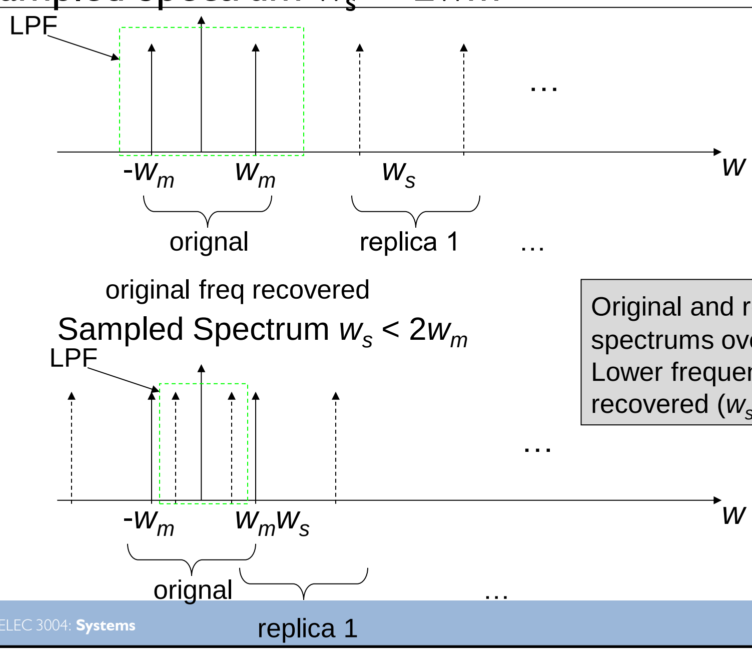


Spectrum of sampled signal



Reconstruction filter removes the replica spectrums & leaves only the original

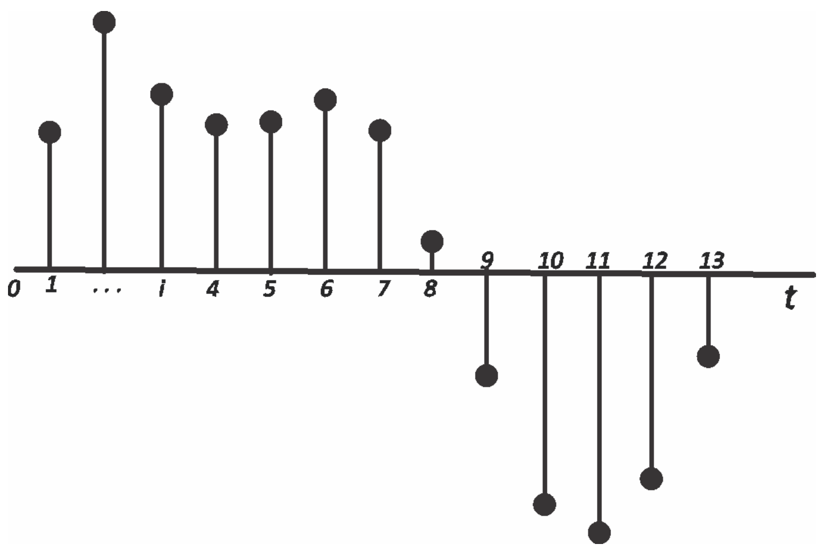
Sampled Spectrum w_s > 2wm



Original and replica spectrums overlap Lower frequency recovered ($w_s - w_m$)

RECONSTRUCTION

Reconstruction



Sampling and Reconstruction Theory and Practice

- Signal is bandlimited to bandwidth WB
  - Problem: real signals are not bandlimited
    - Therefore, require (non-ideal) anti-aliasing filter
- Signal multiplied by ideal impulse train
  - problems: sample pulses have finite width
  - and not ⊗ in practice, but sample & hold circuit
- Samples discrete-time, continuous valued
  - Problem: require discrete values for DSP
    - Therefore, require A/D converter (quantisation)
- Ideal lowpass reconstruction ('sinc' interpolation)
  - problems: ideal lowpass filter not available
    - Therefore, use D/A converter and practical lowpass filter

Time Domain Analysis of Reconstruction

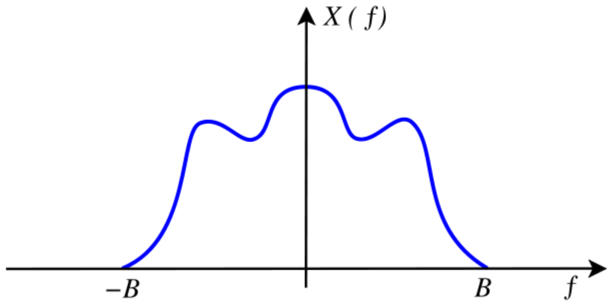
- Frequency domain: multiply by ideal LPF
  - ideal LPF: 'rect' function (gain Δt, cut off w_c)
  - removes replica spectrums, leaves original
- Time domain: this is equivalent to
  - convolution with 'sinc' function
  - as $F^{-1}\{\Delta t \ \text{rect}(w/w_c)\} = \Delta t\, w_c \,\text{sinc}(w_c t/\pi)$
  - i.e., weighted sinc on every sample
- Normally, $w_c = w_s/2$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(n\Delta t)\Delta t w_c \,\text{sinc}\left(\frac{w_c(t-n\Delta t)}{\pi}\right)$$

