http://elec3004.com

Signals as Vectors
Systems as Maps
&
Data Acquisition

ELEC 3004: **Systems**: Signals & Controls

Dr. Surya Singh

Lecture 4

elec3004@itee.uq.edu.au

http://robotics.itee.uq.edu.au/~elec3004/

March 10, 2016

Lecture Schedule:

Week	Date	Lecture Title
1	29-Feb	Introduction
	3-Mar	Systems Overview
2	7-Mar	Systems as Maps & Signals as Vectors
	10-Mar	Data Acquisition & Sampling
3	14-Mar	Sampling Theory
	17-Mar	Antialiasing Filters
4	21-Mar	Discrete System Analysis
	24-Mar	Convolution Review
	28-Mar	Holiday
	31-Mar	
6	11-Apr	Digital Filters
	14-Apr	Digital Filters
7	18-Apr	Digital Windows
	21-Apr	FFT
8	25-Apr	Holiday
	28-Apr	Feedback
9	3-May	Introduction to Feedback Control
	5-May	Servoregulation/PID
10	9-May	Introduction to (Digital) Control
	12-May	Digitial Control
11	16-May	Digital Control Design
	19-May	Stability
12	23-May	Digital Control Systems: Shaping the Dynamic Response & Estimation
	26-May	Applications in Industry
13	30-May	System Identification & Information Theory
	31-May	Summary and Course Review

Systems as Maps

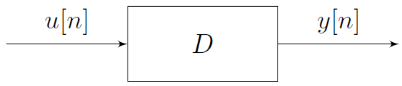
Basis Spaces of a Signal

- A basis of a vector space is a sequence of vectors that has two properties at once:

1. The vectors are linearly independent
2. The vectors span the space

➔ The vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are a basis for $\mathbb{R}^n$ exactly when they are the columns of a $n \times n$ invertible matrix. Thus $\mathbb{R}^n$ has infinitely many bases.

Then a System is a **MATRIX**

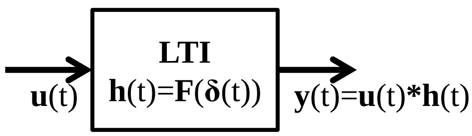


$$y = Du.$$

$$\begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[M] \end{bmatrix} = \begin{bmatrix} D_{11} & D12 & \cdots & D_{1N} \\ D_{21} & D_{22} & \cdots & D_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ D_{M1} & D_{M2} & \cdots & D_{MN} \end{bmatrix} \begin{bmatrix} u[1] \\ u[2] \\ \vdots \\ u[N] \end{bmatrix}.$$

$$y[i] = \sum_j D_{ij} u[j].$$

Linear Time Invariant



- Linear & Time-invariant (of course - tautology!)
- Impulse response: **h**(t)=**F**(**δ**(t))
- Why?
  - Since it is linear the output response (**y**) to any input (**x**) is:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$
$$y(t) = F\left[\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right] \overset{linear}{\rightarrow} \int_{-\infty}^{\infty} x(\tau)\,F\left[\delta(t-\tau)\right]d\tau$$
$$h(t-\tau) \overset{TI}{\equiv} F\left[\delta(t-\tau)\right]$$
$$\Rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = x(t) * h(t)$$

- The output of any continuous-time LTI system is the **convolution** of input **u**(t) with the impulse response **F**(**δ**(t)) of the system.

Linear Dynamic [Differential] System

≡ LTI systems for which the input & output are linear ODEs

$$a_0 y + a_1 \frac{dy}{dt} + \cdots + a_n \frac{d^n y}{dt^n} = b_0 x + b_1 \frac{dx}{dt} + \cdots + b_m \frac{d^m x}{dt^m}$$

Laplace:

$$a_0 Y(s) + a_1 sY(s) + \cdots + a_n s^n Y(s) = b_0 X(s) + b_1 sX(s) + \cdots + b_m s^m X(s)$$
$$A(s)Y(s) = B(s)X(s)$$

- Total response = $\underbrace{\text{Zero-input response}}_{\text{Initial conditions}}$ + $\underbrace{\text{Zero-state response}}_{\text{External Input}}$

Linear Systems and ODE's

- Linear system described by differential equation

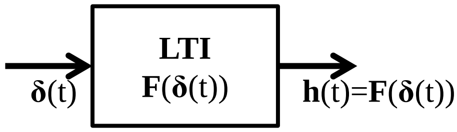
$$a_0 y + a_1 \frac{dy}{dt} + \cdots + a_n \frac{d^n y}{dt^n} = b_0 x + b_1 \frac{dx}{dt} + \cdots + b_m \frac{d^m x}{dt^m}$$

