http://elec3004.com

Systems Overview

ELEC 3004: **Systems**: Signals & Controls

Dr. Surya Singh

Lecture 2

elec3004@itee.uq.edu.au

http://robotics.itee.uq.edu.au/~elec3004/

March 2, 2017

Lecture Schedule:

Week	Date	Lecture Title
1	28-Feb	Introduction
	2-Mar	Systems Overview
2	7-Mar	Systems as Maps & Signals as Vectors
	9-Mar	Data Acquisition & Sampling
3	14-Mar	Sampling Theory
	16-Mar	Antialiasing Filters
4	21-Mar	Discrete System Analysis
	23-Mar	Convolution Review
5	28-Mar	Frequency Response
	30-Mar	Filter Analysis
5	4-Apr	Digital Filters (IIR)
	6-Apr	Digital Windows
6	11-Apr	Digital Filter (FIR)
	13-Apr	FFT
	18-Apr	**Holiday**
	20-Apr	
	25-Apr	
7	27-Apr	Active Filters & Estimation
8	2-May	Introduction to Feedback Control
	4-May	Servoregulation/PID
10	9-May	Introduction to (Digital) Control
	11-May	Digitial Control
11	16-May	Digital Control Design
	18-May	Stability
12	23-May	Digital Control Systems: Shaping the Dynamic Response
	25-May	Applications in Industry
13	30-May	System Identification & Information Theory
	1-Jun	Summary and Course Review

Prere-***quiz***-ite Solutions ☺

Q1: Complex Solutions to Real Problems

Can an ODE with only real constant coefficients have a complex solution?

- Yes, because the coefficients do not give the solution, but rather setup an equation that instead gives a solution
- For example:

$$y'' + y = 0$$

- Has solutions:

$$e^{ix} \text{ and } e^{-ix}$$

Q2: Transfer Functions and the s-Domain [1]

Final Value Theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

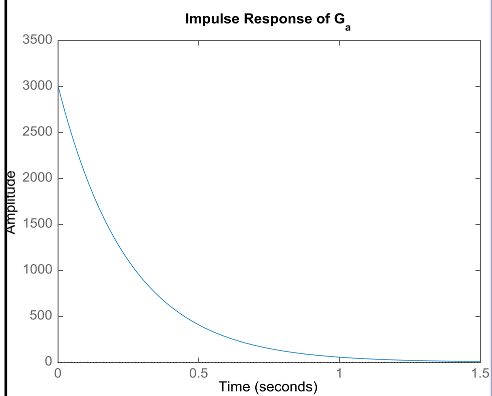
Latex Version:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

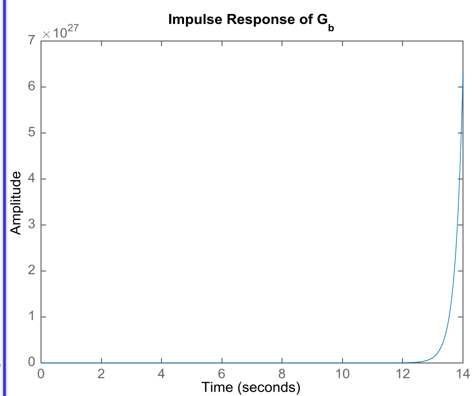
- For systems that are valid (i.e., stable):
  - Roots of the denominator of $\boldsymbol{H(s)}$ must have negative real parts.
  - $\boldsymbol{H(s)}$ must not have more than one pole at the origin.

Q2: Transfer Functions and the s-Domain [2]

- $G_a(s) = \frac{3004}{s+4}$

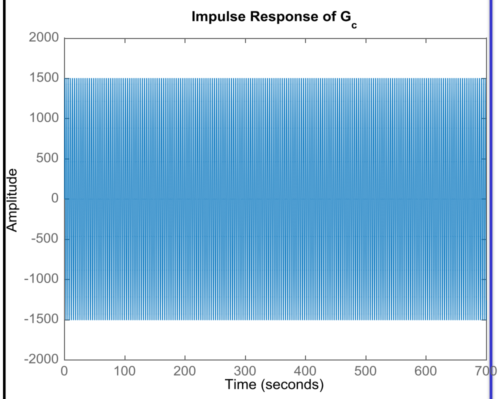


- $G_b(s) = \frac{3004}{s-4}$

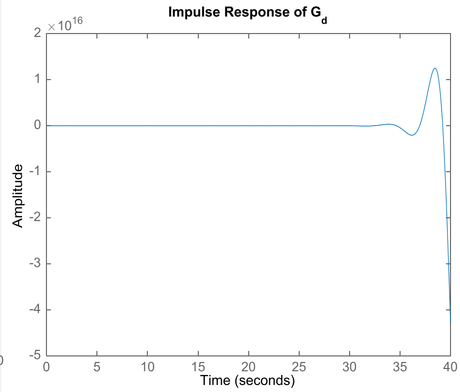


Q2: Transfer Functions and the s-Domain [3]

