http://elec3004.com

Summary & Course Review

ELEC 3004: **Systems**: Signals & Controls

Dr. Surya Singh

Lecture 24

(with material from many!)

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http://robotics.itee.uq.edu.au/~elec3004/

June 1, 2017

Lecture Schedule:

Week	Date	Lecture Title
1	28-Feb	Introduction
	2-Mar	Systems Overview
2	7-Mar	Systems as Maps & Signals as Vectors
	9-Mar	Systems: Linear Differential Systems
3	14-Mar	Sampling Theory & Data Acquisition
	16-Mar	Aliasing & Antialiasing
4	21-Mar	Discrete Time Analysis & Z-Transform
	23-Mar	Second Order LTID (& Convolution Review)
5	28-Mar	Frequency Response
	30-Mar	Filter Analysis
6	4-Apr	Digital Filters (IIR) & Filter Analysis
	6-Apr	Digital Filter (FIR)
7	11-Apr	Digital Windows
	13-Apr	FFT
	18-Apr	Holiday
	20-Apr	
	25-Apr	
8	27-Apr	Active Filters & Estimation
9	2-May	Introduction to Feedback Control
	4-May	Servoregulation/PID
10	9-May	PID & State-Space
	11-May	State-Space Control
11	16-May	Digital Control Design
	18-May	Stability
12	23-May	State Space Control System Design
	25-May	Shaping the Dynamic Response
13	30-May	System Identification & Information Theory
	1-Jun	Summary and Course Review

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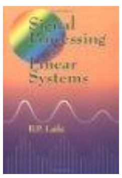
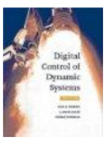
13	1-Jun	Summary and Course Review

Follow Along Reading:



B. P. Lathi
Signal processing and linear systems
1998
TK5102.9.L38 1998



G. Franklin, J. Powell, M. Workman
Digital Control of Dynamic Systems
1990

TJ216.F72 1990
[Available as UQ Ebook]

Today

- Everything in Lectures! ☺
- Lathi:
  - Ch. 5: **Sampling**
  - Ch. 7: Frequency Response and Analog Filters
  - Ch. 8: Discrete-Time Signals and Systems
  - Ch. 12: Frequency Response and Digital Filters
  - Ch. 13: State-Space Analysis
- FPW:
  - Ch. 2: Linear, Discrete, Dynamic-Systems Analysis: The **z-Transform**
  - Ch. 3: Sampled-Data Systems
  - Ch. 4: Discrete Equivalents to Continuous Transfer Functions: The Digital Filter
  - Ch. 5: Design of Digital Control Systems Using Transform Techniques
  - Ch. 6: Design of Digital Control Systems Using State-Space Methods
- Final Exam 2015

Next Time

- Final Exam 2016 ☺

Review Materials at:
http://robotics.itee.uq.edu.au/~elec3004/tutes.html#Final

Final Exam Information

- Date: **Saturday, June/10**
  (remember buses/parking on Saturday schedule)

- **Time:**
  4:30-7:30 pm

- **Location:**
  Connel Gym
  (Bldg. 26 Next to UQ Centre)

- **Parking:**
  Try Conifer Knoll (maybe!)
  (It's harder than the exam! ☺)

- UQ Exams are now "ID Verified"
  ➔ Please remember your ID! 🡐

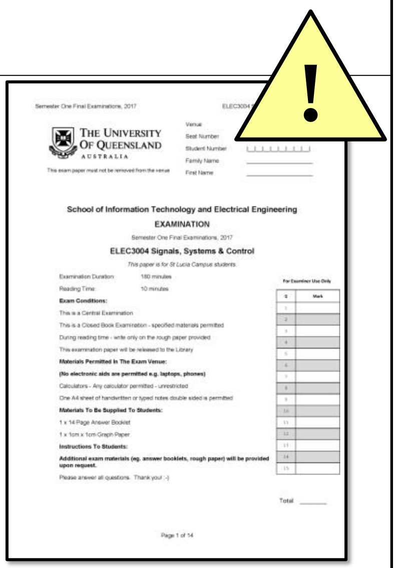
ELEC 3004 Final² Review

Tuesday, June 6

- 4-6 pm
- In: 8-139
  (Tuesday Lecture Spot/Thurs. Lecture Time)

Thursday, June 8

- 9a-12 noon
- In: 50-T203
- EBESS BBQ Afterwards



- Review Notes (Summarized from Course Textbooks)

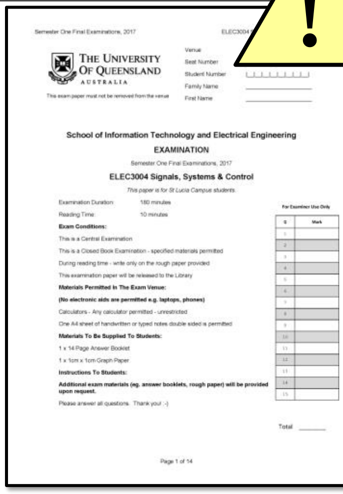
➔ http://robotics.itee.uq.edu.au/~elec3004/tutes.html

Final Exam Information

- Section 1:
  - Digital Linear Dynamical **Systems**
  - 5 Questions
  - 60 Points (33 %)
- Section 2:
  - Digital Processing / Filtering of **Signals**
  - 5 Questions
  - 60 Points (33 %)
- Section 3:
  - Digital & State-Space **Control**
  - 5 Questions
  - 60 Points (33 %)
- Online materials:

➔ Supplied Equation Sheet

➔ Some Review Notes


Semester One Final Examinations, 2017

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

This exam paper must not be removed from the venue

Venue
Seat Number
Student Number
Family Name
First Name

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2017

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

Please answer all questions. Thank you! :-)

For Examiner Use Only

Q	Mark
1	
2	
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14	
15	

Total ______

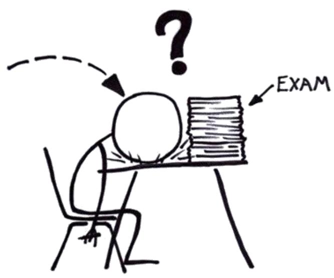
Page 1 of 14

Fun Fact: With this, 45% of the exam is already "public" ☺

Announcements

ELEC 3004 Grading:

- We're working on it!
- You can preview grades by completing peer reviews.



Please don't make this our fate in ~2 weeks!

ELEC 3004: A Review

AKA ELEC 3004:
What do I need to know about *.* ???

