http://elec3004.org

Digital Control Design

ELEC 3004: **Digital Linear Systems**: Signals & Controls

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Lecture 10-Part A (Slide 13, Revised May 13)

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May 11, 2015

Schedule

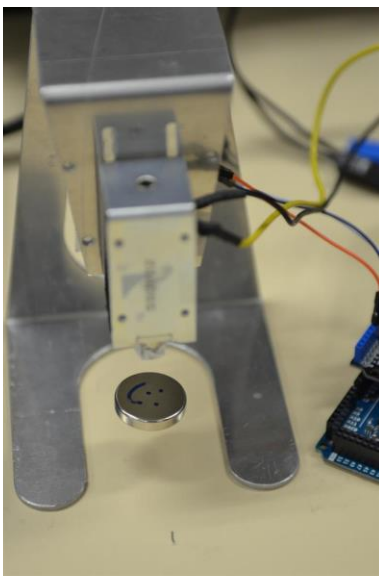
Week	Date	Lecture Title
1	2-Mar	Introduction
	3-Mar	Systems Overview
2	9-Mar	Signals as Vectors & Systems as Maps
	10-Mar	[Signals]
3	[illegible]	[illegible]ata Acquisition & Antialiasing Filters
	17-Mar	[Sampling]
4	23-Mar	System Analysis & Convolution
	[illegible]	[Convolution & FT]
5	30-Mar	Discrete Systems & Z-Transforms
	31-Mar	[Z-Transforms]
6	13-Apr	Frequency Response & Filter Analysis
	14-Apr	[Filters]
[illegible]	20-Apr	Digital Filters
	[illegible]	[illegible]
8	27-Apr	Discrete Systems Analysis
	28-Apr	[Feedback]
9	4-May	Introduction to (Digital) Control
	5-May	[Digitial Control]
10	11-May	Digital Control Design & State-Space
	12-May	Controllability & Observability
11	18-May	Stability of Digital Systems
	19-May	[Stability]
12	25-May	Applications in Industry
	26-May	Digitial Control System Hardware
13	1-Jun	System Identification & Information Theory + Communications
	2-Jun	Summary and Course Review

Outline:
(1) Review: 2nd Order Systems
(2) PID
(3) Tuning Controllers
(4) State-Space
(5) Least Squares
(6) Quantization Effects (and Handling this via Least Squares)

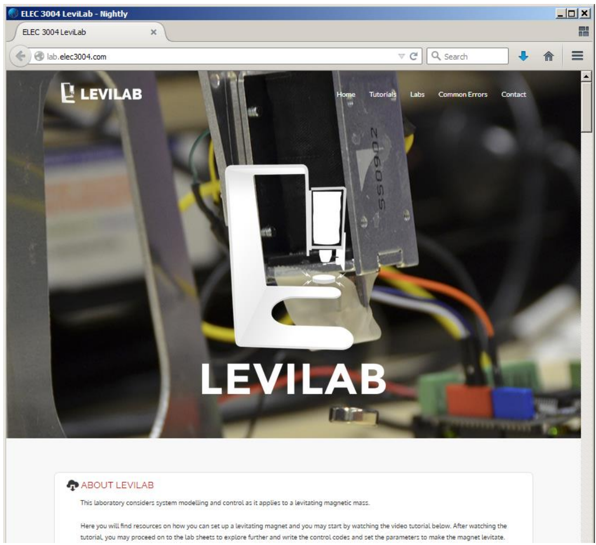
Announcements:

- **PS 3 Grading:**
  - Let's see if we can get this done in 24-hours
  - We're working on it
- **PS 4:**
  - Working on it too!
- **PS 5:**
  - Working on it too too!!
- Final Exam Logistics:
  - Friday 26/6/2014 at 2:30pm
  - Location: TBA
  - Closed-book
  - Practice exam will be posted soon (exams from 2014, 2013, 2012 are online!) (Working on it too too too!!)