- Which using Laplace Transforms can be written as

$$a_0 Y(s) + a_1 sY(s) + \cdots + a_n s^n Y(s) = b_0 X(s) + b_1 sX(s) + \cdots + b_m s^m X(s)$$
$$A(s)Y(s) = B(s)X(s)$$

where *A(s)* and *B(s)* are polynomials in *s*

Unit Impulse Response



- δ(t): Impulsive excitation
- h(t): characteristic mode terms

Ex:

EXAMPLE 2.4

Determine the unit impulse response $h(t)$ for a system specified by the equation

$$(D^2 + 3D + 2)\, y(t) = Dx(t) \qquad (2.25)$$

This is a second-order system ($N = 2$) having the characteristic polynomial
$(\lambda^2 + 3\lambda + 2) = (\lambda + 1)(\lambda + 2)$

The characteristic roots of this system are $\lambda = -1$ and $\lambda = -2$. Therefore

$$y_n(t) = c_1 e^{-t} + c_2 e^{-2t} \qquad (2.26a)$$

Differentiation of this equation yields

$$\dot{y}_n(t) = -c_1 e^{-t} - 2c_2 e^{-2t} \qquad (2.26b)$$

The initial conditions are [see Eq. (2.24b) for $N = 2$]
$\dot{y}_n(0) = 1$ and $y_n(0) = 0$

Setting $t = 0$ in Eqs. (2.26a) and (2.26b), and substituting the initial conditions just given, we obtain
$0 = c_1 + c_2$

$1 = -c_1 - 2c_2$

Solution of these two simultaneous equations yields
$c_1 = 1$ and $c_2 = -1$

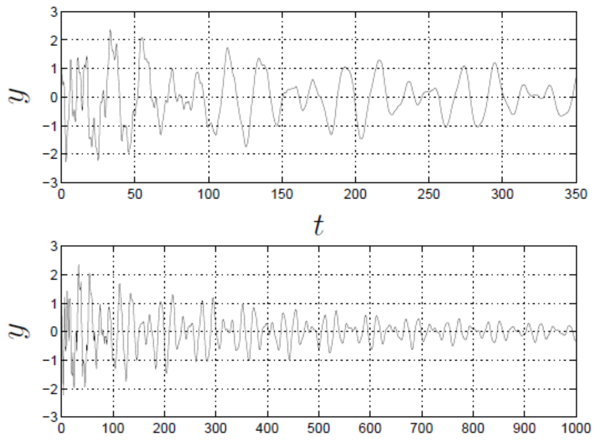
Therefore
$y_n(t) = e^{-t} - e^{-2t}$

Moreover, according to Eq. (2.25), $P(D) = D$, so that
$P(D)y_n(t) = Dy_n(t) = \dot{y}_n(t) = -e^{-t} + 2e^{-2t}$

Also in this case, $b_0 = 0$ [the second-order term is absent in $P(D)$]. Therefore
$h(t) = [P(D)y_n(t)]u(t) = (-e^{-t} + 2e^{-2t})u(t)$

This can help simplify matters… An Example

Consider the following system:

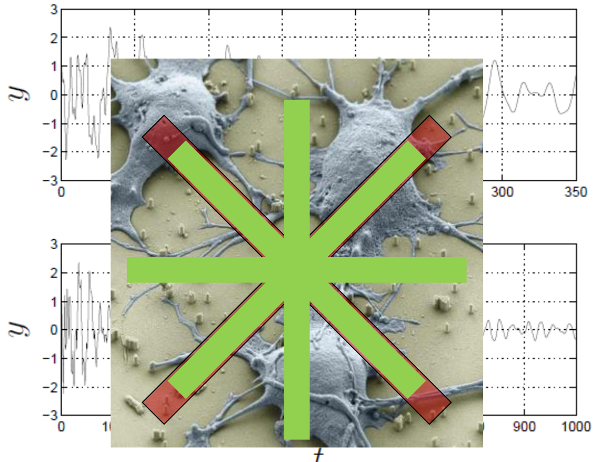


- How to model and predict (and control the output)?

Source: EE263 (s.1-13)

This can help simplify matters… An Example

Consider the following system:



- How to model and predict (and control the output)?

Source: EE263 (s.1-13)

This can help simplify matters… An Example

- Consider the following system:

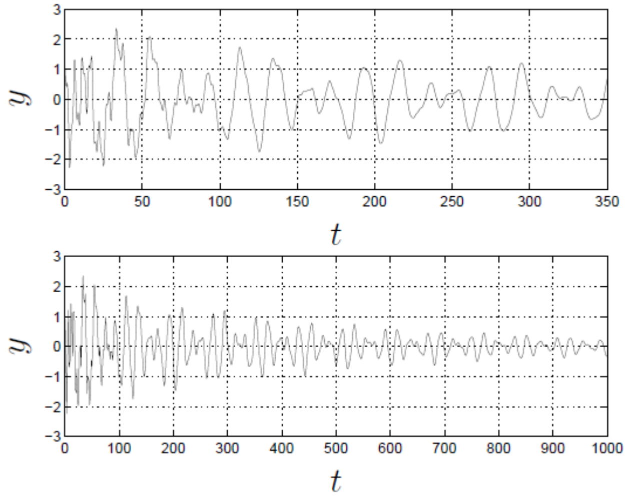
$$\dot{x} = Ax, \qquad y = Cx$$

- $x(t) \in \mathbb{R}^8$, $y(t) \in \mathbb{R}^1$ → 8-state, single-output system
- Autonomous: No input yet! ($u(t) = 0$)