PS 1

ELEC 3004 / 7312 – Systems: Signals & Controls **2017**

Problem Set 1: An Introduction to Signals and Systems

Total marks: 100 **Due Date:** March 25, 2017 at 23:59 AEST [end of week 4]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and submitted directly in Platypus (preferred) or as PDF (n.b., Microsoft Word documents and/or scanned images of handwritten pages are specifically disallowed). The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

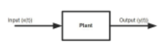
Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. Planting a First Step **[20 points]**

Consider an open loop system with an input signal ($x(t)$), a plant with a given transfer function and an output signal ($y(t)$). In this problem $u(t)$ is the unit step function.



For each pair of input and output systems below
- Plot the input and output signals[1].
- State whether the system is linear, and why.
- State whether the system is causal, and why.

[Please recall that $u(t)$ is the unit step function]

(a) $x(t) = u(t) + 2u(t-2)$
$y(t) = u(t)\left(1 - e^{-t}\right) + u(t-2)\left(2 - 2e^{-(t-2)}\right)$

(b) $x(t) = u(t) - u(t-2)$
$y(t) = u(t)e^{t}\sin(50\pi t) - u(t-2)e^{t-2}sin(50\pi(t-2))$

(c) $x(t) = u(t) + u(t-1)$
$y(t) = u(t)\left(1 - e^{-t}\right) + u(t+1)\left(1 - e^{-(t+1)}\right)$

(d) $x(t) = u(t) + 2u(t-1)$
$y(t) = u(t)\left(1 - e^{-t}\right) + u(t-1)\left(1 - e^{-(t-1)}\right)$

[1] In MATLAB the 'step()' and/or 'heaviside()' functions may be helpful. See also doc step or doc heaviside for more information.

1

PS 2

ELEC 3004 / 7312 – Systems: Signals & Controls **2017**

Problem Set 2: Sampling and Filters (Digital & Analog)

Total marks: 100 **Due Date:** May 2, 2017 at 23:59 AEST [end of week 8++]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and submitted directly in Platypus (preferred) or as PDF (n.b., Microsoft Word documents, scanned images of handwritten pages or items the clearly identify the author are specifically disallowed). The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Finally, the tutors will *not* assist you further unless there is real evidence you have attempted the questions.

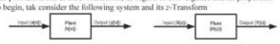
Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. An e-z Transformation into Problem Set 2 **[20 points]**

This problem looks at the z-Transform, its region of convergence and its properties. To begin, tak consider the following system and its z-Transform



(a) Assume $x[n] = a^n u[n]$ and $h[n] = \delta[n] - a\delta[n-1]$.
What are the z-Transforms of $x[n]$, $h[n]$ and $y[n]$
(i.e., what are $X[z]$, $H[z]$, and $Y[z]$)? What is the region of convergence of $Y[z]$?

(b) Derivative Problems
One solution, sometimes creates another problem. Starting with the solution in (a), what is the z-Transform of the derivative of the input state (i.e., $nx[n]$)?

(c) Are z-Transforms Unique?
Consider a new given z-Transform for a signal $X_C[z]$.
Is the inverse ($x_C^1[n]$) signal unique?
Meaning is it possible to have another signal (call it $x_C^2[n]$) that has the same z-Transform, but a different value and/or region of convergence?

(d) DTFT It!
Consider the following z-Transform of a new given signal
$$X_D[z] = \frac{4z + z^{-4}}{1 - 3z^{-5} + 4z^{-8}}$$
Please find the DTFT at the Nyquist Rate ($\Omega = \pi$).
(Assume, if needed, normalized or unit sampling (i.e., $T_s = 1s$))

1

PS 3

ELEC 3004 / 7312 – Systems: Signals & Controls **2017**

Problem Set 3: Digital Feedback Control

Total marks: 75 **Due Date:** Friday, June 2, 2017 at 23:59 AEST [end of week 13]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and submitted directly in Platypus (preferred) or as PDF (n.b., Microsoft Word documents, scanned images of handwritten pages or items the clearly identify the author are specifically disallowed). The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Finally, the tutors will *not* assist you further unless there is real evidence you have attempted the questions.

Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

> For Questions Q1 to Q4: Please answer **3 out of the 4** questions (your choice). If all four questions are submitted, *the tutors will only mark three chosen at random.*
> **Thus, of these, three questions should be answered.**
>
> For Question Q5: This question is optional and is for extra credit only.

Q1. State Space Delivers (Outputs) **[25 points]**

This problem considers the output of various state-space processes. For this please consider a discrete-time LTI system given by:

$$\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \mathbf{D} = [1]$$

where : $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad \mathbf{f}[n] = \mathbf{u}_{input}[n] = \mathbf{u}_{unit\ step}[n]$

(a) Please find the output in the Laplace Domain (i.e., $\mathbf{Y}(s)$)
[hint: $\mathbf{Y}(s) = \mathbf{C\Phi}(s)\mathbf{x}(0) + [\mathbf{C\Phi}(s)\mathbf{B} + \mathbf{D}]\mathbf{F}(s)$]

(b) Please find the output in the Z-Domain (i.e., $\mathbf{Y}[z]$)
[hint: $\mathbf{Y}[z] = \mathbf{C}(\mathbf{I} - z^{-1}\mathbf{A})^{-1}x[0] + \mathbf{H}[z]\mathbf{F}[z]$]

(c) Please find the output in the Discrete-Time Domain (i.e., $y[k]$)
[hint: $\mathbf{y}[k] = \mathbf{CA}^k x[0] + \sum_{j=0}^{k-1} \mathbf{CA}^{k-1-j}\mathbf{Bf}[j] + \mathbf{Df}$]

[**Textbook (Lathi) Reference Note**: This is based on Lathi, Question 13.6-1. The equations in the above hints are Eq. 13.37, 13.103 and 13.94 respectively]

1

To Review:
Back to the **Beginning…Lecture 1 Slide 27**

- Systems
- Signal Abstractions
- Signals as Vectors / Systems as Maps

- Linear Systems and Their Properties
- LTI Systems
- Autonomous Linear Dynamical Systems

- Convolution
- FIR & IIR Systems
- Frequency domain
- Fourier Transform (CT)
- Fourier Transform (DT)

- Even and Odd Signals
- Likelihood
- Causality

- Impulse Response
- Root Locus
- Bode Functions

- Left-hand Plane

- Frequency Response

- Discrete Time
- Continuous Time

- Laplace Transformation
- Feedback and Control
- Additional Applications

- Linear Functions
- Linear Algebra Review
- Least Squares
- Least Squares Problems
- Least Squares Applications
- Matrix Decomposition and Linear Algebra
- Regularized Least Squares

- Least-squares
- Least-squares applications
- Orthonormal sets of vectors
- Eigenvectors and diagonalization
- Linear dynamical systems with inputs and outputs
- Symmetric matrices, quadratic forms, matrix norm, and SVD

- Controllability and state transfer
- Observability and state estimation

- And that, of course,
  Linear Systems are Cool! ☺

Lots of Stuff To Cover…

- Systems
- Signal Abstractions
- Signals as Vectors / Systems as Maps

- Linear Systems and Their Properties
- LTI Systems
- Autonomous Linear Dynamical Systems

- Convolution
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- And that, of course,
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Review

- What do you think when you see?

$$\ddot{y} + 2\dot{y} + 3y = u$$

- System?
- ODE?
- Linear Algebra?
- Joy?
- Excitement?
- Shock and Awe??