- $G_c(s) = \frac{3004}{s^2+4}$

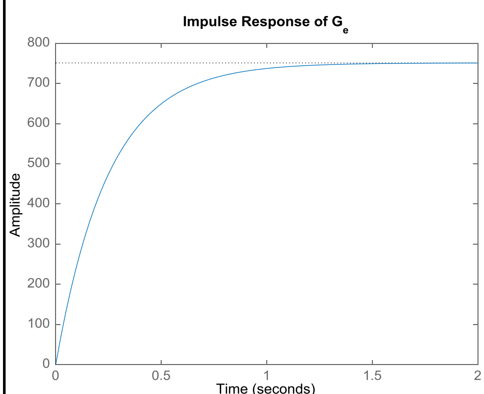


- $G_d(s) = \frac{3004}{s^4+4}$

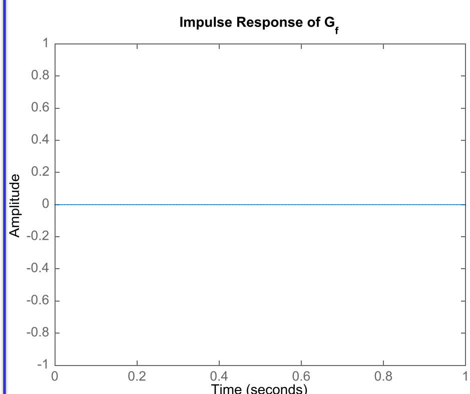


Q2: Transfer Functions and the s-Domain [4]

- $G_e(s) = \frac{3004}{s^2+4s}$

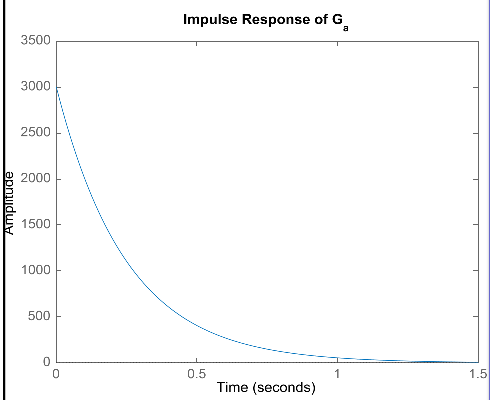


- $G_f(s) = \frac{3004}{4} = 751$
- Not a "dynamic system"

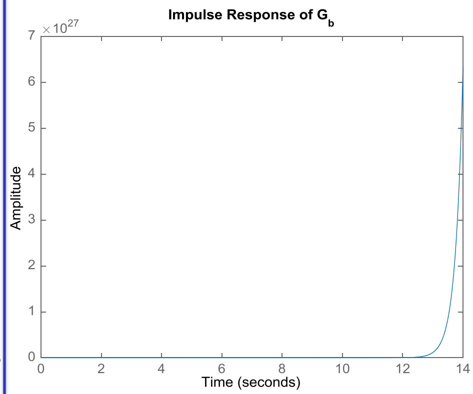


Q2: Transfer Functions and the s-Domain [2]

- $G_a(s) = \frac{3004}{s+4}$



- $G_b(s) = \frac{3004}{s-4}$



Q2: Transfer Functions and the s-Domain [5] Matlab Source for Graphs

```
%% ELEC 3004 Quiz 0 -- Q2
% Ga
a=[3004]; b=[1 4]; Ga=tf(a, b);  figure(10);
impulse(Ga);  title('Impulse Response of G_a');
% Gb
a=[3004]; b=[1 -4]; Gb=tf(a, b);  figure(20);
impulse(Gb); title('Impulse Response of G_b');
% Gc
a=[3004]; b=[1 0 4]; Gc=tf(a, b);  figure(30);
impulse(Gc); title('Impulse Response of G_c');
% Gd
a=[3004]; b=[1 0 0 4]; Gd=tf(a, b);  figure(40);
impulse(Gd); title('Impulse Response of G_d');
% Ge
a=[3004]; b=[1 4 0]; Ge=tf(a, b);  figure(50);
impulse(Ge); title('Impulse Response of G_e');
% Gf
a=[3004]; b=[4]; Gf=tf(a, b);  figure(60);
impulse(Gf); title('Impulse Response of G_f');
```