Reconstruction

- Whittaker–Shannon interpolation formula

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot \text{sinc}\left(\frac{t-nT}{T}\right)$$

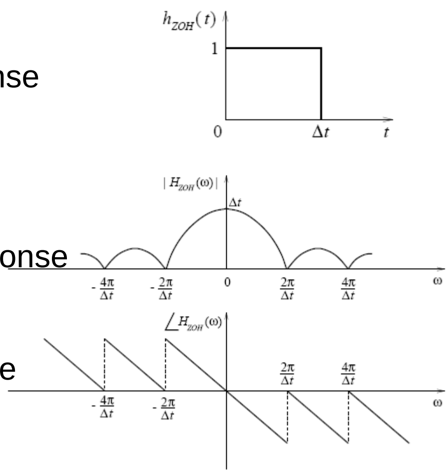


Zero Order Hold (ZOH)

ZOH impulse response

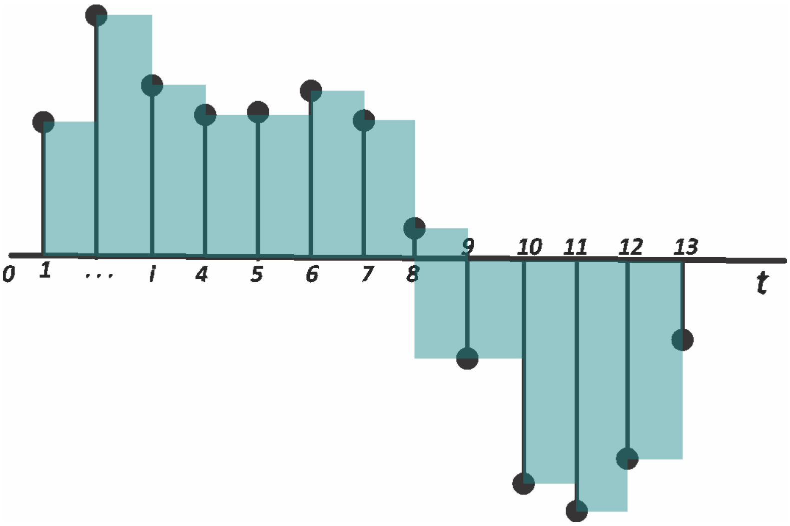
ZOH amplitude response

ZOH phase response



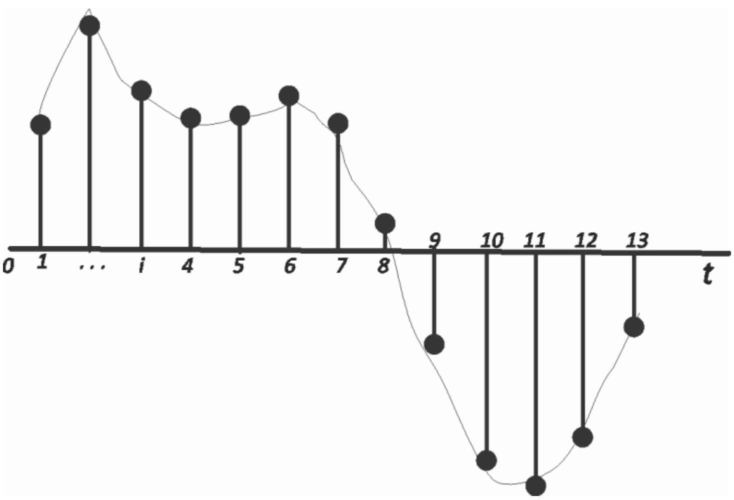
Reconstruction

- Zero-Order Hold [ZOH]

