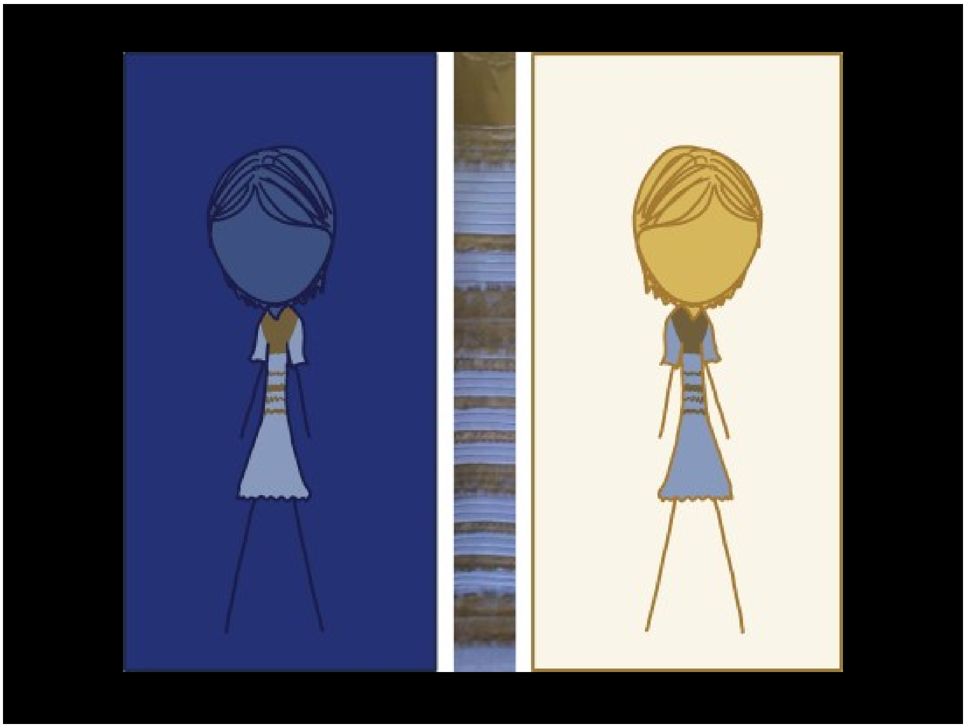


Signals & Systems Terminology

http://elec3004.org

ELEC 3004: **Digital Linear Systems**: Signals & Controls
Dr. Surya Singh

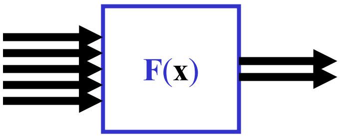
Open Lecture 1

elec3004@itee.uq.edu.au
http://robotics.itee.uq.edu.au/~elec3004/

March 3, 2015

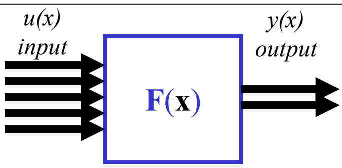
An Overview of Systems

- Today we are going to look at **F**(x)!



- F(x): System Model
  - The rules of operation that describe it's behaviour of a "system"
  - Predictive power of the responses
  - Analytic forms > Empirical ones
    - Analytic formula offer various levels of detail
    - Not everything can be experimented on *ad infinitum*
    - Also offer Design Intuition (let us devise new "systems")
    - Let's us do **analysis**! (determine the outputs for an input)
  - Various Analytic Forms
    - Constant, Polynomial, **Linear**, Nonlinear, Integral, **ODE**, PDE, Bayesian…

Linear Systems



- Model describes the relationship between the input **u**(x) and the output **y**(x)
- If it is a Linear System (wk 3):

$$y(t) = \int_0^t F(t-\tau)\, u(\tau)\, d\tau$$

- If it is also a (Linear and) **lumped**, it can be expressed **algebraically** as:

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t)$$
$$y(t) = C(t)\, x(t) + D(t)\, u(t)$$

- If it is also (Linear and) **time invariant** the matrices can be reduced to:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$\mathscr{L}$aplacian: $$y(s) = F(s)\, u(s)$$