Q3: Free Determination

- False:

$$\det(A+B) \neq \det(A) + \det(B)$$

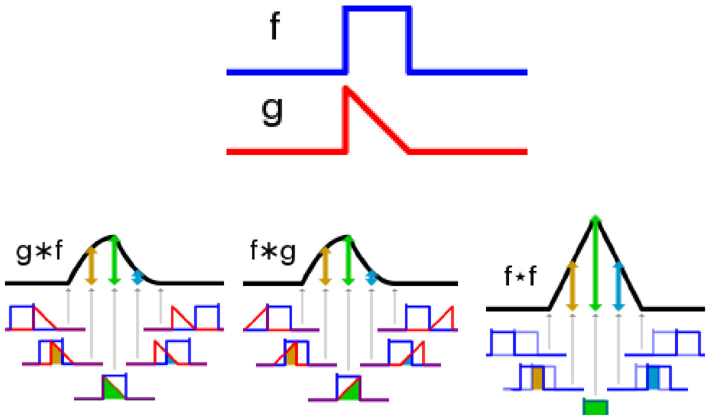
- True:

$$\det(AB) = \det(A) \cdot \det(B)$$

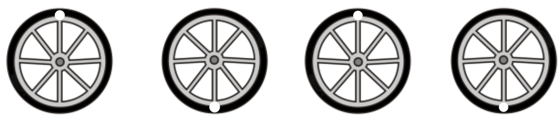
Q4: Free Determination : All TRUE

- True:
  $A = LU$: is a factorization that is basically an elimination
- True:
  If $\boldsymbol{A}$ is invertible, then the only solution to $\boldsymbol{Ax} = \boldsymbol{0}$ is $\boldsymbol{x} = \boldsymbol{0}$.
- True:
  Linear Equations ($Ax = b$) come from steady-state problems.
  eigenvalues ($Ax = \lambda x$) have importance in dynamic problems.

Q5: Convolution!: All TRUE



Q6: A Signal Re-volution!



A. It could be rotating either way (CW or CCW). The angular velocity is $\dot{\theta} = \frac{\Delta\theta}{\Delta t} = \left[\frac{(2n+1)\pi}{\frac{1}{25}}\right] \Rightarrow 12.5\ rev/second$

B. Speeds (m/s):

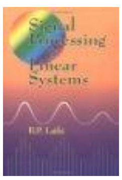
$$v = \omega \times r = 25\pi \frac{rad}{s} \cdot (0.32\ m) = 25.1 \frac{m}{s} = 90.5\ kmh$$

C. $\text{Speed}_{car} \stackrel{?}{=} \text{Speed}_{wheel}$:

- Straight line (no turning)
- Full traction
- No suspension effects …
- What is the **frame of reference**? Should be picked with care!

Signals & Systems: A Primer!

Follow Along Reading:

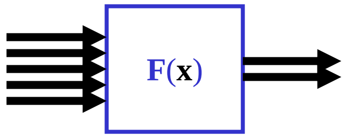


B. P. Lathi
Signal processing and linear systems
1998
TK5102.9.L38 1998

- Chapter 1 (**Introduction to Signals and Systems**)
  - § 1.2: Classification of Signals
  - § 1.2: Some Useful Signal Operations
  - § 1.6 Systems
- Chapter B (Background)
  - B.5 Partial fraction expansion
  - B.6 Vectors and Matrices

An Overview of Systems

- Today we are going to look at **F**(x)!