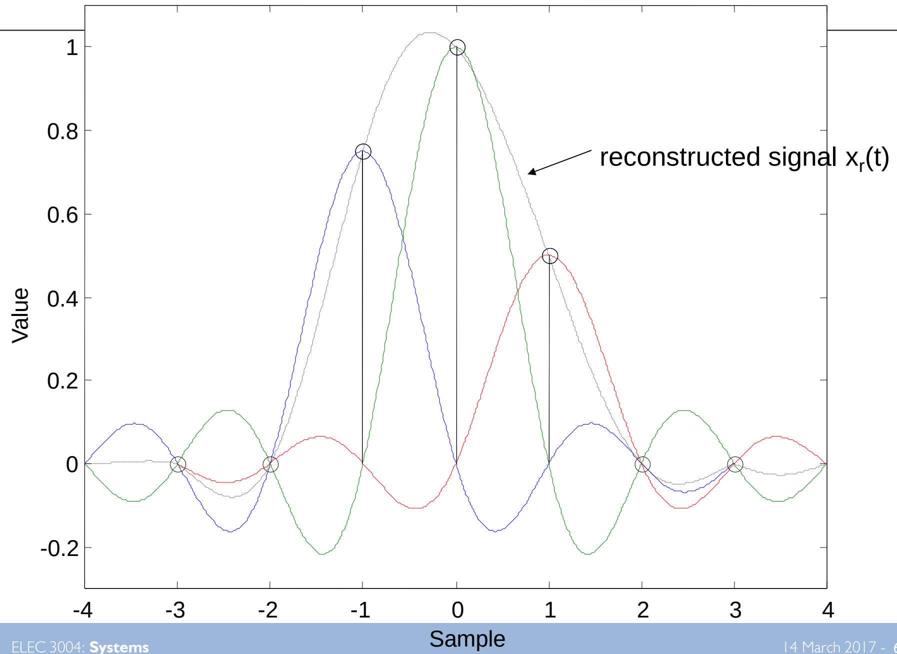


Reconstruction

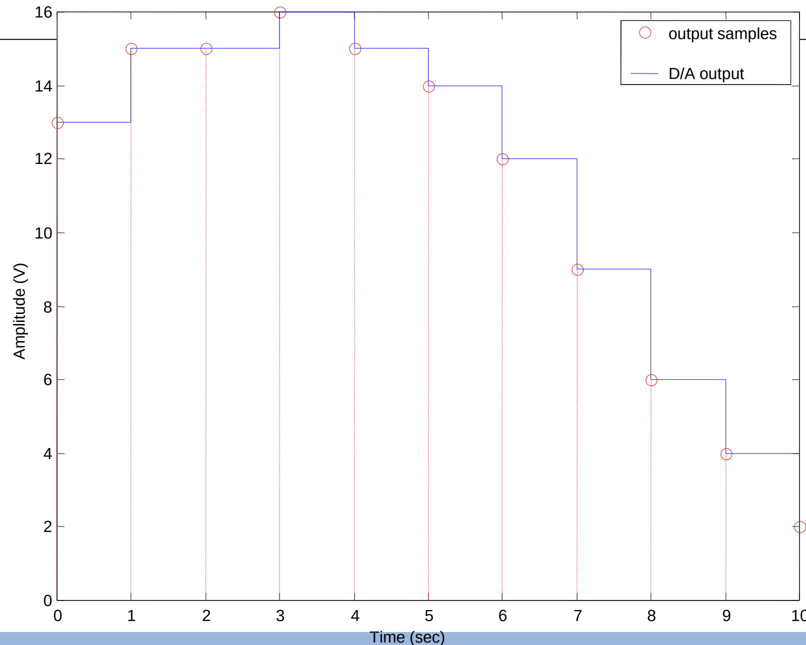
- Whittaker–Shannon interpolation formula



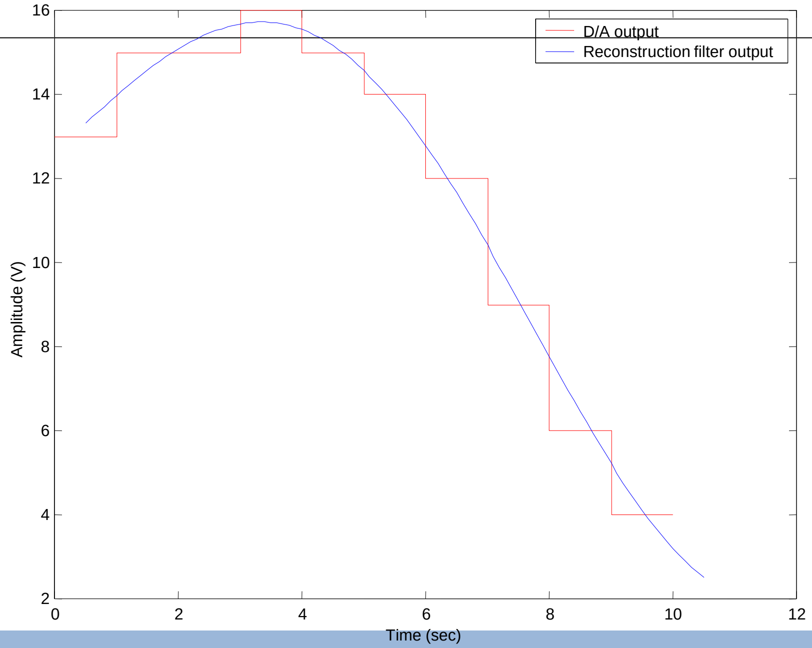
Ideal *sinc* Interpolation of sample values [0 0 0.75 1 0.5 0 0]



