ELEC 3004 / 7312 – Systems: Signals & Controls **2016**

Problem Set 1: An Introduction to Signals and Systems

Total marks: 100 **Due Date:** March 24, 2016 at 23:59 AEST [end of week 4]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. It is requested that solutions, including equations, should be typed please. The final grade is the median of the marks from the peer reviews and the staff (with provisions for review). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions. Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Although a rubric will be provided to guide peer marking, peers may reduce marks if they believe an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. A Good Linear Cause? [10 points]

Are the following systems are (I) Linear? and (II) Causal?
Briefly **justify why** (please specify key conditions/assumptions as needed).

(a) $y[n] = x[n]$

(b) $y[n] = x[n] + 1$

(c) $y[n] = x[-n]$

(d) $Y(s) = \frac{1}{(s+1)}$

(e) An affine system:

$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$, such that $(\alpha + \beta) = 1$

Q2. Nyquist Says! [10 points]

Recall that the Nyquist Sampling Theorem has that for a band-limited signal of frequency ω_M that $\omega_S > 2\omega_M$.

Using the signal, $x(t) = \cos\left(\frac{\omega_s}{2} t + \phi\right)$, show why it is greater than (>) twice the highest frequency as compared to greater than equal to (≥) twice the highest frequency.

Q3. A New Basis **[30 points]**

Remember that a signal (vector) need not only be written in the standard basis (**S** consisting of basis vector $\mathbf{s_1}, \ldots, \mathbf{s_n}$, where $\mathbf{s_i}$ are columns of an identity matrix (i.e., **S**=**I**))

While every vector $\boldsymbol{v}$ in $\mathbb{R}^n$ can be written in exactly one way as a combination of the basis vectors, a new set of basis vectors may be chosen or designed. One, of many, convenient basis is the Wavelet basis. Wavelets are "small waves" (from the French *ondelette*) that have different lengths and are located at different places and where each basis vector has more "frequency."

Recall from linear algebra that for a set of vectors ($\boldsymbol{w}$) to form a basis in $\mathbb{R}^n$ that:

1. The vectors ($\mathbf{w_1}, \ldots, \mathbf{w_n}$) are linearly independent
2. These basis vectors may be seen as columns and assembled to form a n×n basis matrix (**W**) that is invertible
3. Related to the aforementioned note, a given vector ($\boldsymbol{x}$) may be written as a linear combination of coefficients ($\boldsymbol{c}$) such that $\boldsymbol{x}=\mathbf{W}\boldsymbol{c}$ as:

   $$\mathbf{x} = c_1\mathbf{w_1} + c_2\mathbf{w_2} + \ldots + c_n\mathbf{w_n}$$

(a) Consider $x = \begin{bmatrix} 3 & 0 & 0 & 4 \end{bmatrix}$.
For this case, what are the standard basis vectors $\mathbf{s_1}, \ldots, \mathbf{s_4}$?
Please determine the scalar coefficients $\mathbf{c_1}, \ldots, \mathbf{c_4}$?

(b) Consider a small (4×4) wavelet basis given by:

$$\mathbf{w_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{w_2} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{w_3} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{w_4} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Using this basis, determine the basis matrix **W** and its inverse $\mathbf{W}^{-1}$
[**Hint**: you can do this by hand, consider the properties of **W**]

Now, please find the coefficients for the vectors:

- $\mathbf{x_1} = \begin{bmatrix} 3 & 0 & 0 & 4 \end{bmatrix}$
- $\mathbf{x_2} = \begin{bmatrix} 9 & 7 & 5 & 3 \end{bmatrix}$
- $\mathbf{x_3} = \begin{bmatrix} \text{The first 4 digits of your student number} \end{bmatrix}$

(c) For the case (b) above, it has been postulated that the coefficients, **c**, should always be given by $\mathbf{c} = \mathbf{W}^{-1}\mathbf{x}$. Please prove or disprove this.

(d) For case (b), how does the coefficient c_1 relate to the values $x_1, \ldots, x_4$?

Q4. Reconstruction Efforts in Sinc! **[20 points]**

(a) Show that the signal ***f(t)***, reconstructed from samples ***f[nT]*** using

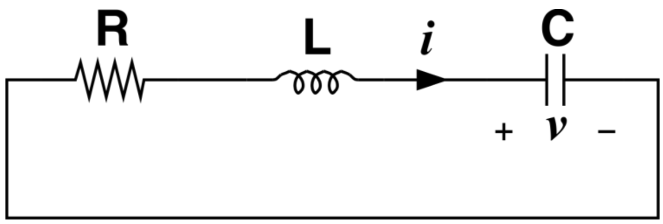
$$f(t) = \sum_n f[nT] \operatorname{sinc}(2\pi Bt - n\pi)$$ has a bandwidth B ≤ ½T Hz

[**Hint**: This is Equation 5.10b [p. 324] in Lathi]

(b) Are our Reconstructions Sunk?
Show that ***f(t)*** is the smallest bandwidth signal that passes through the samples ***f[nT]***.

Q5. Applications: Critical Damping? **[30 points]**

Given the Series RLC circuit below:



Recall by KVL we have: $Ri + L\frac{di}{dt} + v = 0$ and that $i = C\frac{dv}{dt}$

(a) What is the 2nd order ODE that describes this system? Is it linear? Causal?

(b) Please solve this LCCODE.
What is the oscillation frequency (often denoted **ω**) of this system?

(c) A "lazy" engineer specifies a case with **L = 1H**, **C = 1F**, and $\mathbf{V_0}$**=1V** (V_0 is the initial voltage across the capacitor) and $\boldsymbol{i_0}$**=0 mA** (i.e., zero initial current in the circuit). They also specify a resistor of **R=2Ω** (we define this as Case I). Discuss what the result is in this case.

(d) Consider and Discuss what happens for the cases where: **R=1.99Ω** (Case II) and **R=2.01Ω** (Case III).

(e) Imagine this was to be checked by a "low-cost" digital sampling oscilloscope rated at 2 S/s (2 Samples per second). Would this work?
[**Hint**: What is the Nyquist rate required?]

(f) Please plot three voltage waveforms of Cases I, II, and III (using Matlab or an equivalent system of your choice).
[**Hint**: Does this confer with your previous analysis?]

(g) A "crafty" engineer suggests changing this to a parallel configuration (as shown below). Please briefly discuss what the implications of this would be.
[**Hint**: what is the new **ω**?]