- F(x): System Model
  - The rules of operation that describe it's behaviour of a "system"
  - Predictive power of the responses
  - Analytic forms > Empirical ones
    - Analytic formula offer various levels of detail
    - Not everything can be experimented on *ad infinitum*
    - Also offer Design Intuition   (let us devise new "systems")
    - Let's us do **analysis**! (determine the outputs for an input)
  - Various Analytic Forms
    - Constant, Polynomial, **Linear**, Nonlinear, Integral, **ODE**, PDE, Bayesian…

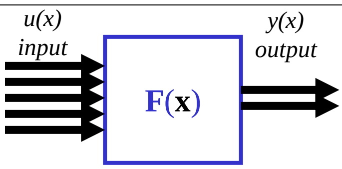
Modelling Ties Back with ELEC 2004

- Linear Circuit Theorems, Operational Amplifiers
- Operational Amplifiers
- Capacitors and Inductors, RL and RC Circuits
- AC Steady State Analysis
- AC Power, Frequency Response
- Laplace Transform
- Reduction of Multiple Sub-Systems
- Fourier Series and Transform
- Filter Circuits

➔ Modelling Tools!

System Terminology

Linear Systems



- Model describes the relationship between the input **u**(x) and the output **y**(x)
- If it is a Linear System (wk 3):

$$y(t) = \int_0^t F(t-\tau)\, u(\tau)\, d\tau$$

- If it is also a (Linear and) **lumped**, it can be expressed **algebraically** as:

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t)$$
$$y(t) = C(t)\, x(t) + D(t)\, u(t)$$

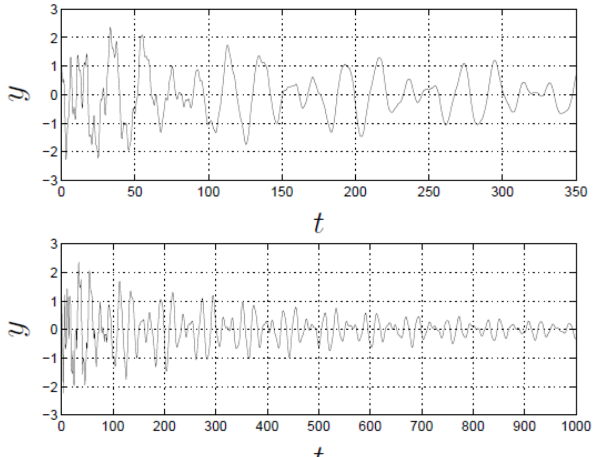
- If it is also (Linear and) **time invariant** the matrices can be reduced to:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$\mathscr{L}$aplacian: $$y(s) = F(s)\, u(s)$$

WHY? This can help simplify matters…

For Example: Consider the following system:

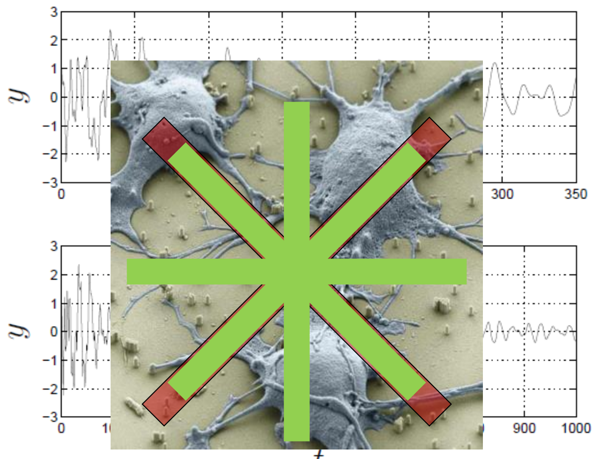


- How to model and predict (and control the output)?

Source: EE263 (s.1-13)

This can help simplify matters…

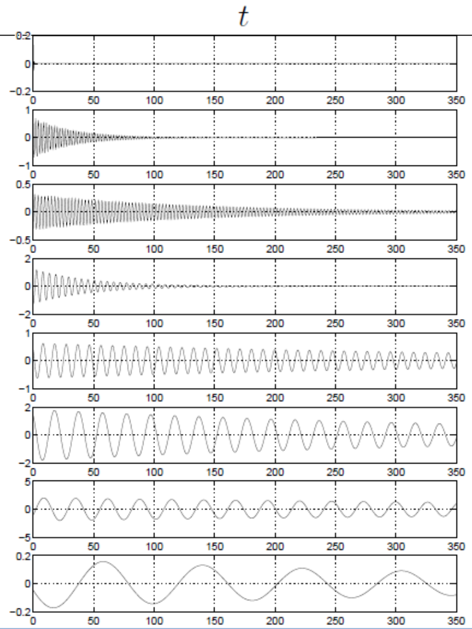
Consider the following system:



- How to model and predict (and control the output)?