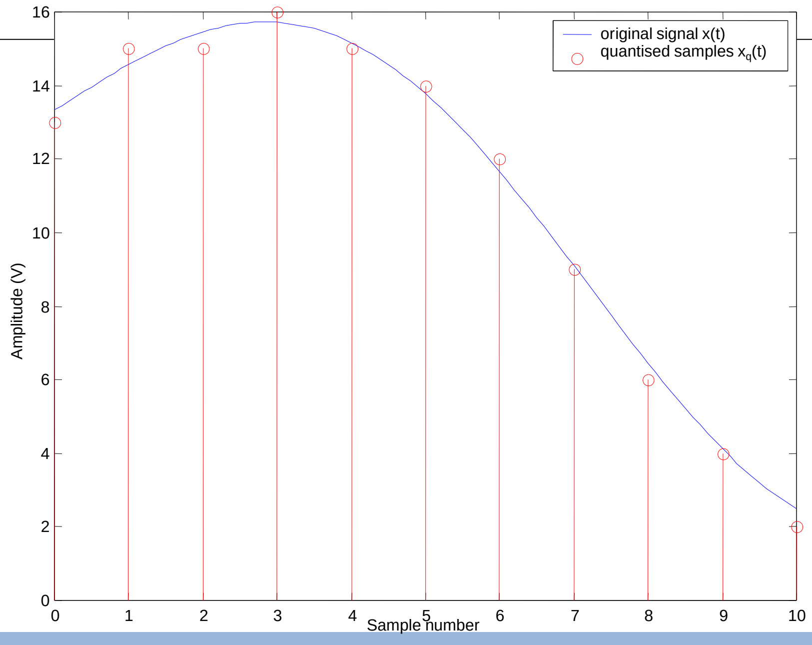
'staircase' output from D/A converter (ZOH)



Smooth output from reconstruction filter



Example: error due to signal quantisation

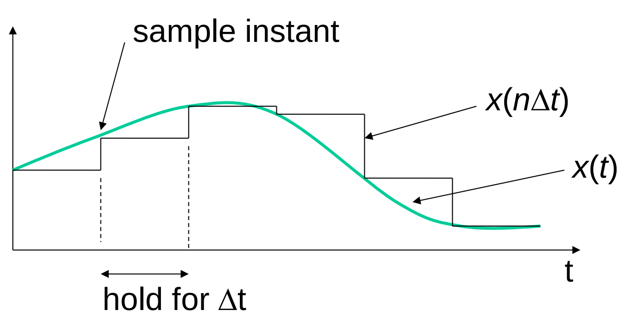


Finite Width Sampling

- Impulse train sampling not realisable
  - sample pulses have finite width (say nanosecs)
- This produces two effects,
- Impulse train has sinc envelope in frequency domain
  - impulse train is square wave with small duty cycle
  - Reduces amplitude of replica spectrums
    - smaller replicas to remove with reconstruction filter ☺
- Averaging of signal during sample time
  - effective low pass filter of original signal
    - can reduce aliasing, but can reduce fidelity ☹
    - negligible with most S/H ☺

Practical Sampling

- Sample and Hold (S/H)
  1. takes a sample every Δt seconds
  2. holds that value constant until next sample
- Produces 'staircase' waveform, $x(n\Delta t)$



Practical Reconstruction

Two stage process:

1. Digital to analogue converter (D/A)
   - zero order hold filter
   - produces 'staircase' analogue output
2. Reconstruction filter
   - non-ideal filter: $w_c = w_s/2$
   - further reduces replica spectrums
   - usually 4^{th} – 6^{th} order e.g., Butterworth
     - for acceptable phase response

D/A Converter

- Analogue output y(t) is
  - convolution of output samples $y(n\Delta t)$ with $h_{ZOH}(t)$

$$y(t) = \sum_n y(n\Delta t) h_{ZOH}(t - n\Delta t)$$

$$h_{ZOH}(t) = \begin{cases} 1, & 0 \le t < \Delta t \\ 0, & \text{otherwise} \end{cases}$$

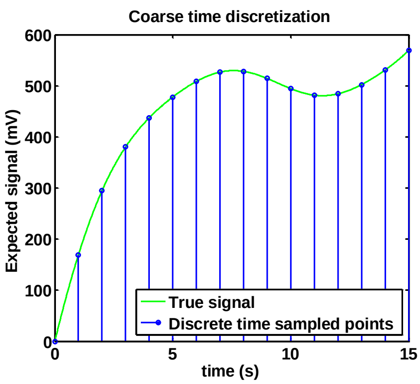
$$H_{ZOH}(w) = \Delta t \exp\left(\frac{-jw\Delta t}{2}\right)\frac{\sin(w\Delta t/2)}{w\Delta t/2}$$