Linear algebra provides the tools/foundation for working with (linear) differential equations.

Signals & Systems

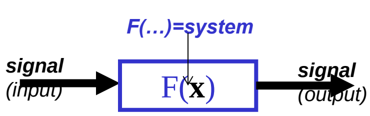
Linear algebra provides the tools/foundation for working with (linear) differential equations.

- Signals are vectors. Systems are matrices.

$$\mathbf{y} = F\mathbf{x} + G\mathbf{u}$$

Linear Systems

Linear Systems in 1-Slide



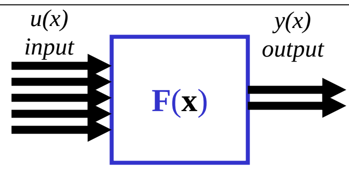
- Signals Are Vectors

- Systems Are Matricies

Linear Systems

- Model describes the relationship between the input **u**(x) and the output **y**(x)
- If it is a <u>Linear</u> System (wk 3):

$$y\left(t\right)=\int_{0}^{t}F\left(t-\tau\right)u\left(\tau\right)d\tau$$



- If it is also a (Linear and) <u>**lumped**</u>, it can be expressed <u>**algebraically**</u> as:

$$\dot{x}\left(t\right)=A\left(t\right)x\left(t\right)+B\left(t\right)u\left(t\right)$$
$$y\left(t\right)=C\left(t\right)x\left(t\right)+D\left(t\right)u(t)$$

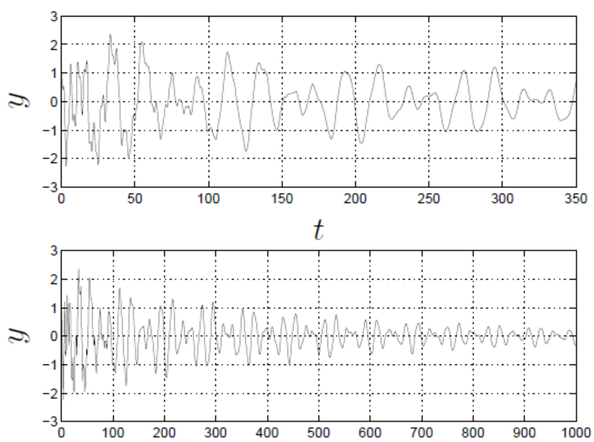
- If it is also (Linear and) <u>**time invariant**</u> the matrices can be reduced to:

$$\dot{x}\left(t\right)=Ax\left(t\right)+Bu\left(t\right)$$
$$y\left(t\right)=Cx\left(t\right)+Du(t)$$

$\mathscr{L}$aplacian: $$y\left(s\right)=F\left(s\right)u\left(s\right)$$

WHY? This can help simplify matters…

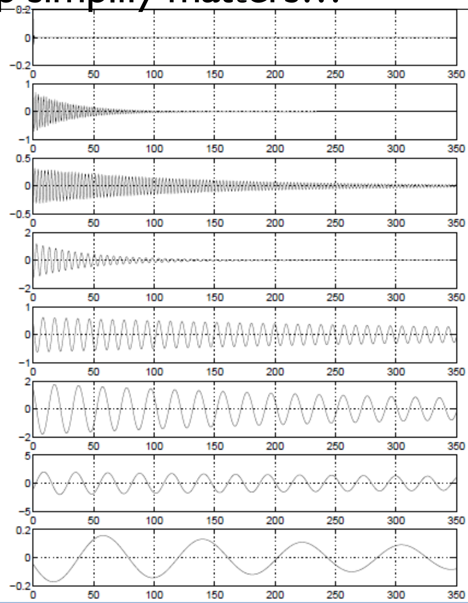
For Example: Consider the following system:



- How to model and predict (and control the output)?

Source: EE263 (s.1-13)

This can help simplify matters…



Source: EE263 (s.1-13)

This can help simplify matters…

- Consider the following system:

$$\dot{x} = Ax, \qquad y = Cx$$

- $x(t) \in \mathbb{R}^8$, $y(t) \in \mathbb{R}^1$ → 8-state, single-output system
- Autonomous: No input yet! ($u(t) = 0$)

Source: EE263 (s.1-13)

System Classifications/Attributes

1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Types of Linear Systems

- LDS:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

To Review:

- Continuous-time linear dynamical system (CT LDS):

$$\frac{dx}{dt} = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)$$

- $t \in \mathbb{R}$ denotes time
- $x(t) \in \mathbb{R}^n$ is the state (vector)
- $u(t) \in \mathbb{R}^m$ is the input or control
- $y(t) \in \mathbb{R}^p$ is the output

Types of Linear Systems

- LDS:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

- $A(t) \in \mathbb{R}^{n\times n}$ is the dynamics matrix
- $B(t) \in \mathbb{R}^{n\times m}$ is the input matrix
- $C(t) \in \mathbb{R}^{p\times n}$ is the output or sensor matrix
- $D(t) \in \mathbb{R}^{p\times m}$ is the feedthrough matrix

➔ state equations, or "*m*-input, *n*-state, *p*-output' LDS

Types of Linear Systems

- LDS:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

- **Time-invariant:** where A(t), B(t), C(t) and D(t) are **constant**
- **Autonomous:** there is no input *u* (B,D are irrelevant)
- **No Feedthrough**: D = 0

- SISO: *u(t)* and *y(t)* are scalars
- MIMO: *u(t)* and *y(t)*: They're vectors: Big Deal ?