Source: EE263 (s.1-13)

This can help simplify matters…



Source: EE263 (s.1-13)

This can help simplify matters…

- Consider the following system:

$$\dot{x} = Ax, \qquad y = Cx$$

- $x(t) \in \mathbb{R}^8$, $y(t) \in \mathbb{R}^1$ → 8-state, single-output system
- Autonomous: No input yet! ($u(t) = 0$)

Source: EE263 (s.1-13)

System Classifications/Attributes

1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Expanding on this: Types of Linear Systems

- LDS:

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t)$$
$$y(t) = C(t)\, x(t) + D(t)\, u(t)$$

- LTI – LDS:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Types of Linear Systems

- LDS:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

To Review:

- Continuous-time linear dynamical system (CT LDS):

$$\frac{dx}{dt} = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)$$

- $t \in \mathbb{R}$ denotes time
- $x(t) \in \mathbb{R}^n$ is the state (vector)
- $u(t) \in \mathbb{R}^m$ is the input or control
- $y(t) \in \mathbb{R}^p$ is the output

Types of Linear Systems

- LDS:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

- $A(t) \in \mathbb{R}^{n\times n}$ is the dynamics matrix
- $B(t) \in \mathbb{R}^{n\times m}$ is the input matrix
- $C(t) \in \mathbb{R}^{p\times n}$ is the output or sensor matrix
- $D(t) \in \mathbb{R}^{p\times m}$ is the feedthrough matrix

➔ state equations, or "*m*-input, *n*-state, *p*-output' LDS

Types of Linear Systems

- LDS:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

- **Time-invariant:** where A(t), B(t), C(t) and D(t) are **constant**
- **Autonomous:** there is no input *u* (B,D are irrelevant)
- **No Feedthrough**: D = 0

- SISO: *u(t)* and *y(t)* are scalars
- MIMO: *u(t)* and *y(t)*: They're vectors: Big Deal ?

Discrete-time Linear Dynamical System

- Discrete-time Linear Dynamical System (DT LDS) has the form:

$$x(t+1) = A(t)x(t) + B(t)u(t), \qquad y(t) = C(t)x(t) + D(t)u(t)$$

- ***t*** ∈ ℤ denotes time index : ℤ={0, ±1, …, ± **n**}

- *x(t), u(t), y(t)* ∈ are sequences

- Differentiation handled as difference equation:
  ➔ first-order vector recursion

Discrete Variations & Stability

$$y\left(s\right) = F\left(s\right)u\left(s\right)$$

- Is in continuous time …
- To move to discrete time it is more than just "sampling" at: 2 × (biggest Frequency)

- Discrete-Time Exponential

$$F(t) \to F\left[kT\right]$$

$$e^{\frac{k}{T}} = \gamma^{k}$$

$$\frac{1}{T} = \ln\gamma$$

- SISO to MIMO
  - Single Input, Single Output
  - Multiple Input, Multiple Output
- BIBO:
  - Bounded Input, Bounded Output

- Lyapunov:
  - Conditions for Stability

  ➔ Are the results of the system asymptotic or exponential

Linear Systems

Linearity:

- **A most desirable property for many systems to possess**
- Ex: Circuit theory, where it allows the powerful technique or voltage or current superposition to be employed.

Two requirements must be met for a system to be *linear:*

- *Additivity*
- *Homogeneity or Scaling*

Additivity **∪** ***Scaling*** **➔ Superposition**

Linear Systems: Additivity

- **Given** input $x_1(t)$ produces output $y_1(t)$ and input $x_2(t)$ produces output $y_2(t)$
- **Then** the input $x_1(t) + x_2(t)$ must produce the output $y_1(t) + y_2(t)$ for arbitrary $x_1(t)$ and $x_2(t)$
- Ex:
  - Resistor
  - Capacitor
- **Not** Ex:
  - $y(t) = \sin[x(t)]$

Linear Systems: Homogeneity or Scaling

- **Given** that $x(t)$ produces $y(t)$
- **Then** the scaled input $\mathrm{a} \cdot x(t)$ must produce the scaled output $\mathrm{a} \cdot y(t)$ for an arbitrary $x(t)$ and a
- Ex:
  - $y(t) = 2x(t)$
- **Not** Ex:
  - $y(t) = x^2(t)$
  - $y(t) = 2x(t) + 1$

Linear Systems: Superposition

- **Given** input $x_1(t)$ produces output $y_1(t)$ and input $x_2(t)$ produces output $y_2(t)$