Source: EE263 (s.1-13)

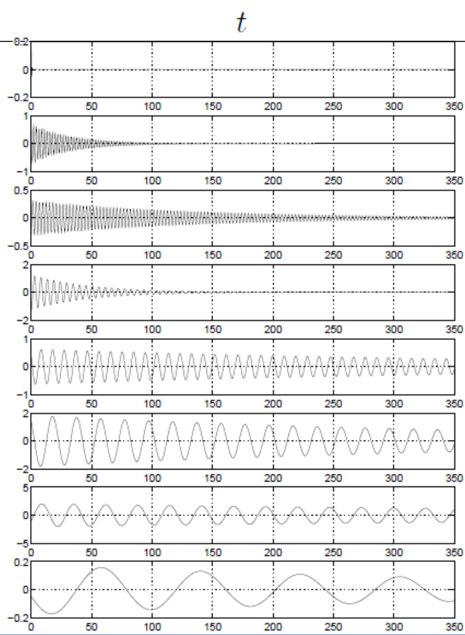
This can help simplify matters… An Example

- Consider the following system:



Source: EE263 (s.1-13)

This can help simplify matters… An Example



Source: EE263 (s.1-13)

Example: Let's consider the control…

Expand the system to have a control input…

- $B \in \mathbb{R}^{8\times 2}$, $C \in \mathbb{R}^{2\times 8}$ (note: the 2nd dimension of C)

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad x(0) = 0$$

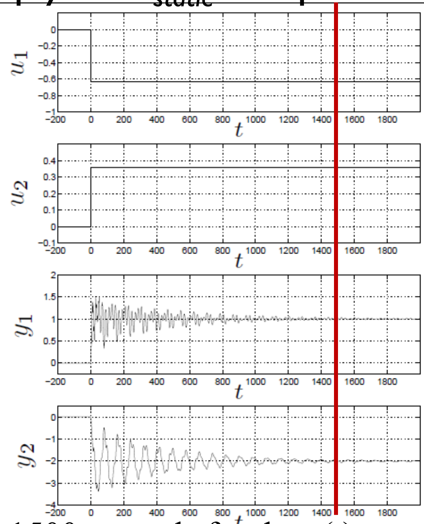
- Problem: Find **u** such that $y_{des}(t)=(1,-2)$
- A simple (and rational) approach:
  - solve the above equation!
  - Assume: static conditions (u, x, y constant)

$$\dot{x} = 0 = Ax + Bu_{\text{static}}, \quad y = y_{\text{des}} = Cx$$

➔ Solve for u:

$$u_{\text{static}} = \left(-CA^{-1}B\right)^{-1} y_{\text{des}} = \begin{bmatrix} -0.63 \\ 0.36 \end{bmatrix}$$

Example: Apply $u=u_{static}$ and presto!

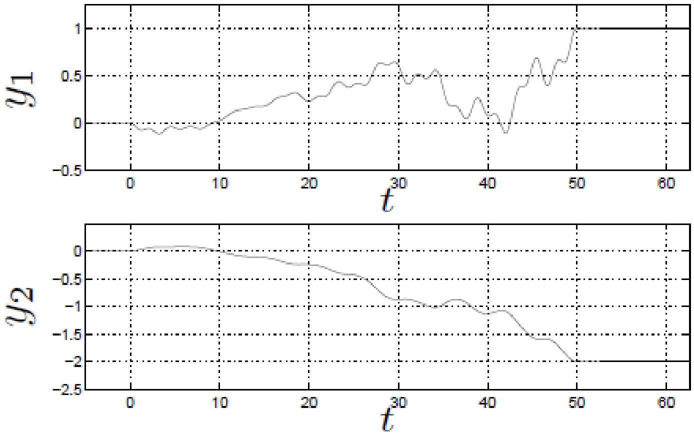


- Note: It takes 1500 seconds for the y(t) to converge …
  but that's natural … can we do better?

Source: EE263 (s.1-13)

Example: Yes we can!