Discrete-time Linear Dynamical System

- Discrete-time Linear Dynamical System (DT LDS) has the form:

$$x(t+1) = A(t)x(t) + B(t)u(t), \qquad y(t) = C(t)x(t) + D(t)u(t)$$

- $t \in \mathbb{Z}$ denotes time index : $\mathbb{Z}=\{0, \pm 1, \ldots, \pm \mathbf{n}\}$
- *x(t), u(t), y(t)* ∈ are sequences
- Differentiation handled as difference equation:
  ➔ first-order vector recursion

Discrete Variations & Stability

$$y(s) = F(s)\,u(s)$$

- Is in continuous time …
- To move to discrete time it is more than just "sampling" at: 2 × (biggest Frequency)
- Discrete-Time Exponential

$$F(t) \rightarrow F[kT]$$

$$e^{\frac{k}{T}} = \gamma^k$$

$$\frac{1}{T} = \ln \gamma$$

- SISO to MIMO
  - Single Input, Single Output
  - Multiple Input, Multiple Output
- BIBO:
  - Bounded Input, Bounded Output
- Lyapunov:
  - Conditions for Stability
  - ➔ Are the results of the system asymptotic or exponential

Linear Systems

Linearity:

- **A most desirable property for many systems to possess**
- Ex: Circuit theory, where it allows the powerful technique or voltage or current superposition to be employed.

Two requirements must be met for a system to be *linear:*

- *Additivity*
- *Homogeneity or Scaling*

Additivity **∪** ***Scaling*** **➔ Superposition**

Linear Systems: Additivity

- **Given** input $x_1(t)$ produces output $y_1(t)$ and input $x_2(t)$ produces output $y_2(t)$
- **Then** the input $x_1(t) + x_2(t)$ must produce the output $y_1(t) + y_2(t)$ for arbitrary $x_1(t)$ and $x_2(t)$
- Ex:
  - Resistor
  - Capacitor
- **Not** Ex:
  - $y(t) = \sin[x(t)]$

Linear Systems: Homogeneity or Scaling

- **Given** that $x(t)$ produces $y(t)$

- **Then** the scaled input $\mathrm{a} \cdot x(t)$
  must produce the scaled output $\mathrm{a} \cdot y(t)$
  for an arbitrary $x(t)$ and a

- Ex:
  - $y(t) = 2x(t)$

- **Not** Ex:
  - $y(t) = x^2(t)$
  - $y(t) = 2x(t) + 1$

Linear Systems: Superposition

- **Given** input $x_1(t)$ produces output $y_1(t)$
  and input $x_2(t)$ produces output $y_2(t)$

- **Then**: The linearly combined input
  $$x(t) = ax_1(t) + bx_2(t)$$
  must produce the linearly combined output
  $$y(t) = ay_1(t) + by_2(t)$$
  for arbitrary a and b

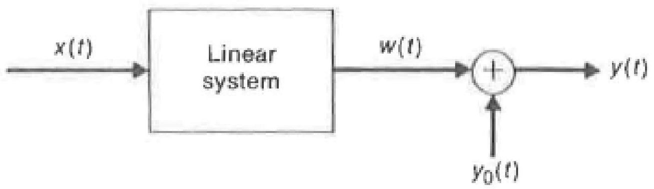
- **Generalizing:**
  - Input: $x(t) = \sum_k a_k x_k(t)$
  - Output: $y(t) = \sum_k a_k y_k(t)$

Linear Systems: Superposition [2]

Consequences:

- Zero input for all time yields a zero output.
  - This follows readily by setting $a = 0$, then $0 \cdot x(t) = 0$

- DC output/Bias ➔ **Incrementally linear**
- Ex: $y(t) = [2x(t)] + [1]$
- Set offset to be added offset [Ex: $y_0(t)$=1]



Dynamical Systems…

- A system with a memory
  - Where past history (or derivative states) are **relevant** in determining the response
- Ex:
  - RC circuit: Dynamical
    - Clearly a function of the "capacitor's past" (initial state) and
    - Time! (charge / discharge)
  - R circuit: is memoryless ∵ the output of the system (recall V=IR) at some time **t** only depends on the input at time **t**

- Lumped/Distributed
  - Lumped: Parameter is constant through the process & can be treated as a "point" in space
- Distributed: System dimensions ≠ small over signal
  - Ex: waveguides, antennas, microwave tubes, etc.

Causality: Looking at this from the output's perspective…

- **Causal** = The output *before* some time *t* does not depend on the input *after* time *t*.

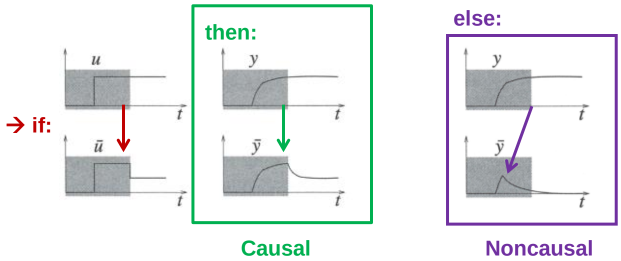
Given: $y(t) = F(u(t))$

For:

$\hat{u}(t) = u(t), \forall 0 \le t < T \text{ or } [0, T)$

Then for a T>0:

$\rightarrow \hat{y}(t) = y(t), \ \forall 0 \le t < T$



Systems with Memory

- A system is said t have *memory* if the output at an arbitrary time $t = t_*$ depends on input values other than, or in addition to, $x(t_*)$

- Ex: Ohm's Law

$$V(t_o) = Ri(t_o)$$

- **Not** Ex: Capacitor

$$V(t_0) = \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$$

Time-Invariant Systems

- **<u>Given</u>** a shift (delay or advance) in the input signal
- **<u>Then/Causes</u>** simply a like shift in the output signal

- If $x(t)$ produces output $y(t)$
- Then $x(t-t_0)$ produces output $y(t-t_0)$

- Ex: Capacitor
- $V(t_0) = \frac{1}{C}\int_{-\infty}^{t} i(\tau - t_0)\, d\tau$

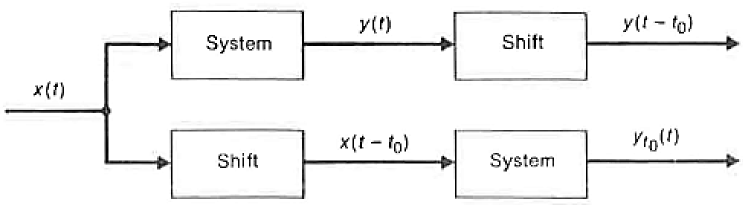
  $= \frac{1}{C}\int_{-\infty}^{t-t_0} i(\tau)\, d\tau$

  $=V(t-t_0)$

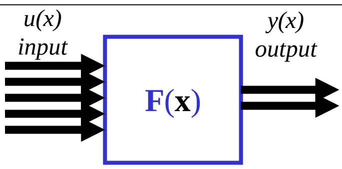
Time-Invariant Systems

- **<u>Given</u>** a shift (delay or advance) in the input signal
- **<u>Then/Causes</u>** simply a like shift in the output signal

