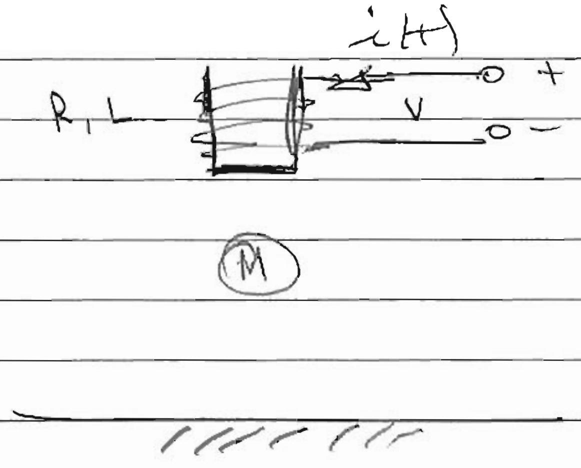
SPNS - 5/18/2015 — LECTURE 11 — EXAMPLE



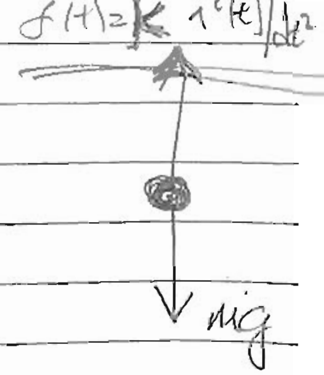
Input: $v(t)$: Voltage to control circuit.
Output: $y(t)$: Distance Between The Magnet & the coil

$\boxed{1}$ KVL

$$R\, i(t) + L \frac{di(t)}{dt} = v(t)$$

$\boxed{2}$ N-2nd-Law : $f = ma$

$$M \frac{d^2 y(t)}{dt^2} = -k \frac{i^2(t)}{y(t)} + Mg$$



$x_1(t) = i(t)$
$x_2(t) = y(t)$
$x_3(t) = \dot{y}(t)$
$u(t) = v(t)$

$x_1(t) = i(t)$, $x_2 = y$, $x_3 = \dot{y}$, $u(t) = v(t)$

$$\dot{\vec{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \dot{i} \\ \dot{y} \\ \ddot{y} \end{bmatrix}$$

$$\vec{y} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \boxed{I} \\ \boxed{II} \\ \boxed{III} \end{bmatrix}$$

what is $\boxed{I}$: what is $\dot{i}$

$$R\,i(t) + L\dot{i}(t) = v(t)$$

$$\dot{i}(t) = (v(t) - R\,i(t))/L$$

$$= \frac{u(t) - R x_1(t)}{L} \qquad \boxed{I}$$

what is $\boxed{II}$:

$$x_2(t) = y \quad \therefore \dot{x}_2 = \dot{y}$$

$$\Rightarrow \dot{x}_2 = x_3 \qquad \boxed{II}$$

What is $\boxed{III}$ $= \dot{x}_3$

$X_3 = \dot{y} \quad \therefore \quad \dot{x}_3 = \ddot{y}$

$M\ddot{y} = -k\,\cancel{i^2}\,/y + Mg$

$\therefore \quad \dot{x}_3 = \ddot{y} = \left[-k\dfrac{i^2}{y} + g\right]\left[\dfrac{1}{M}\right]$

$= -\dfrac{\cancel{k}\,x_1^2(t)}{Mx_2(t)} + g \qquad \boxed{III}$

<u>LINEARIZE</u> $\Rightarrow$ ABOUT A CONTROL POINT y_0

$$
\left[
\begin{array}{l}
0 = \dfrac{\bar{u}_0 - R\bar{x}_1}{L} \qquad\qquad \bar{q} = \bar{q}_0 \\
0 = \overline{x_3} \\
0 = \dfrac{\overline{x_1^2}}{M\overline{x_2}} + g \\
y_0 = \overline{x_2}
\end{array}
\right.
$$

$$\bar{u}_0 = R\sqrt{Mg\bar{y}_0}$$

$$\cancel{\bar{X} = \begin{bmatrix}\bar{x}_1\\ \bar{x}_2\\ \bar{x}_3\end{bmatrix} = \begin{bmatrix}\sqrt{Mg} \\ \\ \end{bmatrix}}$$

$$\vec{X} = \begin{bmatrix}\bar{x}_1\\ \bar{x}_2\\ \bar{x}_3\end{bmatrix} = \begin{bmatrix}\sqrt{Mg\bar{y}_0}\\ y_0\\ 0\end{bmatrix}$$

Voltage; $u(t) = R\sqrt{Mg\bar{y}_0}$

$$u(t) = u(t) - u(0) = u(t) - R\sqrt{Mg\bar{y}_0}$$

$$\tilde{x} = x(t) - x_0 = \begin{bmatrix} x(t) - \sqrt{Mgy_0} \\ x(t) - y_0 \\ x_3(t) \end{bmatrix}$$

Linearize the model

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y = Cx + du$$

Where;

$$A = \begin{bmatrix} -R/L & 0 & 0 \\ 0 & 0 & 1 \\ -2\sqrt{\frac{g}{My_0}} & g/y_0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$