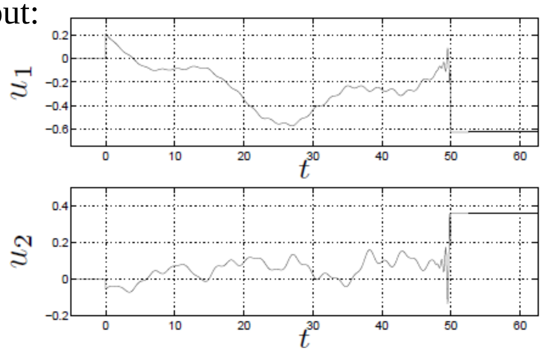
- How about:



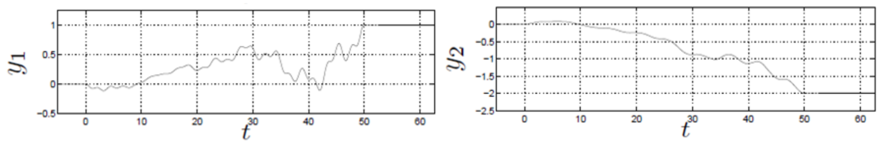
Source: EE263 (s.1-13)

Example: **How?** How about a more clever input?

- How about:



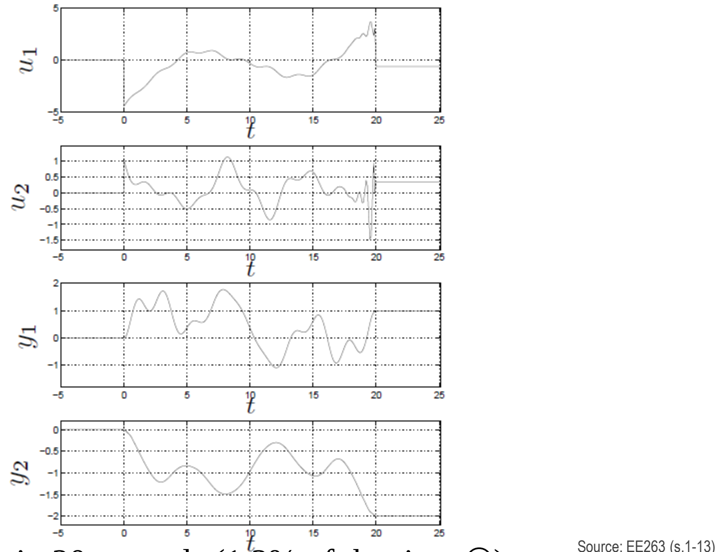
➔



- Converges in 50 seconds (3.3% of the time ☺)

Source: EE263 (s.1-13)

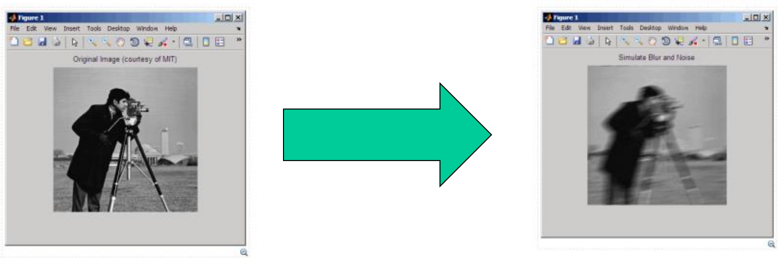
Example: Can we beat it? Larger inputs & LDS



- Converges in 20 seconds (1.3% of the time ☺)

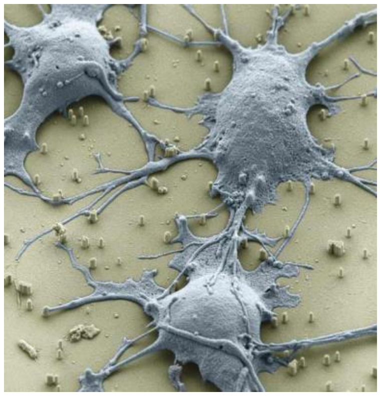
Ex: Deblurring

perfect image → Moving Camera Optics → blurry image

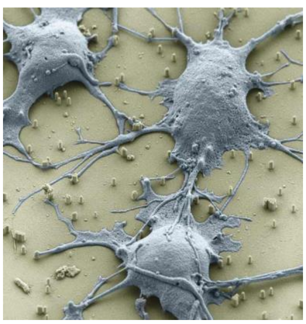


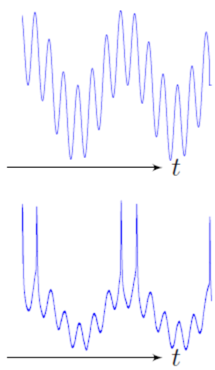
- Matlab: **deconvwnr**

What about …



What about …





- For small current inputs, neuron membrane potential output response is surprisingly **linear**.
- Though this has limits …
  neurons “spike” are (quite) nonlinear (truly)

Source: ELEC6.003 (s.3-49)