- If $x(t)$ produces output $y(t)$
- Then $x(t-t_0)$ produces output $y(t-t_0)$



Recap: Linear Systems



- Model describes the relationship between the input **u**(x) and the output **y**(x)
- If it is a <u>Linear</u> System (wk 3):

$$y(t) = \int_0^t F(t-\tau)\,u(\tau)\,d\tau$$

- If it is also a (Linear and) **<u>lumped</u>**, it can be expressed **<u>algebraically</u>** as:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

- If it is also (Linear and) **<u>time invariant</u>** the matrices can be reduced to:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$\mathscr{L}$aplacian: $$y(s) = F(s)\,u(s)$$

Equivalence Across Domains

Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

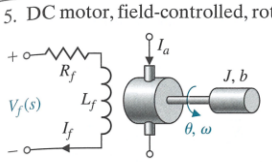
Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2^2$	v_1 = constant
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2^2$	ω_1 = constant
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}^2$	
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	$\mathcal{T}_1$ = constant
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}^2$	
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}^2$	
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 74

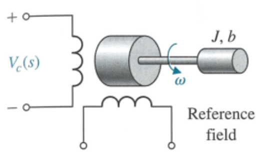
Motors

5. DC motor, field-controlled, rotational actuator



$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s(Js + b)(L_fs + R_f)}$$

7. AC motor, two-phase control field, rotational actuator



$$\frac{\theta(s)}{V_c(s)} = \frac{K_m}{s(\tau s + 1)}$$

$$\tau = J/(b - m)$$

m = slope of linearized torque-speed curve (normally negative)

First Order Systems

First order systems

$$ay' + by = 0 \quad \text{(with } a \neq 0\text{)}$$

righthand side is zero:

- called *autonomous system*
- solution is called *natural* or *unforced response*

can be expressed as

$$Ty' + y = 0 \quad \text{or} \quad y' + ry = 0$$

where

- $T = a/b$ is a *time* (units: seconds)
- $r = b/a = 1/T$ is a *rate* (units: $1/$sec)

First Order Systems

Solution by Laplace transform

take Laplace transform of $Ty' + y = 0$ to get

$$T(\underbrace{sY(s) - y(0)}_{\mathcal{L}(y')}) + Y(s) = 0$$

solve for $Y(s)$ (algebra!)

$$Y(s) = \frac{Ty(0)}{sT + 1} = \frac{y(0)}{s + 1/T}$$

and so $y(t) = y(0)e^{-t/T}$

First Order Systems

solution of $Ty' + y = 0$: $y(t) = y(0)e^{-t/T}$

if $T > 0$, y *decays* exponentially

- T gives time to decay by $e^{-1} \approx 0.37$
- $0.693T$ gives time to decay by half ($0.693 = \log 2$)
- $4.6T$ gives time to decay by 0.01 ($4.6 = \log 100$)

if $T < 0$, y *grows* exponentially

- $|T|$ gives time to grow by $e \approx 2.72$;
- $0.693|T|$ gives time to double
- $4.6|T|$ gives time to grow by 100

Second Order Systems

Second order systems

$$ay'' + by' + cy = 0$$

assume $a > 0$ (otherwise multiply equation by -1)

solution by Laplace transform:

$$a(\underbrace{s^2Y(s) - sy(0) - y'(0)}_{\mathcal{L}(y'')}) + b(\underbrace{sY(s) - y(0)}_{\mathcal{L}(y')}) + cY(s) = 0$$

solve for Y (just algebra!)

$$Y(s) = \frac{asy(0) + ay'(0) + by(0)}{as^2 + bs + c} = \frac{\alpha s + \beta}{as^2 + bs + c}$$

where $\alpha = ay(0)$ and $\beta = ay'(0) + by(0)$

Second Order Systems

so solution of $ay'' + by' + cy = 0$ is

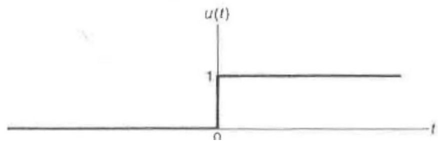
$$y(t) = \mathcal{L}^{-1}\left(\frac{\alpha s + \beta}{as^2 + bs + c}\right)$$

- $\chi(s) = as^2 + bs + c$ is called *characteristic polynomial* of the system
- form of $y = \mathcal{L}^{-1}(Y)$ depends on roots of characteristic polynomial χ
- coefficients of numerator $\alpha s + \beta$ come from initial conditions

Signal Terminology

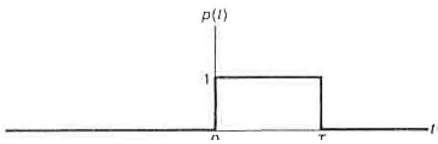
Unit Step Function

- $u(t) = \begin{cases} 0, t < 0 \\ 1, t > 0 \end{cases}$



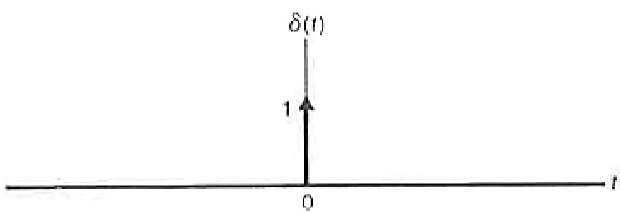
"Rectangular Pulse"

- $p(t) = u(t) - u(t - T)$



Unit-Impulse Function

1. $\delta(t) = 0$ for $t \neq 0$.
2. $\delta(t)$ undefined for $t = 0$.
3. $\int_{t_1}^{t_2} \delta(t)\,dt = \begin{cases} 1, & \text{if } t_1 < 0 < t_2 \\ 0, & \text{otherwise.} \end{cases}$



Break ☺

Is it Useful?

Yes.

(For example … Next Year – ELEC/METR 4810)

It Can Rock Your Boat Gently Down The Stream: IMU Deaduced Reckoning (Navigation)

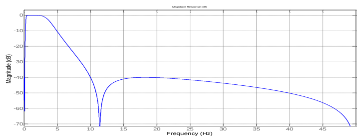
Idea: Integrate your motion (twice for $\ddot{x} \rightarrow x$ and once for $\dot{\theta} \rightarrow \theta$)

Problem:

- (DC) bias in accelerometer → <u>drift</u>

Solution:

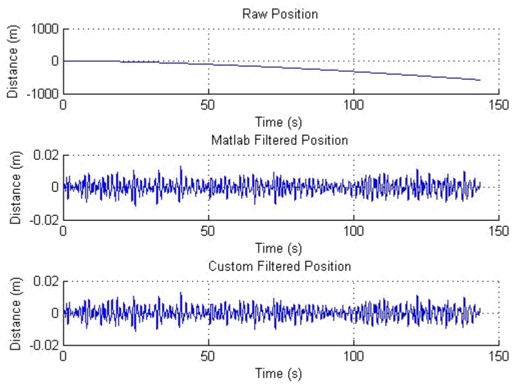
- IIR Bandpass filter (0.1-10 Hz)



```
A = [1.0000   -5.8235   14.1332  -18.2966   13.3259   -5.1772    0.8382];
B = [0.9155   -5.4933   13.7331  -18.3109   13.7331   -5.4933    0.9155];
```


It Can Rock Your Boat Gently Down The Stream: IMU Deaduced Reckoning (Navigation) [2]

Solution:



Today's Lecture is Brought To You By the Number 5

SECATs: Let's look back at the topic list from Lecture 1

The course is has a huge mandate:

- It is really 3 · ½ courses in one !
  - Linear Systems
  - Signal Processing
  - Controls & Digital Controls

- ∴ It is **b r o a d !!**

- There is a logic to it
  - They share the same mathematical nature (poles & zeros)
  - The math is common to more than just circuits!