D/A is lowpass filter with sinc type frequency response
It does not completely remove the replica spectrums
Therefore, additional reconstruction filter required

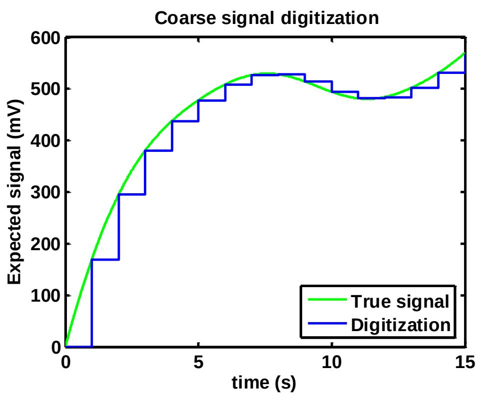
Data Acquisition
(A/D Conversion)

Representation of Signal

- Time Discretization



- Digitization



Quantisation

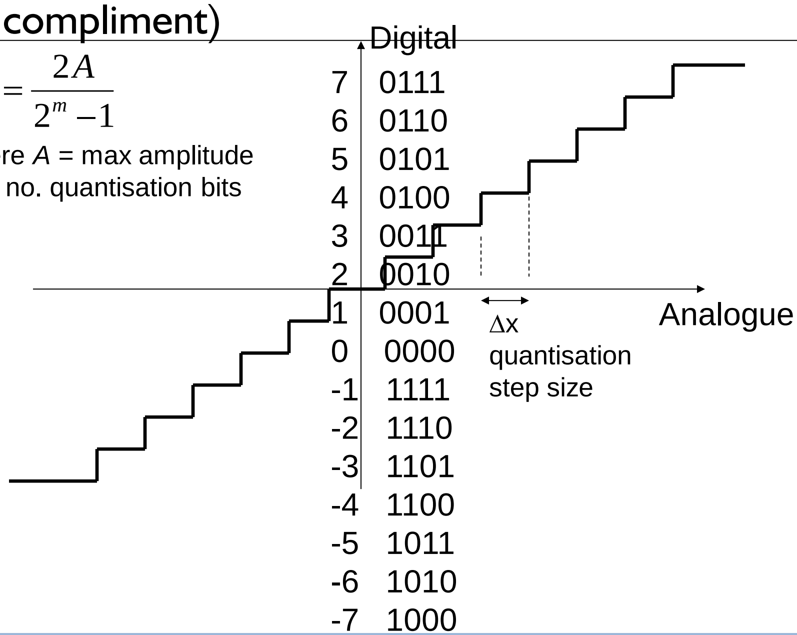
- Analogue to digital converter (A/D)
  - Calculates nearest binary number to $x(n\Delta t)$
    - $x_q[n] = q(x(n\Delta t))$, where q() is non-linear rounding fctn
  - output modeled as $x_q[n] = x(n\Delta t) + e[n]$
- Approximation process
  - therefore, loss of information (unrecoverable)
  - known as 'quantisation noise' ($e[n]$)
  - error reduced as number of bits in A/D increased
    - i.e., Δx, quantisation step size reduces

$$|e[n]| \le \frac{\Delta x}{2}$$

Input-output for 4-bit quantiser (two's compliment)

$$\Delta x = \frac{2A}{2^m - 1}$$

where A = max amplitude
m = no. quantisation bits



Signal to Quantisation Noise

- To estimate SQNR we assume
  - $e[n]$ is uncorrelated to signal and is a
  - uniform random process
- assumptions not always correct!
  - not the only assumptions we could make…
- Also known a 'Dynamic range' (R_D)
  - expressed in decibels (dB)
  - ratio of power of largest signal to smallest (noise)

$$R_D = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

Dynamic Range

Need to estimate:

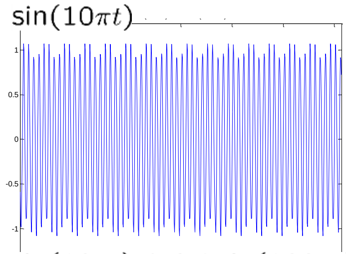
1. Noise power
   - uniform random process: $P_{noise} = \Delta x^2/12$
2. Signal power
   - (at least) two possible assumptions
   1. sinusoidal: $P_{signal} = A^2/2$
   2. zero mean Gaussian process: $P_{signal} = \sigma^2$
      - Note: as $\sigma \approx A/3$: $P_{signal} \approx A^2/9$
      - where σ^2 = variance, A = signal amplitude

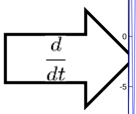
1 extra bit halves Δx
i.e., 20log10(1/2) = 6dB

Regardless of assumptions: R_D increases by 6dB for every bit that is added to the quantiser

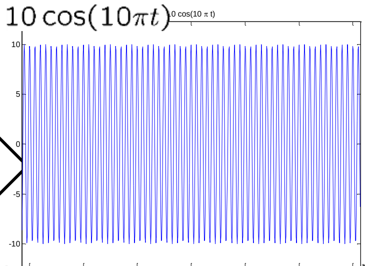
Derivatives magnify noise!