Linear Dynamical Systems Review

Linear Differential Systems

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_1\frac{dy}{dt} + a_0 y(t) =$$
$$b_m\frac{d^m f}{dt^m} + b_{m-1}\frac{d^{m-1}f}{dt^{m-1}} + \cdots + b_1\frac{df}{dt} + b_0 f(t) \qquad (2.1a)$$

where all the coefficients a_i and b_i are constants. Using operational notation D to represent d/dt, we can express this equation as

$$(D^n + a_{n-1}D^{n-1} + \cdots + a_1 D + a_0)\, y(t)$$
$$= (b_m D^m + b_{m-1}D^{m-1} + \cdots + b_1 D + b_0)\, f(t) \qquad (2.1b)$$

or

$$Q(D)y(t) = P(D)f(t) \qquad (2.1c)$$

where the polynomials $Q(D)$ and $P(D)$ are

$$Q(D) = D^n + a_{n-1}D^{n-1} + \cdots + a_1 D + a_0 \qquad (2.2a)$$
$$P(D) = b_m D^m + b_{m-1}D^{m-1} + \cdots + b_1 D + b_0 \qquad (2.2b)$$

Linear Differential System Order

$$Q(D)y(t) = P(D)f(t)$$

$$Q(D) = D^n + a_{n-1}D^{n-1} + \cdots + a_1 D + a_0$$
$$P(D) = b_m D^m + b_{m-1}D^{m-1} + \cdots + b_1 D + b_0$$

y(t)=P(D)/Q(D) f(t)
P(D): M
Q(D): N
(yes, N is de**N**ominator)

- In practice: $m \leq n$

∵ **if** m > *n:*
then the system is an
(m - n)th -order differentiator of high-frequency signals!

- Derivatives magnify noise!

Second Order Systems

Second order systems

$$ay'' + by' + cy = 0$$

assume $a > 0$ (otherwise multiply equation by -1)

solution by Laplace transform:

$$a(\underbrace{s^2Y(s) - sy(0) - y'(0)}_{\mathcal{L}(y'')}) + b(\underbrace{sY(s) - y(0)}_{\mathcal{L}(y')}) + cY(s) = 0$$

solve for Y (just algebra!)

$$Y(s) = \frac{asy(0) + ay'(0) + by(0)}{as^2 + bs + c} = \frac{\alpha s + \beta}{as^2 + bs + c}$$

where $\alpha = ay(0)$ and $\beta = ay'(0) + by(0)$

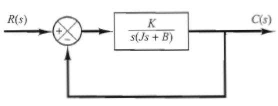
Second Order Systems

so solution of $ay'' + by' + cy = 0$ is

$$y(t) = \mathcal{L}^{-1}\left(\frac{\alpha s + \beta}{as^2 + bs + c}\right)$$

- $\chi(s) = as^2 + bs + c$ is called *characteristic polynomial* of the system
- form of $y = \mathcal{L}^{-1}(Y)$ depends on roots of characteristic polynomial χ
- coefficients of numerator $\alpha s + \beta$ come from initial conditions

Second Order Response



$$\frac{C(s)}{R(s)} = \frac{\frac{K}{J}}{\left[s + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]\left[s + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]}$$

$$\frac{K}{J} = \omega_n^2, \qquad \zeta = \frac{B}{B_c} = \frac{B}{2\sqrt{JK}}$$

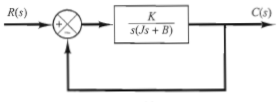
Three Types:

- I: Underdamped: $(0 < \zeta < 1)$:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \zeta\omega_n + j\omega_d)(s + \zeta\omega_n - j\omega_d)} \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\begin{aligned}\mathcal{L}^{-1}[C(s)] &= c(t)\\ &= 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right)\\ &= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right),\end{aligned}$$

Second Order Response



$$\frac{C(s)}{R(s)} = \frac{\frac{K}{J}}{\left[s + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]\left[s + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]}$$

$$\frac{K}{J} = \omega_n^2, \qquad \zeta = \frac{B}{B_c} = \frac{B}{2\sqrt{JK}}$$

Three Types:

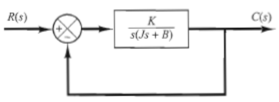
- II: Critically Damped: $(\zeta = 1)$

For a unit-step input, $R(s) = 1/s$ and $C(s)$ can be written

$$C(s) = \frac{\omega_n^2}{(s + \omega_n)^2 s}$$

$$\lim_{\zeta \to 1} \frac{\sin \omega_d t}{\sqrt{1 - \zeta^2}} = \lim_{\zeta \to 1} \frac{\sin \omega_n \sqrt{1 - \zeta^2}\, t}{\sqrt{1 - \zeta^2}} = \omega_n t$$

Second Order Response



$$\frac{C(s)}{R(s)} = \frac{\frac{K}{J}}{\left[s + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]\left[s + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]}$$

$$\frac{K}{J} = \omega_n^2, \qquad \zeta = \frac{B}{B_c} = \frac{B}{2\sqrt{JK}}$$

