ELEC3004 Open Lecture 16/03/2015

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1 System Attributes

Let us consider a system, represented by:

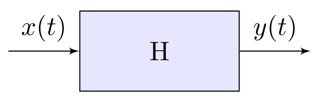

Figure 1: The system under consideration

1.1 Linearity

Linear systems satisfy the principle of **Superposition**. Let:

$$x_1(t) \xrightarrow{H} y_1(t)$$
$$x_2(t) \xrightarrow{H} y_2(t)$$

Where $\xrightarrow{H}$ represents the system acting on the input x to produce output y.

The system is linear iff:

1. The system response to the input $x_1(t) + x_2(t)$ is equal to the output $y_1(t) + y_2(t)$.
2. The system response to the input $\alpha x_1(t)$ is equal to the output $\alpha y_1(t)$.

Property 1 is the *additivity property*. Property 2 is the *homogeneity property*, also known as the scaling property.

Combining these two properties gives the definition of superposition:

$$\boxed{\alpha_1 x_1(t) + \alpha_2 x_2(t) \xrightarrow{H} \alpha_1 y_1(t) + \alpha_2 y_2(t)} \tag{1}$$

Test the following two systems for linearity:

$$y(t) = e^{x(t)}$$
$$y(t) = k\frac{dx(t)}{dt}$$

1.1.1 Example 1

Define

$$y_1(t) = e^{x_1(t)}$$
$$y_2(t) = e^{x_2(t)}$$

Now substitute the LHS of (1) into the original equation, rearrange and simplify

$$\begin{aligned} y(t) &= e^{x(t)} \\ &= e^{\alpha_1 x_1(t) + \alpha_2 x_2(t)} \\ &= e^{\alpha_1 x_1(t)} e^{\alpha_2 x_2(t)} \\ &= [y_1(t)]^{\alpha_1} [y_2(t)]^{\alpha_2} \end{aligned}$$

As $y(t) \neq \alpha_1 y_1(t) + \alpha_2 y_2(t)$ this system is non-linear. An example of this kind of system is a diode.

1.1.2 Example 2

Define

$$y_1(t) = k\frac{dx_1(t)}{dt}$$
$$y_2(t) = k\frac{dx_2(t)}{dt}$$

Now substitute the LHS of (1) into the original equation, rearrange and simplify

$$\begin{aligned} y(t) &= k\frac{dx(t)}{dt} \\ &= k\frac{d}{dt}[\alpha_1 x_1(t) + \alpha_2 x_2(t)] \\ &= \alpha_1 k\frac{dx_1(t)}{dt} + \alpha_2 k\frac{dx_2(t)}{dt} \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned}$$

As $y(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$ this is system is linear. Can you think of an example of this kind of system?

1.2 Time-Invariance

Time invariant systems are systems whose output does not depend on the time of the input signal applied. To test for time-invariance, use the following steps:

Step 1 Let $y_1(t)$ be the output corresponding to $x_1(t)$.

Step 2 Consider a 2nd input $x_2(t)$ detained by a shift of $x_1(t)$ i.e. $x_2(t) = x_1(t - t_0)$.

Step 3 From step 1, find $y_1(t - t_0)$.

Step 4 If $y_2(t) = y_1(t - t_0)$ the system is time-invariant.

Test the following two systems for linearity:

$$y(t) = cosx(t)$$
$$y(t) = x(t)cos(t)$$

1.2.1 Example 3

Step 1 Let $y_1(t) = cosx_1(t)$.

Step 2 Let $x_2(t) = x_1(t - t_0)$, then $y_2(t) = cosx_2(t) = cosx_1(t - t_0)$.

Step 3 $y_1(t - t_0) = cosx_1(t - t_0)$.

Step 4 $y_2(t) = y_1(t - t_0) = cosx_1(t - t_0)$.

As $y_1(t - t_0) = y_2(t)$ the system is time-invariant.

1.2.2 Example 4

Step 1 Let $y_1(t) = x_1(t)cos(t)$.

Step 2 Let $x_2(t) = x_1(t - t_0)$, then $y_2(t) = x_1(t - t_0)cos(t)$.

Step 3 $y_1(t - t_0) = x_1(t - t_0)cos(t - t_0)$.

Step 4 $y_2(t) \neq y_1(t - t_0)$.

As $y_1(t - t_0) \neq y_2(t)$ the system is not time-invariant.

1.3 Closing Remarks

Linearity and time-invariance are two very important properties of systems. Consider why they are important properties. What do they allow us to do?

The examples worked through here were drawn from Soliman and Srinath [1].

References

[1] S.S. Soliman and M.D. Srinath. *Continuous and Discrete Signals and Systems.* International edition. Prentice-Hall International, 1998.