- $\sin(10\pi t)$

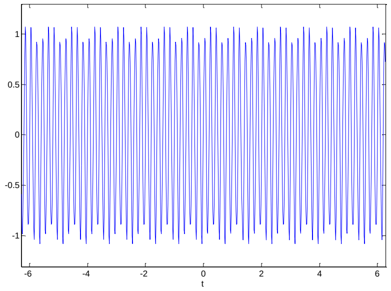




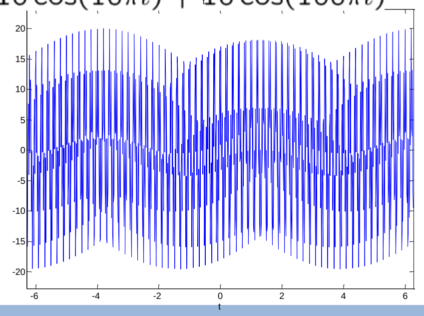
- $10\cos(10\pi t)$



- $\sin(10\pi t) + 0.1\sin(100\pi t)$



- $10\cos(10\pi t) + 10\cos(100\pi t)$



Summary

- Theoretical model of Sampling
  - bandlimited signal (wB)
  - multiplication by ideal impulse train (ws > 2wB)
    - convolution of frequency spectrums (creates replicas)
  - Ideal lowpass filter to remove replica spectrums
    - wc = ws /2
    - Sinc interpolation
- Practical systems
  - Anti-aliasing filter (wc < ws /2)
  - A/D (S/H and quantisation)
  - D/A (ZOH)
  - Reconstruction filter (wc = ws /2)

Don't confuse theory and practice!

Next Time…

- **Aliasing and Anti-Aliasing**

- Review:
  - Chapter 5 of Lathi

- A signal has many signals ☺
  [Unless it's bandlimited]

Alliasing

- Aliasing - through sampling, two entirely different analog sinusoids take on the same "discrete time" identity

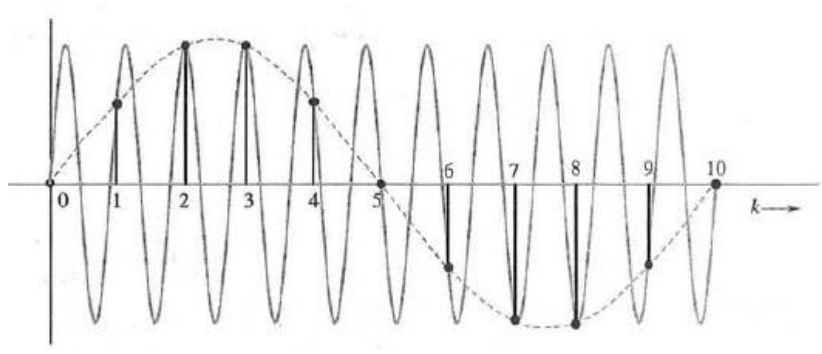
For $f[k]=\cos\Omega k$, $\Omega=\omega T$:

The period has to be less than Fh (highest frequency): $T \le \frac{1}{2\mathcal{F}_h}$

Thus: $0 \le \mathcal{F} \le \frac{\mathcal{F}_s}{2}$

ω_f: aliased frequency: $\omega T = \omega_f T + 2\pi m$

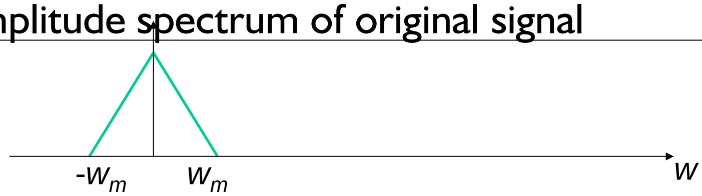
Aliasing: Another view of this



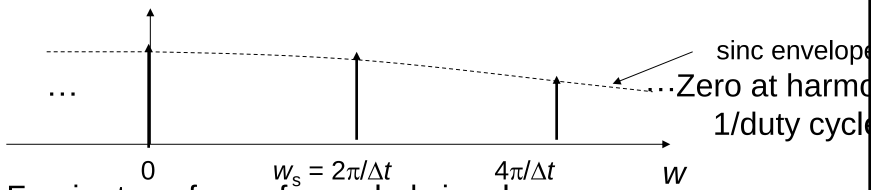
Practical Anti-aliasing Filter

- Non-ideal filter
  - wc = ws /2
- Filter usually 4th – 6th order (e.g., Butterworth)
  - so frequencies > wc may still be present
  - not higher order as phase response gets worse
- Luckily, most real signals
  - are lowpass in nature
    - signal power reduces with increasing frequency
  - e.g., speech naturally bandlimited (say < 8KHz)
  - Natural signals have a (approx) 1/f spectrum
  - so, in practice aliasing is not (usually) a problem