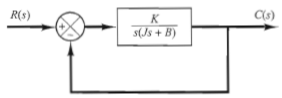
Three Types:

- III: Over Damped: $(\zeta > 1)$

For a unit-step input, $R(s) = 1/s$ and $C(s)$ can be written

$$C(s) = \frac{\omega_n^2}{\left(s + \zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\right)\left(s + \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\right)s}$$

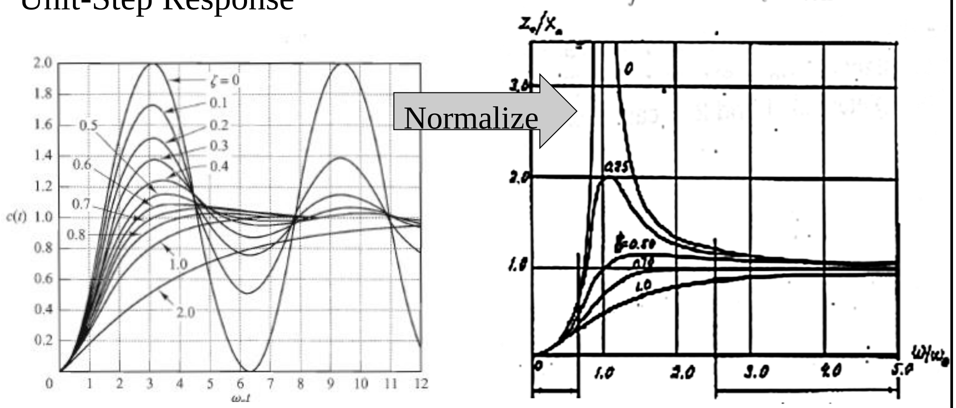
Second Order Response



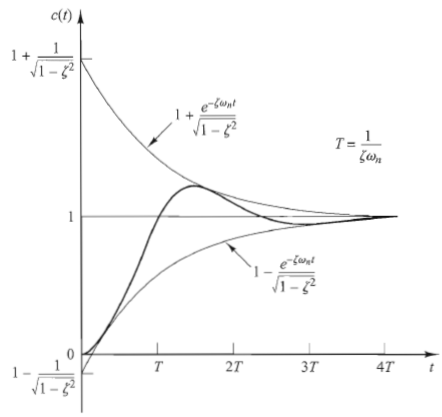
$$\frac{C(s)}{R(s)} = \frac{\frac{K}{J}}{\left[s + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]\left[s + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]}$$

$$\frac{K}{J} = \omega_n^2, \qquad \zeta = \frac{B}{B_c} = \frac{B}{2\sqrt{JK}}$$

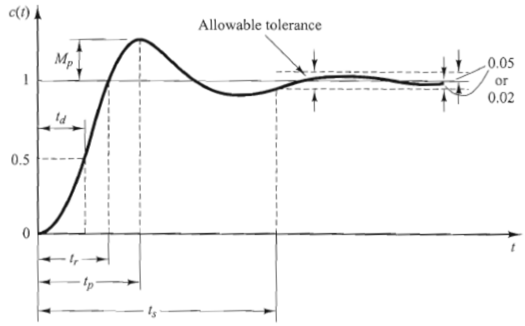
Unit-Step Response



Second Order Response
Envelope Curves



Second Order Response
Unit Step Response Terms

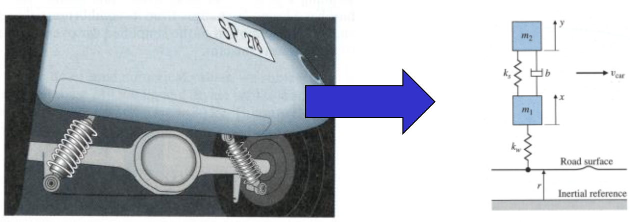


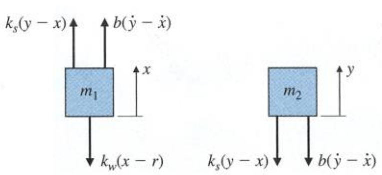
- Delay time, t_d: The time required for the response to reach half the final value
- Rise time, t_r: The time required for the response to rise from 10% to 90%
- Peak time, t_p:The time required for the response to reach the first peak of the overshoot
- Maximum (percent) overshoot, Mp:

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

- Settling time, t_s: The time to be within 2-5% of the final value

Example: Quarter-Car Model





Example: Quarter-Car Model (2)

$$\ddot{x} + \frac{b}{m_1}(\dot{x} - \dot{y}) + \frac{k_s}{m_1}(x - y) + \frac{k_w}{m_1}x = \frac{k_w}{m_1}r,$$

$$\ddot{y} + \frac{b}{m_2}(\dot{y} - \dot{x}) + \frac{k_s}{m_2}(y - x) = 0.$$

$$s^2X(s) + s\frac{b}{m_1}(X(s) - Y(s)) + \frac{k_s}{m_1}(X(s) - Y(s)) + \frac{k_w}{m_1}X(s) = \frac{k_w}{m_1}R(s),$$

$$s^2Y(s) + s\frac{b}{m_2}(Y(s) - X(s)) + \frac{k_s}{m_2}(Y(s) - X(s)) = 0,$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{\frac{k_w b}{m_1 m_2}\left(s + \frac{k_s}{b}\right)}{s^4 + \left(\frac{b}{m_1} + \frac{b}{m_2}\right)s^3 + \left(\frac{k_s}{m_1} + \frac{k_s}{m_2} + \frac{k_w}{m_1}\right)s^2 + \left(\frac{k_w b}{m_1 m_2}\right)s + \frac{k_w k_s}{m_1 m_2}}.}$$

Data Acquisition

Digital Signal

- Representation of a signal against a discrete set
- The se$s \in \mathbb{I}$d in by computing hardware
- Can be scaled or normalized, but is limited

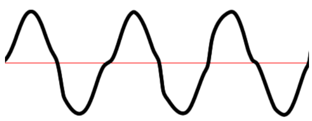
$$s \in \mathbb{I}(0, \ldots, 2^{16})$$

- Time i

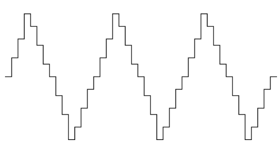
$$s' \in \frac{\mathbb{I}(0,\ldots,2^{16})}{2^{16}}$$

Analog vs Digital

- *Analog Signal*: An analog or analogue signal is any variable signal **continuous** in both time and amplitude



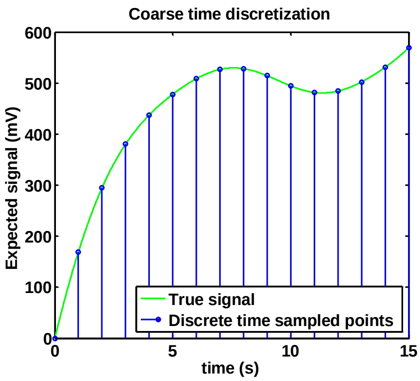
- *Digital Signal*: A digital signal is a signal that is both **discrete** and quantized



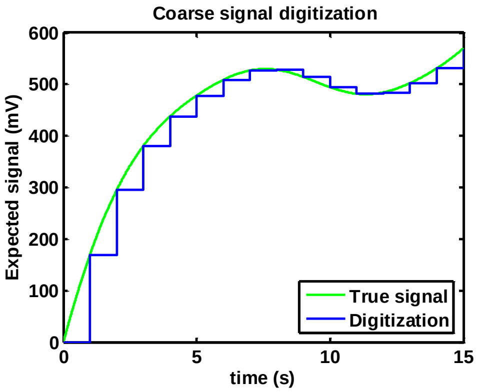
E.g. Music stored in a CD: 44,100 Samples per second and 16 bits to represent amplitude

Representation of Signal

- Time Discretization



- Digitization



Signal: A carrier of (desired) information [1]

- Need **NOT** be electrical:
  - Thermometer
  - Clock hands
  - Automobile speedometer

- Need **NOT** always being given
  - "Abnormal" sounds/operations
  - Ex: "pitch" or "engine hum" during machining as an indicator for feeds and speeds

Signal: A carrier of (desired) information [2]

- Electrical signals
  - Voltage
  - Current

- **Digital signals**
  - **Convert analog electrical signals to an appropriate digital electrical message**
  - **Processing by a microcontroller or microprocessor**

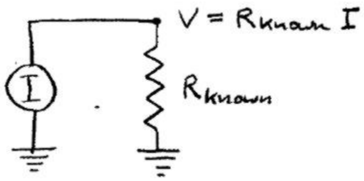
Transduction (sensor to an electrical signal)

- Sensor reacts to environment (physics)
- Turn this into an electrical signal:
  - V: voltage source
  - I: current source
- Measure this signal
  - Resistance
  - Capacitance
  - Inductance

