# Problem Set 2: Sampling and Filters (Digital & Analog)

**Total marks:** 100 **Due Date:** May 2, 2017 at 23:59 AEST [end of week 8++]

**Note:** This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and submitted directly in Platypus (preferred) or as PDF (n.b., Microsoft Word documents, scanned images of handwritten pages or items the clearly identify the author are specifically disallowed). The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

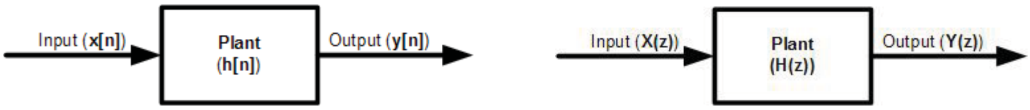
*Thank you.* **:-)**

## Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

### Q1. An e-z Transformation into Problem Set 2 [20 points]

This problem looks at the *z*-Transform, its region of convergence and its properties.
To begin, tak consider the following system and its *z*-Transform




(a) Assume $x[n] = a^n u[n]$ and $h[n] = \delta[n] - a\delta[n-1]$.
What are the *z*-Transforms of $x[n]$, $h[n]$ and $y[n]$
(i.e., what are $X[z]$, $H[z]$, and $Y[z]$)? What is the region of convergence of $Y[z]$?

(b) Derivative Problems
One solution, sometimes creates another problem. Starting with the solution in (a), what is the *z*-Transform of the derivative of the input state (i.e., $nx[n]$)?

(c) Are *z*-Transforms Unique?
Consider a new given *z*-Transform for a signal $X_C[z]$.
Is the inverse ($x_C^1[n]$) signal unique?
Meaning is it possible to have another signal (call it $x_C^2[n]$) that has the same *z*-Transform, but a different value and/or region of convergence?

(d) DTFT It!
Consider the following *z*-Transform of a new given signal

$$X_D[z] = \frac{4z + z^{-4}}{1 - 3z^{-5} + 4z^{-8}}$$

Please find the DTFT at the Nyquist Rate ($\Omega = \pi$).
(Assume, if needed, normalized or unit sampling (i.e., $T_s = 1s$))

**Q2. FIR: Filtering: Interpretations and Realizations** **[20 points]**

FIlters are a type of system. Thus, we can start with a system model of a desired logic and then translate this logic (i.e., its transfer function) into hardware for implementation. Historically, a filter has been constructed with electronic components (op-amps, resistors, capacitors, inductors), but, of course, the logic may be achieved via other means, such as a digital computer that has been programed instead. This gives rise to FIR methods, which are a class of filters with no immediate, direct continuous/analog equivalent.

(a) Briefly explain what are **two fundamental similarities** and **two fundamental differences** between FIR and IIR filters?

(b) The passband and stopband ripple may be attenuated through window selection. Briefly, what is the relative relationship between the stopband ripple and the transition bandwidth and filter order? [**hint:** this should not be more than a few sentences].

(c) Lathi (p. 763) notes that a rectangular window gives the smallest transition band. What order of a Hamming windowed Low-Pass Filter (LPF) would be needed to equal or better the transition band of a $10^{th}$ order Rectangular Window LPF of the same cut-off frequency?

(d) Bilinear Transformations and Equivalence
One approach to realizing a digital filter is to transform the analog filter function (a complex function in the s-domain) to a digital one (a complex function in the z-domain). This may be accomplished via a bilinear transformation (or a conformal mapping) of the form:

$$s = K\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

Where $K$ is a design parameter. A common choice for this is $K = \frac{2}{T_S}$; that is, Tustin's Rule. Using this approach, design an equivalent digital filter to a $4^{th}$-order Chebyshev Type I LPF with a cutoff set at 1 rad/sec and with normalized or unit sampling (i.e., $T_s = 1s$ or $f_s = 1Hz$). Please plot/sketch the Magnitude Response of both the Analog and Digital LPFs for $\Omega = 0 \ldots \pi$ rad/sec.

(e) Extending on (d), design the best (highest signal/noise ratio) $100^{th}$ order non-recursive (FIR) you can for the 50-Hz notch filter application of Problem 2 in Problem Set 1. Please explain the design (i.e., justify which design strategy was used, if a Window is used, justify which window, etc.) in addition to the coefficients.
[Note: The best filter submitted against a signal with random white noise will receive/share a small prize in class].

## Q3. Filter Design **[20 points]**

FIlters run the gamut.  Let us consider two cases.

(a) Butterworth: Smooth & Creamy?
Design a low pass, Butterworth filter with a passband finishing at $0.4\pi$ rads/sample, a stopband which starts at $0.6\pi$ rads/sample, and a minimum drop of 50dB between them.

What is the lowest order filter you can have which satisfies these criterion?
Please plot the poles and zeros of this filter in the *z*-plane.

[**See Also**: Matlab functions: `buttord`, `butter`, `bode`, `tf`, `zplane`, and `freqz`]

(b) Chebyshev: Sharp, but Rough?
Please now design a Chebyshev filter to the same specifications. What's the lowest order it can take, while satisfying the criterion?   Please justify your choice of Chebyshev filter.

[**See Also**: The Matlab functions: `cheby1`, `cheb1ord`, `cheb1ap`, and `cheby2`]

## Q4. Golden's Glittering Reconstruction Efforts! **[20 points]**

Dr. Golden of Pyrite Labs claims his newest whizzbang invention, the SINC (Special, Infinitely Numeric Converter) Digital to Analog converter, is the perfect inverse of his 32-bit analog to digital converter (the CNIS [Colossal Number of Input Samples]), and can perfectly reconstruct any signal digitised by it.

(a) Barring your suspicion of whether Dr. Golden has cracked sinc reconstruction, and that Pyrite Labs has mentioned nothing of band limiting the input, there are three key issues that you assert will break their "perfect inverse" claim:
   a. Quantisation
   b. Finite length sampled sequence
   c. Sampling clock jitter

Discuss why each results in a reconstruction which is imperfect.

(b) Not ZOH Fast?
Dr. Kronecker-Delta on the other hand suggests that it is possible to use the ZOH and then to use a Smoothing filter to remove the higher order harmonics from the output of the ZOH, and that the cutoff frequency should be set such that such that it is no greater than half the sampling frequency.

**Please discuss if this would work.** In your discussion please include a plot/sketch of the magnitude response of the ZOH from $-3\Omega_s \ldots 3\Omega_s$ and a plot of the response after the filter.  Briefly comment if an alternative filter could be used instead to give an even better reconstruction.

**Q5. High-Flying Filters!** **[20 points]**

You have been asked to help Daedalus & Sons UAVs. They have shipment of ultra-fast IMUs, used to measure the pan and tilt of their new craft, the Labyrinth. Unfortunately when operating in their fastest mode (1,000 Hz), there is a lot of noise in acceleration and gyroscope readings. In order to maintain stable flight, the controller has a proportional feedback control (servo-regulator) implemented which reacts to the sensor output. Their engineer, Icarus Daedalus, proposes the following: “Let us average the previous 20 measurements to reduce the noise in the signal, and use the average as our current acceleration and gyroscope values.”

(a) Is this enough samples to be robust to noise? Should we use more or less samples for noise rejection?

(b) In what situations besides those already stated, would it be desired to reduce the number of samples? Can you have too many samples?

(c) Do you have a more optimum solution than the a moving average filter? If so what filter would this be? Please provide and justify a design.