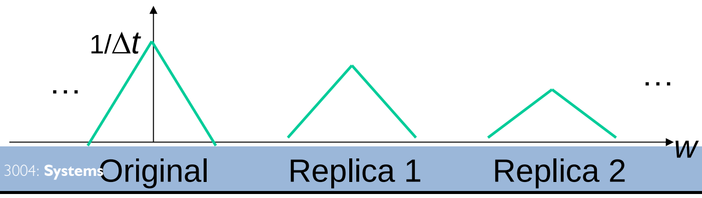
Amplitude spectrum of original signal



Fourier transform of sampling signal (pulses have finite width

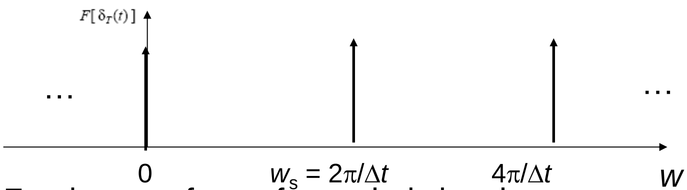


Fourier transform of sampled signal

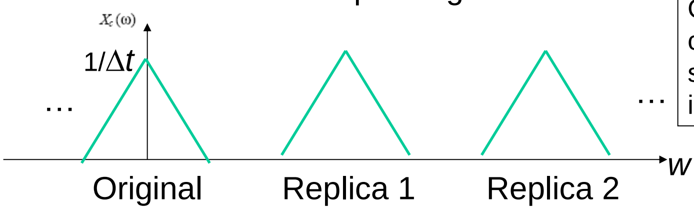


Fourier transform of original signal X(ω) (signal spectrum) **[AGAIN!]**

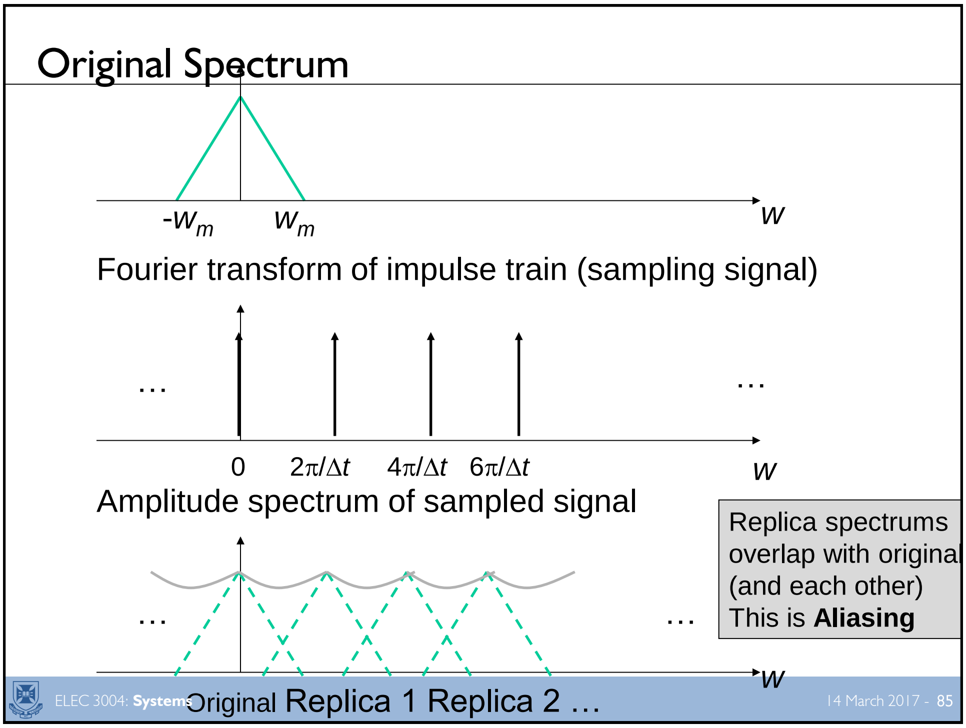
Fourier transform of impulse train $\delta_T(\omega/2\pi)$ (sampling signal)