Discrete Variations & Stability

$$y\left(s\right) = F\left(s\right) u\left(s\right)$$

- Is in <u>continuous time</u> …
- To move to <u>discrete time</u> it is more than just "sampling" at: 2 × (biggest Frequency)
- Discrete-Time Exponential

$$F(t) \rightarrow F\left[kT\right]$$

$$e^{\frac{k}{T}} = \gamma^{k}$$

$$\frac{1}{T} = \ln \gamma$$

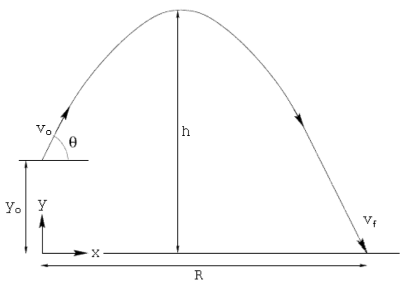
- SISO to MIMO
  - Single Input, Single Output
  - Multiple Input, Multiple Output
- BIBO:
  - Bounded Input, Bounded Output
- Lyapunov:
  - Conditions for Stability
  - ➔ Are the results of the system asymptotic or exponential

Further Classifications of Systems

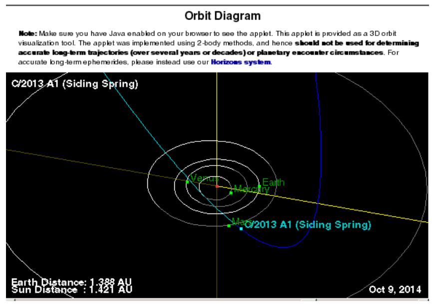
1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Modelling order

- Modelling order depends on what you are trying to achieve



The parabolic trajectory of a projectile

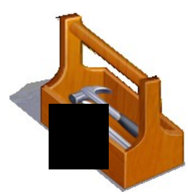


http://ssd.jpl.nasa.gov/

JPL Small-Body Database Browser.url

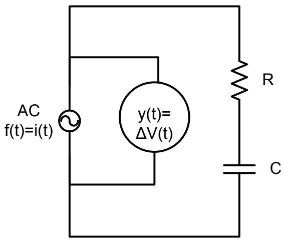
Modelling Ties Back with ELEC 2004

- Linear Circuit Theorems, Operational Amplifiers
- Operational Amplifiers
- Capacitors and Inductors, RL and RC Circuits
- AC Steady State Analysis
- AC Power, Frequency Response
- Laplace Transform
- Reduction of Multiple Sub-Systems
- Fourier Series and Transform
- Filter Circuits



➔ Modelling Tools!

Example: RC Circuits



$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{t} f(\tau)\,d\tau$$

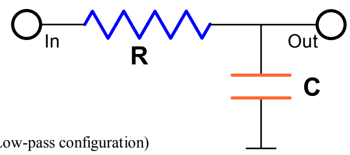
$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{0} f(\tau)\,d\tau + \frac{1}{C}\int_{0}^{t} f(\tau)\,d\tau$$

$$y(t) = v_C(0) + Rf(t) + \frac{1}{C}\int_{0}^{t} f(\tau)\,d\tau$$

$$y(t) = v_C(t_0) + Rf(t) + \frac{1}{C}\int_{t_0}^{t} f(\tau)\,d\tau$$

First Order RC Filter

- Passive, First-Order **R**esistor-**C**apacitor Design:



(Low-pass configuration)