Lots of Stuff To Cover…

- Systems
- Signal Abstractions
- Signals as Vectors / Systems as Maps

- Linear Systems and Their Properties
- LTI Systems
- Autonomous Linear Dynamical Systems

- Convolution
- FIR & IIR Systems
- Frequency domain
- Fourier Transform (CT)
- Fast Fourier Transform (DT)

- Even and Odd Signals
- Likelihood
- Causality

- Impulse Response
- Root Locus
- Bode Functions

- Left-hand Plane

- Frequency Response

- Discrete Time
- Continuous Time

- Laplace Transformation
- Feedback and Control
- Additional Applications

- Linear Functions
- Linear Algebra Review
- Least Squares
- Least Squares Problems
- Least Squares Applications
- Matrix Decomposition and Linear Algebra
- Regularized Least Squares

- Least-squares
- Least-squares applications
- Orthonormal sets of vectors
- Eigenvectors and diagonalization
- Linear dynamical systems with inputs and outputs
- Symmetric matrices, quadratic forms, matrix norm, and SVD

- Controllability and state transfer
- Observability and state estimation

- And that, of course,
  Linear Systems are Cool! ☺

Yes, this is a Theoretical Approach! Why?

- Theory wins because the importance of any one application seems limited

- Breath
  - Books, books, everywhere, yet we're all on Wikipedia!!

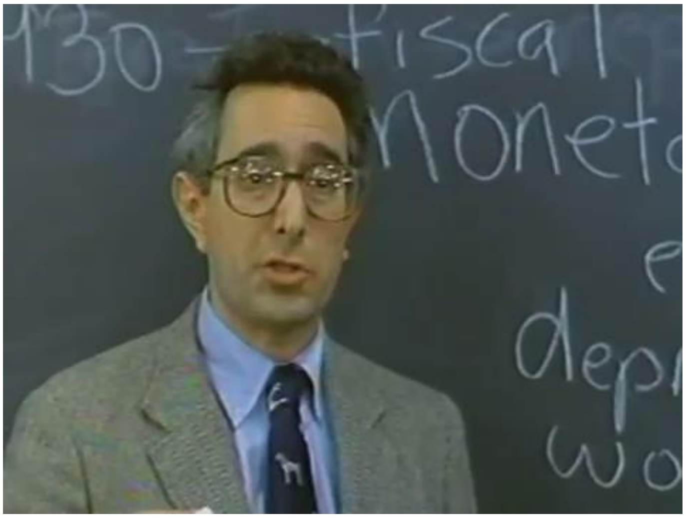
- Assumptions:
  - Numerous conditions that need to be remembered

- Tacit Details:
  →The need for examples (but these are few and always seem the same)

- Time consuming

“4” Is Average

- What is a 3?



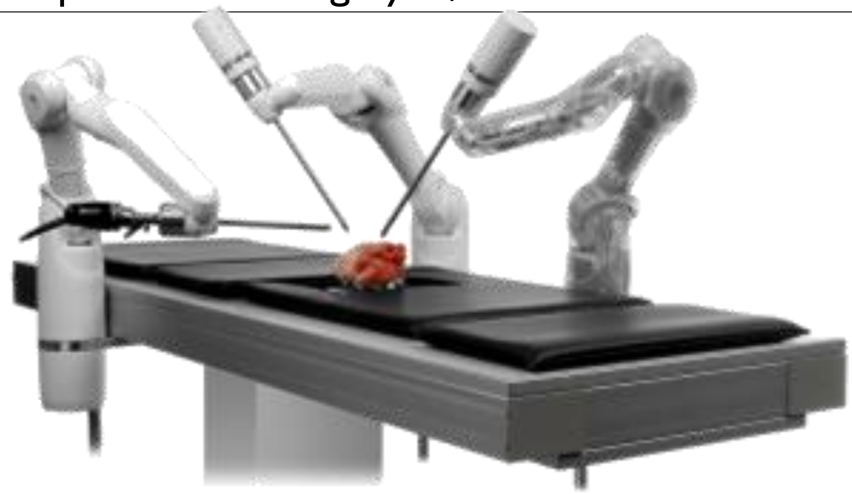
SECaTs: Some Lessons in the Works for Next Year

- I shall only use my own slides
- Less is more!
  - Smaller assignments
  - More time for Examples
- Better organization
  - Better tutorials
  - More examples!!
  - I get that. But, we’ve come a long way

➔ To make this happen I need your support!

Now, What's Next?

Computer Aided Surgery: R/C Toolholders?