Fourier transform of sampled signal



Original spectrum convolved with spectrum of impulse train


Original Spectrum
-wm
wm
w
Fourier transform of impulse train (sampling signal)
…
…
0
2π/Δt
4π/Δt
6π/Δt
w
Amplitude spectrum of sampled signal
Replica spectrums overlap with original (and each other)
This is **Aliasing**
…
…
w
Original Replica 1 Replica 2 …
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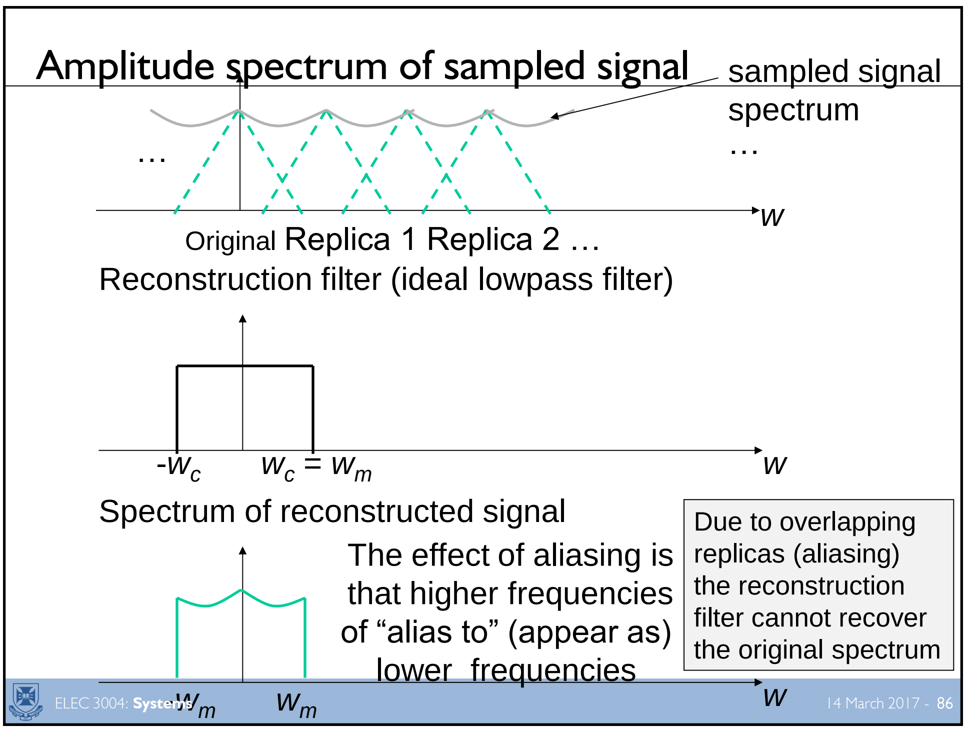
Amplitude spectrum of sampled signal
sampled signal spectrum
…
…
w
Original Replica 1 Replica 2 …
Reconstruction filter (ideal lowpass filter)
-wc
wc = wm
w
Spectrum of reconstructed signal
The effect of aliasing is that higher frequencies of "alias to" (appear as) lower frequencies
Due to overlapping replicas (aliasing) the reconstruction filter cannot recover the original spectrum
-wm
wm
w
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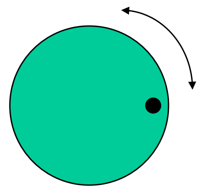
Another way to see Aliasing Too!

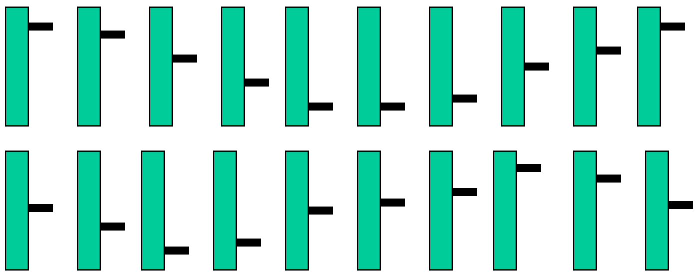
Rotating wheel and peg



Need both top and front view to determine rotation

Top View

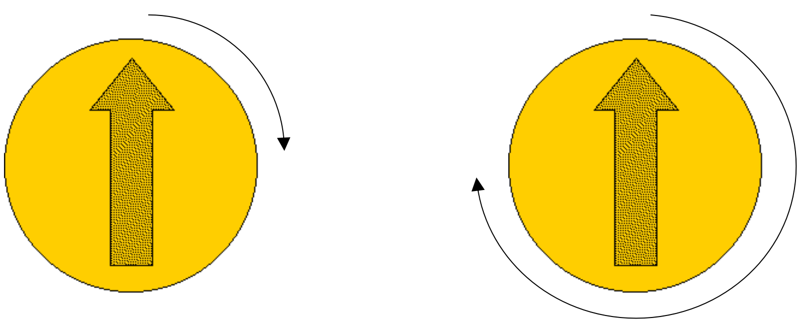
Front View



Temporal Aliasing

90^o clockwise rotation/frame
clockwise rotation perceived

270^o clockwise rotation/frame
(90^o) anticlockwise rotation perceived i.e., aliasing



Require LPF to 'blur' motion