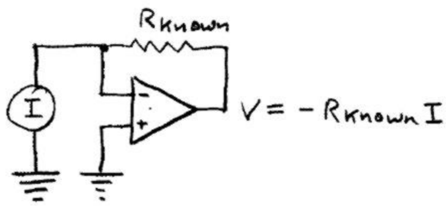
Ex: Current-to-voltage conversion

- simple:
  Precision Resistor

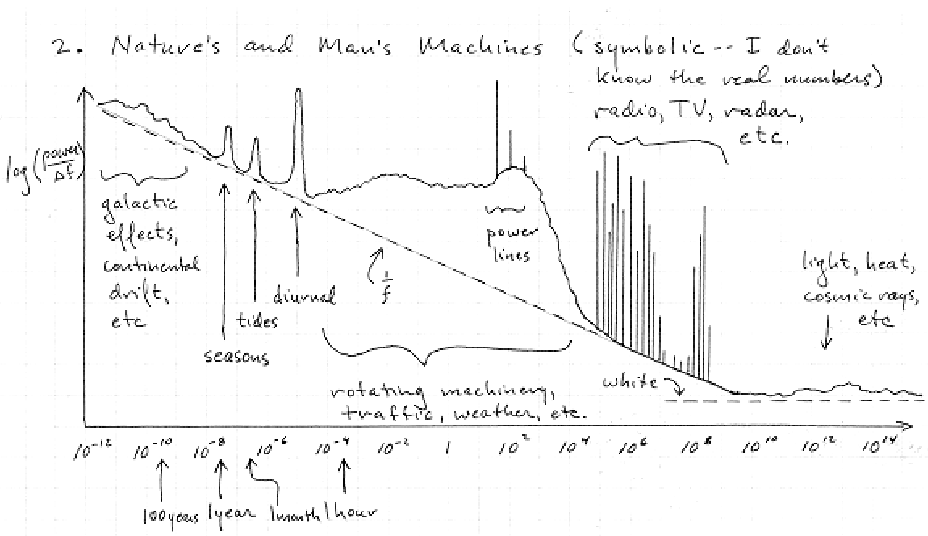
$$i = \frac{V_{\text{measured}}}{R_{\text{known}}}$$



- better:
  Use an "op amp"



BUT there is <u>Noise</u> …



Note: this picture illustrates the concepts but it is not quantitatively precise

Source: Prof. M. Siegel, CMU

Noise: "Unwanted" Signals Carrying Errant Information

- Cross-coupled measurements
- Cross-talk (at a restaurant or even a lecture)
- A bright sunny day obstructing picture subject
- Strong radio station near weak one
- observation-to-observation variation
  - Measurement fluctuates (ex: student)
  - Instrument fluctuates (ex: quiz !)
- Unanticipated effects / variation (Temperature)
- **One man's noise might be another man's signal**

Noise: Fundamental Natural Sources

- Voltage (EMF) – Capacitive & Inductive Pickup
- Johnson Noise – thermal / Brownian
- 1/f ($V_j = \sqrt{4k_bTR}$
- Shot noise (interval-to-interval statistical count)

$$V_f = \sqrt{\frac{\alpha V_R^2}{Nf}}$$

SNR : Signal to Noise Ratio

$$V = V_s + V_n$$

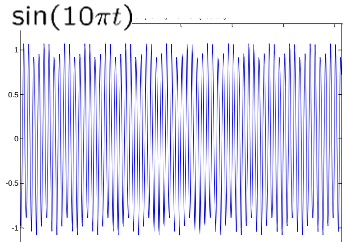
Magnitude: $\bar{V^2} = \bar{V_s^2} + \bar{V_n^2} + \bar{V_s V_n}$

$$\frac{S}{N} = \frac{V_s^2}{V_n^2}$$

in dB: $10 \log \left(\frac{\bar{V_s}^2}{\bar{V_n}^2} \right) = 20 \log \left(\frac{V_s^{rms}}{V_n^{rms}} \right)$

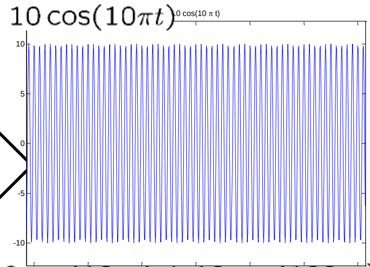
Derivatives magnify noise!

- $\sin(10\pi t)$

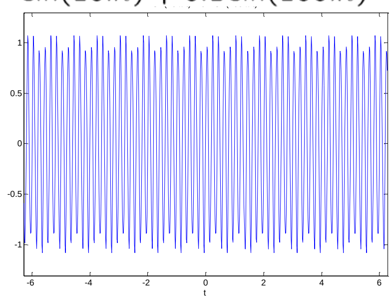


$\xrightarrow{\frac{d}{dt}}$

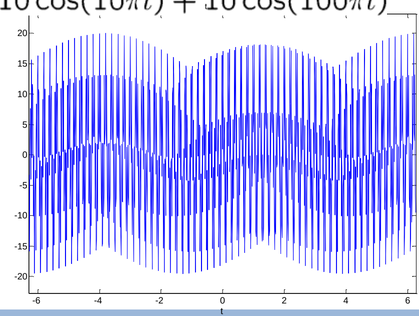
- $10\cos(10\pi t)$



- $\sin(10\pi t) + 0.1\sin(100\pi t)$



- $10\cos(10\pi t) + 10\cos(100\pi t)$



Next Time…

- Register on Platypus
- Try the practise assignment
- We will talk about Data Acquisition / Sampling