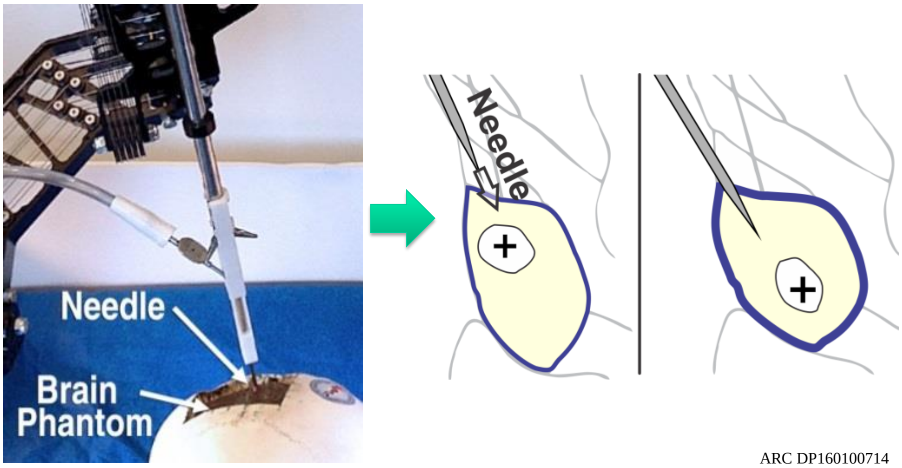


➔ Move in tandem with heart: Cardiac procedures without stopping it

- Unstructured environment (patient) makes this harder

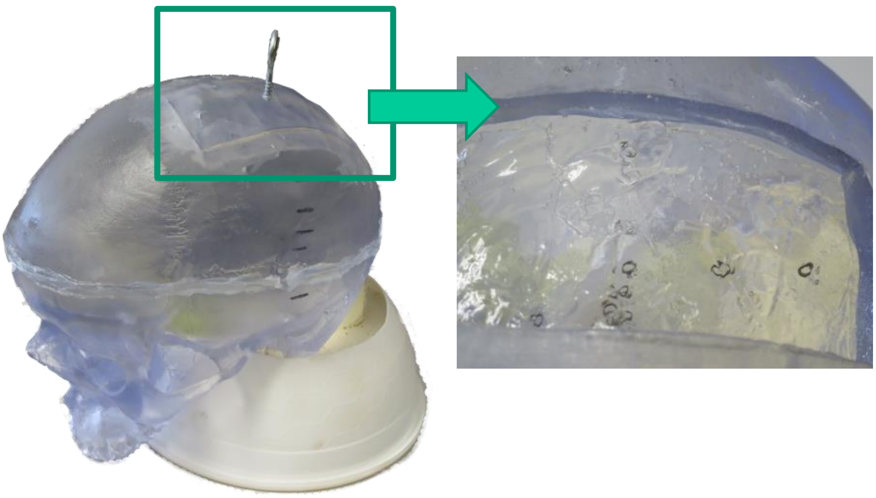
Modern (Tele)Surgical Robotics:

- **Biomechanics approach**: Predict expected tissue trajectories
- (Stochastic) **Robot Motion Planning / Control Methods**!

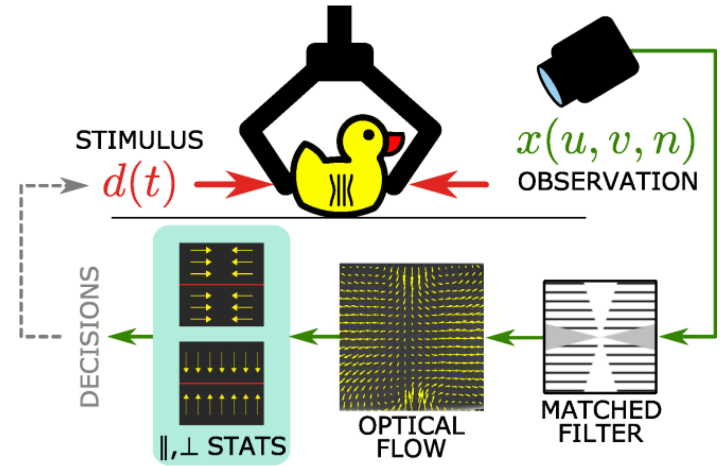


ARC DP160100714

Computer Aided Surgery: "**Soft**" is "**Hard**"!

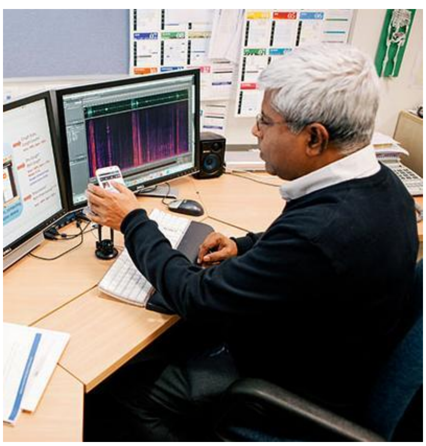


Research: Incorporating Stiffness (Haptics): Visual Deformable Object Analysis



Dansereau, Singh, Leitner, *ICRA 2016*

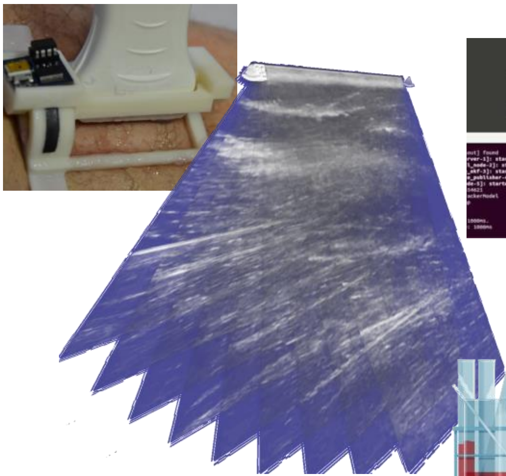
Iceberg to Titanic: Take Advantage of Information



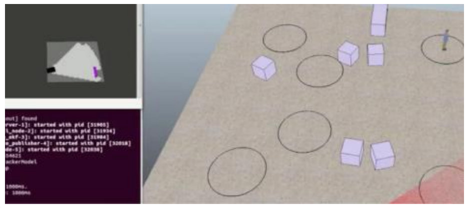
- 30 Min/Day Talking on Phone
  - 5.5 days/year of audio samples
  - Track this (notably the pauses) over time to detect onset of dimentia
- 150 Photos/Month

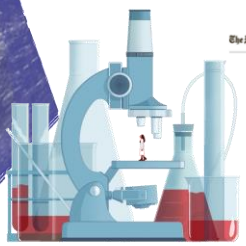
How?

- More Signals



- Stochastic Processing (Think **TAPIR!**)





The New York Times | http://nyti.ms/22UymHG

SundayReview | OPINION

The Tampon of the Future

By PAGAN KENNEDY APRIL 1, 2016

IS it possible to extract blood from people without causing pain? For decades, this problem has stumped the medical industry. In an effort to replace the old-fashioned needle, companies are trying to deploy laser beams and tiny vacuums to draw blood.

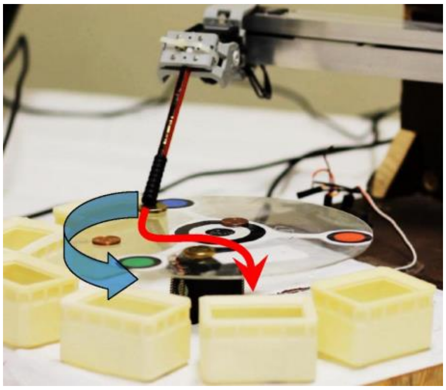
In 2014, an engineer at Harvard named Ridhi Tariyal hit on a far simpler workaround. "I was trying to develop a way for women to monitor their own fertility at home," she told me, and "those kinds of diagnostic tests require a lot of blood. So I was thinking about women and blood. When you put those words together, it becomes obvious. We have an opportunity every single month to collect blood from women, without needles."