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

- 3dB (½ Signal Power):

$$\omega = 2\pi f$$

$$f_c = \frac{1}{2\pi \mathbf{RC}}$$

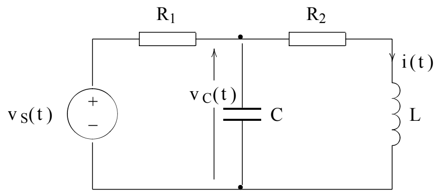
- Magnitude:

$$|V_{\text{out}}| = \sqrt{\frac{1}{(\omega RC)^2}}\,|V_{\text{in}}|$$

- Phase:

$$\phi = \tan^{-1}(-\omega RC)$$

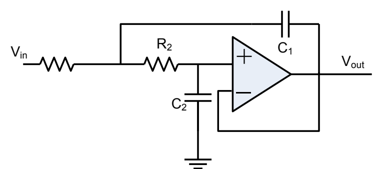
Example of 2nd Order: RLC Circuits



- KCL: $$\frac{V_s(t)-V_c(t)}{R_1}=C\frac{d}{dt}V_c(t)+i(t)$$
- KVL: $$V_c(t)=L\frac{d}{dt}i(t)+R_2 i(t)$$
- Combining: $$V_s(t)=R_1LC\frac{d^2}{dt^2}i(t)+(L+R_1R_2C)\frac{d}{dt}i(t)+(R_1+R_2)\,i(t)$$

2nd Order Active RC Filter (Sallen–Key)

- 2nd Order System Sallen–Key Low-Pass Topology:

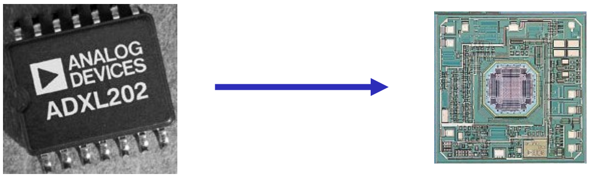


Build this for
Real in
ELEC 4403

- KCL: $$\frac{v_{\text{in}}-v_x}{R_1}=C_1s\left(v_x-v_{\text{out}}\right)+\frac{v_x-v_{\text{out}}}{R_2}$$
- Combined with Op-Amp Law: $$\frac{v_{\text{in}}-v_{\text{out}}(C_2sR_2+1)}{R_1}=C_1sv_{\text{out}}\left(C_2sR_2+1\right)-v_{\text{out}}+\frac{v_{\text{out}}(C_2sR_2+1)-v_{\text{out}}}{R_2}$$
- Solving for Gives a 2nd order System:

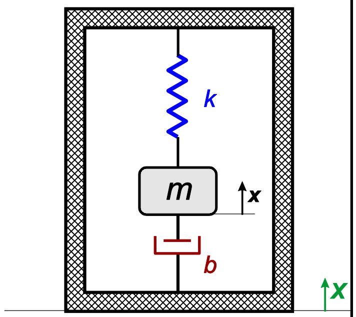
$$\frac{v_{out}}{v_{in}}=\frac{1}{C_1C_2R_1R_2s^2+C_2(R_1+R_2)s+1}$$

Another 2nd Order System: Accelerometer or Mass Spring Damper (MSD)



- General accelerometer:
  - Linear spring (k) (0^{th} order w/r/t o)
  - Viscous damper (b) (1^{st} order)
  - Proof mass (m) (2^{nd} order)

➔ Electrical system analogy:
  - resistor (R) : damper (b)
  - inductance (L) : spring (k)
  - capacitance (C) : mass (m)



Measuring Acceleration: Sense *a* by measuring spring motion *Z*

- Start with Newton's 2^{nd} Law:

$$ma = F$$

- Substitute:

$$m\frac{d^2x}{dt^2} = k(X-x) + b\frac{d(X-x)}{dt}$$

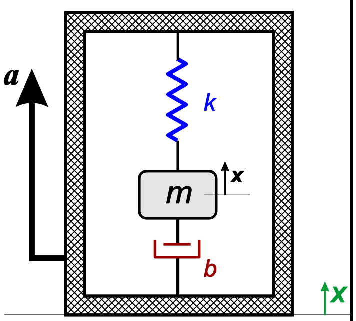
The "displacement" measured by the unit (the motion of *m* relative the accelerometer frame)

$$\mathbf{Z} \equiv (X-x) \rightarrow x = X - \mathbf{Z}$$

$$\Rightarrow m\frac{d^2X}{dt^2} = m\frac{d^2\mathbf{Z}}{dt^2} + k\mathbf{Z} + b\frac{d\mathbf{Z}}{dt}$$

