ELEC 3004 / 7312 – Systems: Signals & Controls **2016**

Problem Set 2: Sampling and Filters (Digital & Analog)

Total marks: 100 **Due Date:** April 29, 2016 at 23:59 AEST [end of week 8]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus directly or as PDF. It is required that solutions, including equations, should be typed please. The final grade is the score determined by the teaching staff directly (which may be formed after peer reviews are entered). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions. Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Although a rubric will be provided to guide marking, marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. Counter Sampling [20 points]

Is a DAC the inverse of an ADC?

(a) Consider the case of a DAC using "ideal" reconstruction via a **`sinc`** function
(b) Consider the case of a DAC using "common" reconstruction via a **`ZOH`** function

Q2. Smoother and Fast-IIR Filters [30 points]

Design a lowpass filter to meet the following specifications:

I. **Passband gain** to lie between **1** and $\mathbf{G_p} = -1$ dB ($\mathbf{G_p} \approx 90\%$) for 0 Hz $\le \boldsymbol{f} <$ 10 Hz (and, if applicable, a ripple, $\mathbf{r} \le 1$ dB)
II. **Stopband gain** not to exceed $\mathbf{G_s}$ = -20 dB ($\mathbf{G_s} \approx 10\%$) for $\boldsymbol{f} \ge$ 15 Hz

In the design process please determine the following:

- Order **n**
- Cutoff frequency, f_c
- The final filter transfer function, **H(s)**
  (**hint**: this is related, but distinct from, the normalized transfer function, ℋ(s))
- A graph of the filter's amplitude response as a function of frequency (*f*)
- A graph of the poles of the filter (on the s-plane).

(a) Use a **Butterworth Filter** to meet the aforementioned specification
(b) Use an **Inverse Chebyshev Filter** to meet the aforementioned specification
(c) What would be the stopband gain for an Inverse Chebyshev filter configuration (as was used in part (b)) with the order of the Butterworth filter (as found in (a))?
(d) Please briefly compare and discuss the two filters, particularly as it relates to order and group delay due.

Hints: [1] A review of Lathi §7.5-7.7 (pp. 505-524, examples 7.6 and C7.9) may help.
[2] Computer codes, such as Matlab's Signal Processing Toolbox (e.g. `cheb2ord` and `cheby2`), may be used, but do not have to be used.
[3] If programs are employed, please include your codes used in determining your solution.

Q3. Frequency Response of Discrete-Time LTI **[20 points]**

A complex exponential, $x[n] = e^{zn}$, is an input signal of particular importance for an LTI system as it an eigenfunction. Thus, the output $y[n] = H(z)z^n$ with $H(z) = \sum_{N=-\infty}^{\infty} h[N]z^{-N}$

With $z = e^{j\Omega}$, the frequency response is given by the function $H(z)$, which also is an eigenvalue of the system (if $H(z)$ converges).

Consider:

I. IIR: The familiar exponential impulse response $h[n] = a^n u[n]$ (for $|a| < 1$), which gives $H_1(z) = \frac{1}{1-az^{-1}}$ (for $|z| > |a|$)

II. FIR: The familiar first-difference operator $H_2(z) = 1 - z^{-1}$ (for $|z| > |0|$)

(a) Determine the equations for the magnitude response and the phase response for the two above cases (I and II) as a function of the normalized radian frequency ***Ω***, where the Nyquist frequency, ***Ω***, is normalized to ***π***, i.e. $\Omega_{Nyquist} = \pi$.
(**hint:** this is $|H(e^{j\Omega})|$ and $\angle H(e^{j\Omega})$))

(b) Please plot the **Magnitude** and **Phase** response for H_1 and H_2 as a function of the normalized radian frequency ***Ω*** over the range $-\pi \le \Omega \le \pi$.
(**hint:** this is nominally four plots)

(c) What is the DC (Ω=0) gain for the two above cases (I and II)?

(d) What is the gain at the Nyquist rate (Ω=π) for the two above cases (I and II)?

Q4. Moving On FIRmly **[30 points]**

The moving average filter is a crude, but common, technique for smoothing a noisy data sequence. For an input signal vector, $x[n]$, it operates by averaging the sequence over N+1 samples such that the output, $y[n]$, is computed as the average of $x[n]$ and the N preceding samples $x[n-1], x[n-2], ..., x[n-N]$, which may be described as

$$y[n] = \frac{1}{N+1}\sum_{k=0}^{N} x[n-k]$$

(a) Determine the corresponding impulse response, $h[n]$, of this system

(b) Determine the system function, $H(z)$, for this system

(c) Determine the zero and/or pole locations for $H(z)$

(d) Using the result from part (c), plot the pole/zero impulse for the case where **N=3**.

(e) Plot the magnitude response and phase response over the range $-\pi \le \Omega \le \pi$
(**hint**: it may help to analytically determine the magnitude, $H(e^{j\Omega})$)

(f) Repeat case (e) and plot the magnitude responses (not phase) for the cases where N_X=34. N_Y=304, N_Z=3004.
(**hint**: it may help to generate this with the help of a computer code)