Together with her business partner, Stephen Gire, she has patented a method

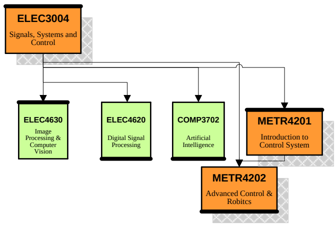
Robotics & Health: A Friendly Touch!

What's Next?

- METR 4202:



- ELEC/METR Programs

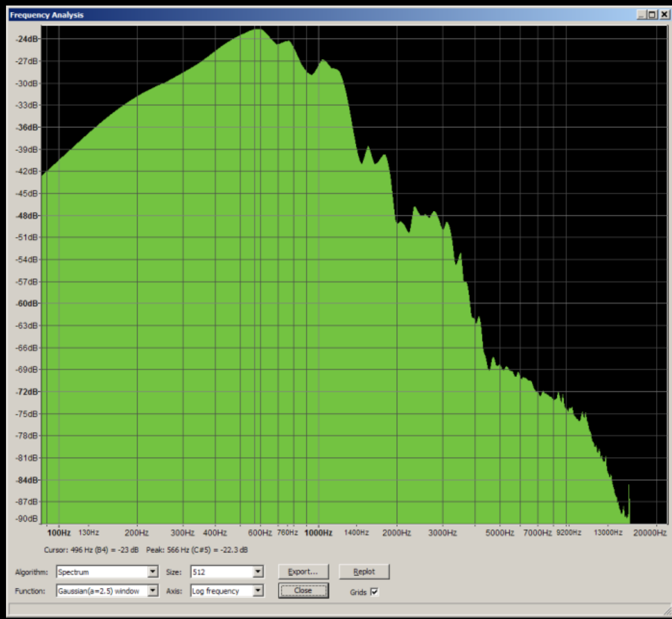


What is … a **Signal?**

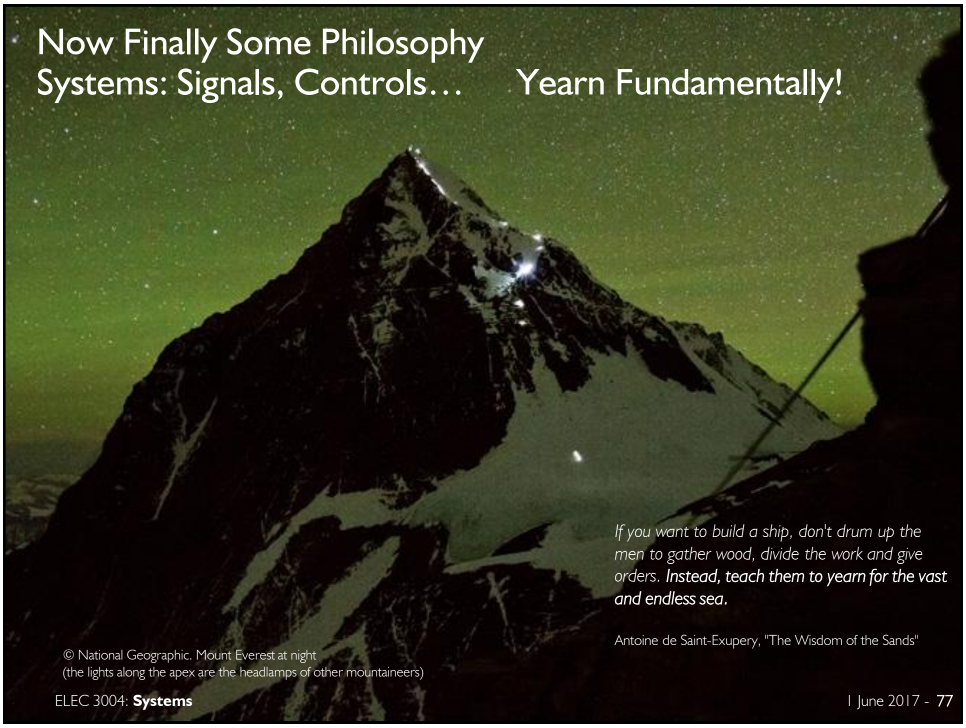
What is … a **System**?

ELEC 3004: **Systems** 1 June 2017 - 73

ELEC 3004: **Systems** 1 June 2017 - 74


Frequency Analysis
-24dB
-27dB
-30dB
-33dB
-36dB
-39dB
-42dB
-45dB
-48dB
-51dB
-54dB
-57dB
-60dB
-63dB
-66dB
-69dB
-72dB
-75dB
-78dB
-81dB
-84dB
-87dB
-90dB
100Hz 130Hz 200Hz 300Hz 400Hz 600Hz 760Hz 1000Hz 1400Hz 2000Hz 3000Hz 5000Hz 7000Hz 9200Hz 13000Hz 20000Hz
Cursor: 496 Hz (B4) = -23 dB Peak: 566 Hz (C#5) = -22.3 dB
Algorithm: Spectrum
Size: 512
Export...
Replot
Function: Gaussian(a=2.5) window
Axis: Log frequency
Close
Grids


101
rit.
I
II
rit.
attacca



Next Time…

- **There is no next time! ☺**

Week	Date	Lecture Title
1	28-Feb	Introduction
	2-Mar	Systems Overview
2	7-Mar	Systems as Maps & Signals as Vectors
	9-Mar	Systems: Linear Differential Systems
3	14-Mar	Sampling Theory & Data Acquisition
	16-Mar	Aliasing & Antialiasing
4	21-Mar	Discrete Time Analysis & Z-Transform
	23-Mar	Second Order LTID (& Convolution Review)
5	28-Mar	Frequency Response
	30-Mar	Filter Analysis
6	4-Apr	Digital Filters (IIR) & Filter Analysis
	6-Apr	Digital Filter (FIR)
7	11-Apr	Digital Windows
	13-Apr	FFT
	18-Apr	**Holiday**
	20-Apr	
	25-Apr	
8	27-Apr	Active Filters & Estimation
9	2-May	Introduction to Feedback Control
	4-May	Servoregulation/PID
10	9-May	PID & State-Space
	11-May	State-Space Control
11	16-May	Digital Control Design
	18-May	Stability
12	23-May	State Space Control System Design
	25-May	Shaping the Dynamic Response
13	30-May	System Identification & Information Theory
	1-Jun	Summary and Course Review

- We're at the End. It's (the) final!
- Thank you folks!

From Brisbane to Perth,
May the Wonder of Linear Dynamical Systems Take You Places & Bring Mirth ☺