- **Then**: The linearly combined input
$$x(t) = ax_1(t) + bx_2(t)$$
must produce the linearly combined output
$$y(t) = ay_1(t) + by_2(t)$$
for arbitrary a and b

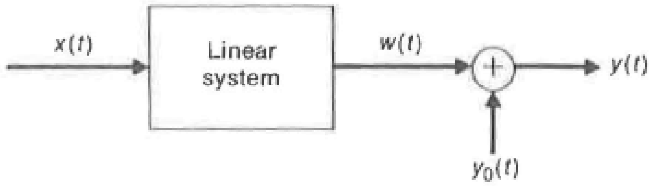
- **Generalizing:**
  - Input: $x(t) = \sum_k a_k x_k(t)$
  - Output: $y(t) = \sum_k a_k y_k(t)$

Linear Systems: Superposition [2]

Consequences:

- Zero input for all time yields a zero output.
  - This follows readily by setting $a = 0$, then $0 \cdot x(t) = 0$

- DC output/Bias ➔ **Incrementally linear**
- Ex: $y(t) = [2x(t)] + [1]$
- Set offset to be added offset [Ex: $y_0(t)$=1]



Dynamical Systems…

- A system with a memory
  - Where past history (or derivative states) are **relevant** in determining the response
- Ex:
  - RC circuit: Dynamical
    - Clearly a function of the "capacitor's past" (initial state) and
    - Time! (charge / discharge)
  - R circuit: is memoryless ∵ the output of the system (recall V=IR) at some time **t** only depends on the input at time **t**

- Lumped/Distributed
  - Lumped: Parameter is constant through the process & can be treated as a "point" in space
- Distributed: System dimensions ≠ small over signal
  - Ex: waveguides, antennas, microwave tubes, etc.

Causality:
Causal (physical or nonanticipative) systems

- **Is one for which the output at any instant t_0 depends only on the value of the input x(t) for $t \le t_0$.** Ex:

$u(t) = x(t-2) \Rightarrow \text{causal}$ $\quad$ $u(t) = x(t-2) + x(t+2) \Rightarrow \text{noncausal}$

- A "*real-time*" system must be causals
  - How can it respond to future inputs?
- A prophetic system: knows future inputs and acts on it (now)
  - The output would begin before t_0
- In some cases Noncausal maybe modelled as causal with delay
- Noncausal systems provide **an upper bound** on the performance of causal systems

Causality:
Looking at this from the output's perspective…

- **Causal** = The output *before* some time *t* does not depend on the input *after* time *t*.

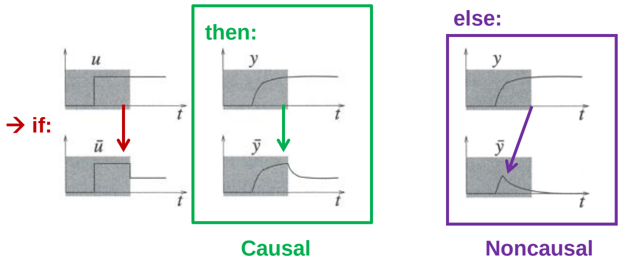
Given: $y(t) = F(u(t))$

For:

$$\hat{u}(t) = u(t),\ \forall 0 \le t < T \ or\ [0, T)$$

Then for a T>0:

$$\rightarrow \hat{y}(t) = y(t),\ \forall 0 \le t < T$$



Systems with Memory

- A system is said t have *memory* if the output at an arbitrary time $t = t_*$ depends on input values other than, or in addition to, $x(t_*)$

- Ex: Ohm's Law

$$V(t_o) = Ri(t_o)$$

- **Not** Ex: Capacitor

$$V(t_0) = \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$$

Time-Invariant Systems

- **Given** a shift (delay or advance) in the input signal
- **Then/Causes** simply a like shift in the output signal

- If $x(t)$ produces output $y(t)$
- Then $x(t-t_0)$ produces output $y(t-t_0)$

- Ex: Capacitor
- $V(t_0) = \frac{1}{C}\int_{-\infty}^{t} i(\tau - t_0)\, d\tau$

$$= \frac{1}{C}\int_{-\infty}^{t-t_0} i(\tau)\, d\tau$$

$$= V(t-t_0)$$

Time-Invariant Systems

- **Given** a shift (delay or advance) in the input signal
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- If $x(t)$ produces output $y(t)$
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