→ SPNS – LECTURE 10 (B) – 5/12/2015

minimize $\|Ax - B\|$

subject to $Cx = D$

→ for a variable x

⇒ least-norm problem

$$\min \quad (1/2)\|Ax - B\|^2$$
$$\text{s/t} \quad Cx = D$$

$$x = (A^T A)^{-1}(A^T B - C^T (C(A^T A)^{-1} C^T)^{-1} \ldots$$
$$(C(A^T A)^{-1} A^T B - D)$$

→ Lagrangian

$$L(x, \lambda) = \frac{1}{2}\|Ax - B\|^2 + \lambda^T (Cx - D)$$
$$= (1/2)\, x^T A^T A x - B^T A x + (1/2)(B^T B) + \ldots$$
$$\lambda^T C x - \lambda^T d$$

OPTIMALITY CONDITIONS ARE:

$$\nabla_x L = A^T A x - A^T B + C^T \lambda = 0$$

and $\nabla_x L = Cx - D = 0$

WRITE This in Block MATRIX form AS:

$$\begin{bmatrix} A^T A & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} A^T B \\ D \end{bmatrix}$$

if the Block Matrix is invertible:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} A^T A & C^T \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} A^T B \\ D \end{bmatrix}$$

Now if $A^T A$ is invertible we can derive a more explicit formula for x

$$x = (A^T A)^{-1} (A^T B - C^T \lambda)$$

if we substitute $Cx = D$

$$\downarrow$$

$$C (A^T A)^{-1} (A^T B - C^T \lambda) = D$$

so $\lambda = \left(C (A^T A)^{-1} C^T \right)^{-1} \left(C (A^T A)^{-1} (A^T B) - D \right)$