- Solve ODE:

$$X(t) = X_0 e^{i\omega t} \qquad \mathbf{Z}(t) = Z_0 e^{i\omega t}$$



Measuring Acceleration [2]

- Substitute candidate solutions:

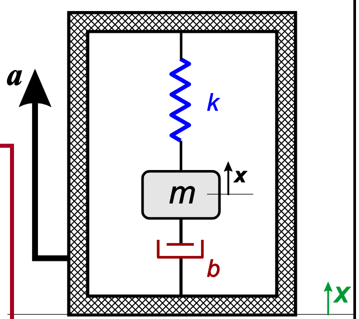
$$m\frac{d^2\left(X_0e^{i\omega t}\right)}{dt^2} = m\frac{d^2\left(Z_0e^{i\omega t}\right)}{dt^2} + k\left(Z_0e^{i\omega t}\right) + b\frac{d\left(Z_0e^{i\omega t}\right)}{dt}$$

$$-m\omega^2 X_0e^{i\omega t} = -m\omega^2 Z_0e^{i\omega t} + kZ_0e^{i\omega t} + (i\omega)\,bZ_0e^{i\omega t}$$

- Define Natural Frequency (ω_0) & Simplify for Z_0 (the spring displacement "magnitude"):

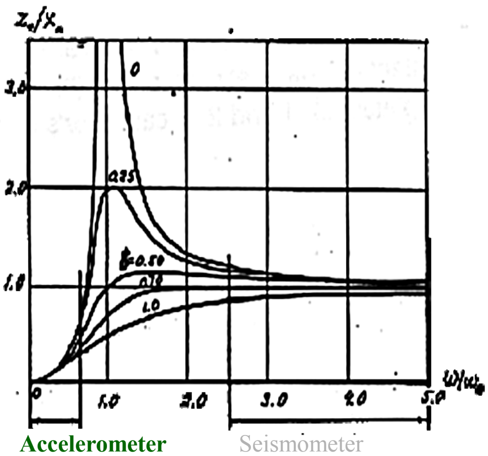
$$\omega_0 \equiv \sqrt{\frac{k}{m}}$$

$$Z_0 = \frac{m\omega^2 X_0}{m\omega^2 - k - i\omega b} = \boxed{\frac{X_0}{\sqrt{1-\frac{{\omega_0}^2}{\omega^2}-\frac{b^2}{m^2\omega^2}}}}$$



Acceleration: 2nd Order System

- Plot for a "unit" mass, etc



- For $\omega \ll \omega_0$:

$$Z_0 \approx \frac{\omega^2 X_0}{\omega_0^2} = \frac{a}{\omega_0^2}$$

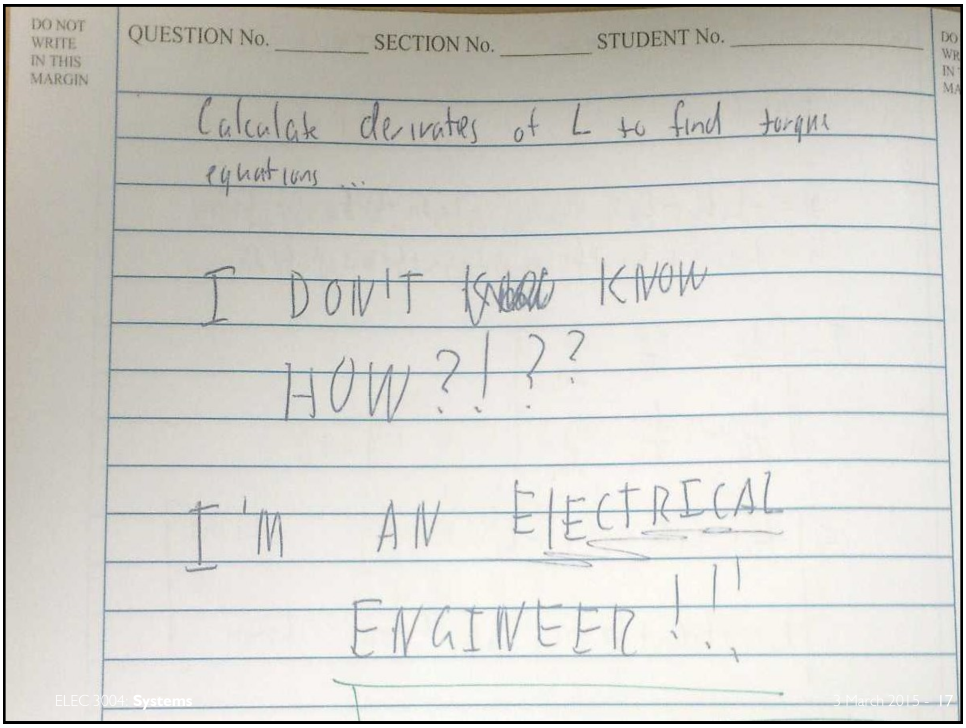
$$\rightarrow a = Z_0\omega_0^2$$

➔ it's an Accelerometer

- For $\omega \sim \omega_0$
  - As: b➔ 0, Z ➔ ∞
  - Sensitivity ↑
- For $\omega \gg \omega_0$:

$$Z_0 \approx X_0$$

➔ it's a Seismometer


DO NOT WRITE IN THIS MARGIN

QUESTION No. ________ SECTION No. ________ STUDENT No. ________________

Calculate derivates of L to find torque equations ...

I DON'T ~~know~~ KNOW HOW?!??

I'M AN ELECTRICAL ENGINEER!!!

Equivalence Across Domains

Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

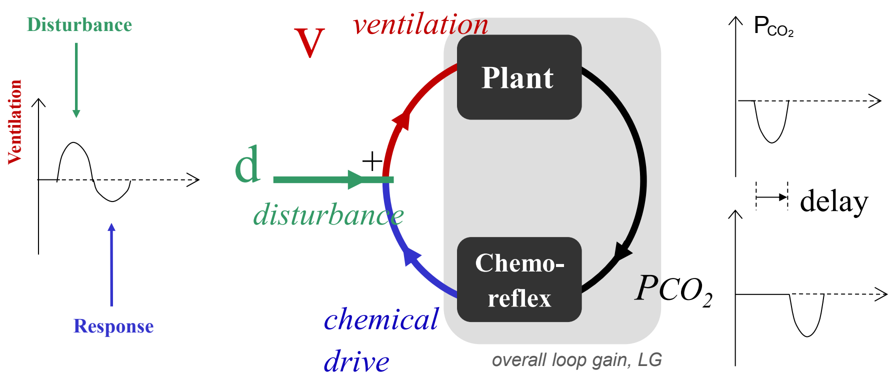
Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	v_2 L i v_1
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	v_2 k v_1 F
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	ω_2 k ω_1 T
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	P_2 I Q P_1
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}{}^2$	v_2 i C v_1
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2{}^2$	F v_2 M $v_1 =$ constant
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2{}^2$	T ω_2 J $\omega_1 =$ constant
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}{}^2$	Q P_2 C_f P_1
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	q $\mathcal{T}_2$ C_t $\mathcal{T}_1 =$ constant
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}{}^2$	v_2 R i v_1
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}{}^2$	F v_2 b v_1
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}{}^2$	T ω_2 b ω_1
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}{}^2$	P_2 R_f Q P_1
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{T}_2$ R_t q $\mathcal{T}_1$

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 74

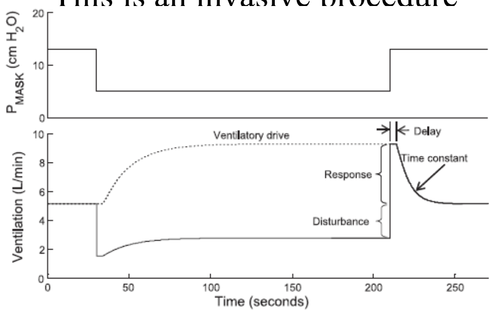
"Loop Gain" to Quantify Ventilatory Stability:



- **Loop Gain =** $\frac{\textit{Response}}{\textit{Disturbance}}$
- **Loop Gain>1 implies an unstable control system**
- **Loop Gain<1 implies a stable control system**
- **Like EEG, disturbance can be characterised by frequency**

Measuring LG – 3min CPAP Drop:

- This is an invasive procedure



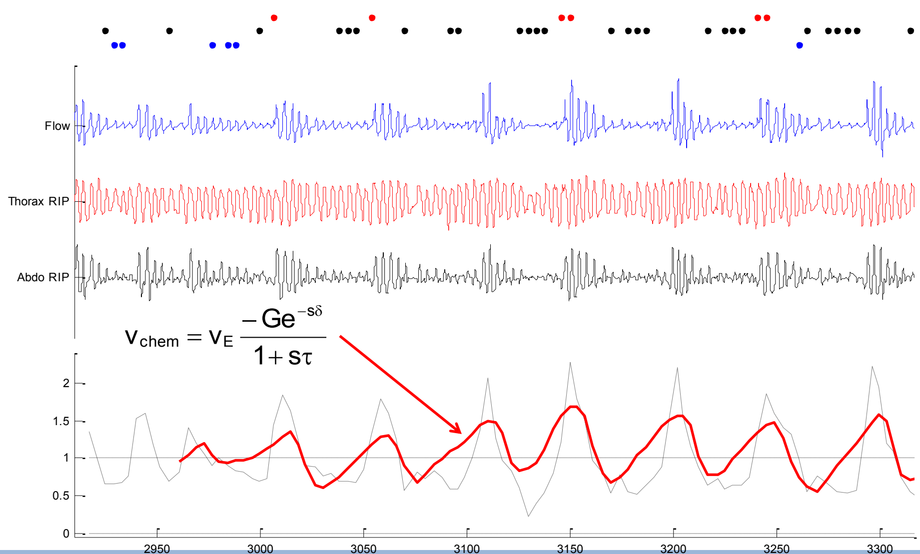
[Wellman, JAP, 2011]

ep lab:
ult

n be applied in a clinical PSG

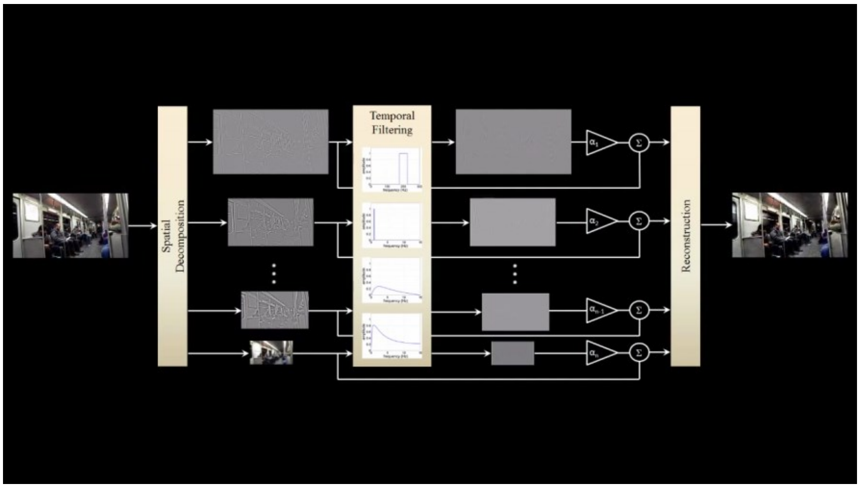
In this study we thus hypothesise that obstructive apnoea's themselves perturb the ventilatory control system, and therefore by fitting an appropriate mathematical model, to the ventilatory response to apnoea's, we can estimate loop gain parameters

Estimating LG from Clinical PSG:



LGn=0.64 LG60=0.9 LG30=0.46

Video Share: Eulerian Video Magnification



Next Time…

- We'll talk about System Models
- Review:
  - Phasers, complex numbers, polar to rectangular, and general functional forms.
  - Chapter 1 of Lathi
    (particularly the first sections on signals & classification thereof)
- Register on Platypus
- Try the practise assignment (will be posted soon)