ELEC 3004 / 7312 – Systems: Signals & Controls **2016**

Problem Set 3:Digital Feedback Control

Total marks: 100 **Due Date:** May 27, 2016 at 23:59 AEST [end of week 12]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus directly or as PDF. It is required that solutions, including equations, should be typed please. The final grade is the score determined by the teaching staff directly (which may be formed based on quality peer reviews). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions. Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Although a rubric will be provided to guide marking, marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

For Questions Q1 to Q3: Please answer **2 out of the 3** questions (your choice). If all three questions are submitted, *the tutors will only mark two chosen at random*. For Questions Q4 and Q5: Please answer **both** of them. **Thus, in total four questions should be answered.**

Q1. Hold On **[20 points]**

Given a transfer function, H(s), there are multiple methods to find a "discrete equivalent" (i.e. a discrete-time with approximately the same characteristics). This problem considers two of these approaches -- **Numerical Integration** (via Trapezoid rule substitution/Tustin's method) and **Hold Equivalence** (via the triangle hold) -- for the case of a first-order filter.

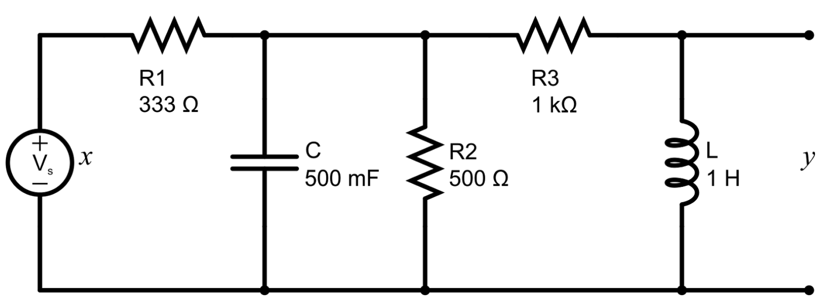
Given: $H(s) = \frac{a}{s+a}$, please find an approximate equivalent $H^*(z)$ via:

(a) Trapezoid rule substitution

(b) Triangle Hold Equivalence
where $H_{tri}(z) = \frac{(z-1)^2}{Tz} Z\{\frac{H(s)}{s^2}\}$
[**Hint**: please review §4.4 of FPW (pp. 149-155) before applying H_{tri} [i.e. Eq. 4.35]]

Q2. Charging Ahead with State Space **[20 points]**

Write the state and output equations for the network below
[**Hint:** this is similar, but different, to Question 13.2-2 in Chapter 13 of Lathi]



Q3. LeviLab: Floating the (Magnetic) Data **[20 points]**

Laboratory 3 and 4 involve involve modelling and control of a levitating magnetic mass. Based on pre-lab analysis and your laboratory experiments, please answer the following:

(a) Using the hall effect sensor and solenoid, measure and plot the relationship between "average input current" and the sensor voltage (i(x,t) and y(x,t) respectively in Laboratory 3). Is this a linear relationship? Please discuss.

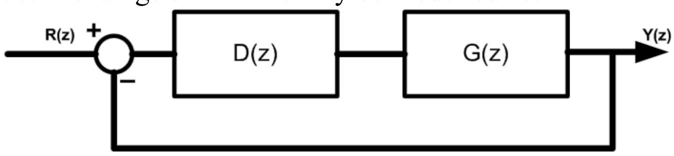
(b) Give the system model
That is, the **function / equation** describing the system's behavior . A set of differential equations or a state-space form is acceptable.

(c) Did it levitate?
Please provide the final control law (equation) with the final tuned values and briefly describe how this was tuned. Please feel free to also include a picture of it levitating.

For Questions Q4 and Q5: Please answer both.
That is, these two questions should be answered.

Q4. A New Spin on Digital Controls **[30 points]**

Consider a digital controller for a spin-coating system in which a platter spins with a constant speed and the has a time between readings of T. This may be modelled as:



Where $G(s) = [ZOH]\left[\frac{5}{s(s+20)}\right] = \left[\frac{1-e^{-sT}}{s}\right]\left[\frac{5}{s(s+20)}\right]$

(a) Determine $G(z)$ [please state your assumptions (e.g. ZOH)]

(b) For $T = 1ms$, a proportional controller ($D(z) = K$), what is the open loop transfer function?
[**Hint:** What happens to G(s) (or how can G(s) be simplified) if the fast pole ($s = -20$) is neglected?]

(c) What is the closed-loop transfer function?
[**Hint:** Apply the same assumptions as was done in Part (b)]

(d) For $K = 4000$, what is response? if $K = 4000$ is stable, what is the settling time? If $K = 4000$ is unstable, what is the maximum K for which the system is stable?

Q5. A New State of Controllability with State-Space Models! [30 points]

Universal Quick Light Dexterous (Uni-QLD) industries has engineered a new robotic manipulator -- the B4-Hand! Chief Engineer Angela Kasner has determined the state-space representation of this system as:

$$A = \begin{bmatrix} 2 & 3 & 2 & 1 \\ -2 & -3 & 0 & 0 \\ -2 & -2 & -4 & 0 \\ -2 & -2 & -2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ -12 \\ 5 \\ 2 \end{bmatrix}$$

$$C = [3 \quad 0 \quad 0 \quad 4] \quad D = [0]$$

(a) What is the state-transition matrix $\Phi(s)$ for this system?
[**Note:** Recall that $\Phi(s) = [sI - A]^{-1}$]

(b) What is the transfer function for this system?
[**Note:** Recall that $H(s) = C\Phi(s)B + D$]

(c) What is order of the system? Do any of the zeros cancel any of the poles? Are (or were) any of these zeros or poles "unstable"?

(d) Is this system controllable?
If so, provide a **Proportional** control law that can serve as a stable regulator
If not, please advise how many of the modes are controllable?
[**Note:** Recall that the Controllability Test Matrix Q for this case would be $Q = [B\ AB\ A^2B\ A^3B\ ]$]

(e) Research engineer Hardrich Mankal hypothesises that designing compensators for unstable systems by cancelling unstable poles by zeros is not feasible even if the cancellation is perfect. Is this true? Please briefly discuss.

(f) Field engineer Justin Quartz wants to field this using a 100 Hz sampler. Is this possible? Please compute $H(z)$ using any a standard approach (i.e. emulation is acceptable).

Some Optional Remarks:
Why is $\Phi(s) = [sI - A]^{-1}$? This comes from the solution to the state space equations $\dot{x} = Ax + Bu$, $y = Cx + Du$ is $X(s) = (sI - A)^{-1}[x(0) + BU(s)]$ and we define $\Phi(s)$ from this. Conversely, the output is $Y(s) = C\Phi(s)x(0) + [C\Phi(s)B + D]U(s)$. For a case where there is zero-input response, this can be simplified and written as a transfer function as $\frac{Y(s)}{U(s)} = H(s) = C\Phi(s)B + D$ (note that this notation is not universal, some define $\frac{Y(s)}{U(s)} = G(s)$). Finally from this we can also obtain a system characteristic polynomial via $|\Phi^{-1}|$ or $|sI - A|$. Similarly, in the Z domain we will get $[zI - \Phi]X(z) = \Gamma U(z)$, and $Y(z) = HX(z)$. Hence, $\frac{Y(z)}{U(z)} = H[zI - \Phi]^{-1}\Gamma$