Lab 3 - LeviLab



LeviLab Interactive – http://lab.elec3004.com



Webpage was a Summer Research Project by Adeline Toh and Isabel Wong

Review: Direct Design: Second Order Digital Systems

Consider the z-transform of a decaying exponential signal:

$$y(t) = e^{-at}\cos(bt)\,\mathcal{U}(t) \qquad (\mathcal{U}(t) = \text{unit step})$$

- ⋆ sample: $y(kT) = r^k \cos(k\theta)\,\mathcal{U}(kT)$ with $r = e^{-aT}$ & $\theta = bT$
- ⋆ transform: $Y(z) = \dfrac{1}{2}\dfrac{z}{(z - re^{j\theta})} + \dfrac{1}{2}\dfrac{z}{(z - re^{-j\theta})}$

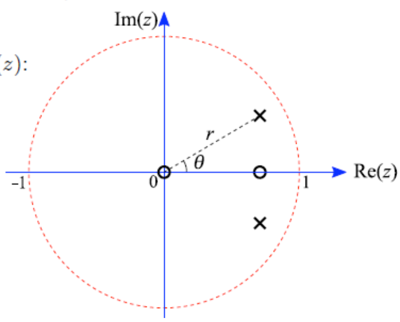
$$= \frac{z(z - r\cos\theta)}{(z - re^{j\theta})(z - re^{-j\theta})}$$

- ⋆ e.g. y_k is the pulse response of $G(z)$:

$$G(z) = \frac{z(z - r\cos\theta)}{(z - re^{j\theta})(z - re^{-j\theta})}$$

poles: $\begin{cases} z = re^{j\theta} \\ z = re^{-j\theta} \end{cases}$

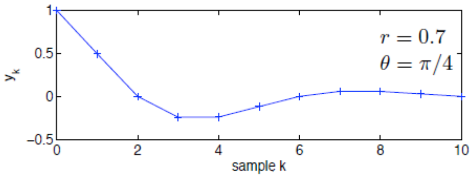
zeros: $\begin{cases} z = 0 \\ z = r\cos\theta \end{cases}$



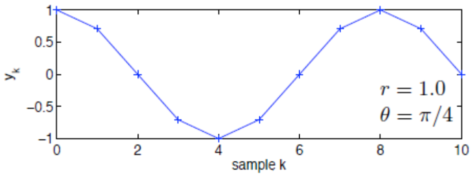
Response of 2nd order system [1/3]

Responses for varying r:

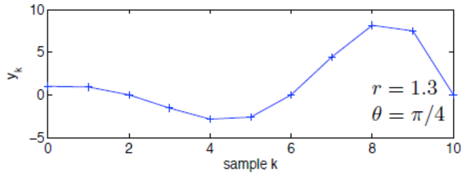
- $r < 1$
  $\Downarrow$
  exponentially decaying envelope



- $r = 1$
  $\Downarrow$
  sinusoidal response with $2\pi/\theta$ samples per period



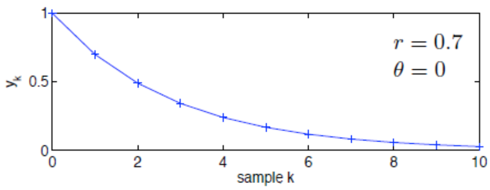
- $r > 1$
  $\Downarrow$
  exponentially increasing envelope



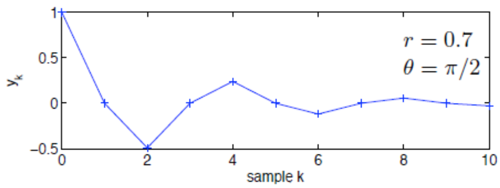
Response of 2nd order system [2/3]

Responses for varying θ:

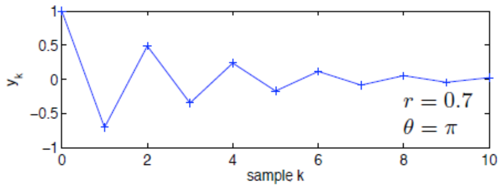
- $\theta = 0$
  $\Downarrow$
  decaying exponential



- $\theta = \pi/2$
  $\Downarrow$
  $2\pi/\theta = 4$ samples per period



- $\theta = \pi$
  $\Downarrow$
  2 samples per period



Response of 2nd order system [3/3]

Some special cases:

- for $\theta = 0$, $Y(z)$ simplifies to:

$$Y(z) = \frac{z}{z - r}$$

$\Longrightarrow$ exponentially decaying response

- when $\theta = 0$ and $r = 1$:

$$Y(z) = \frac{z}{z - 1}$$

$\Longrightarrow$ unit step

- when $r = 0$:

$$Y(z) = 1$$

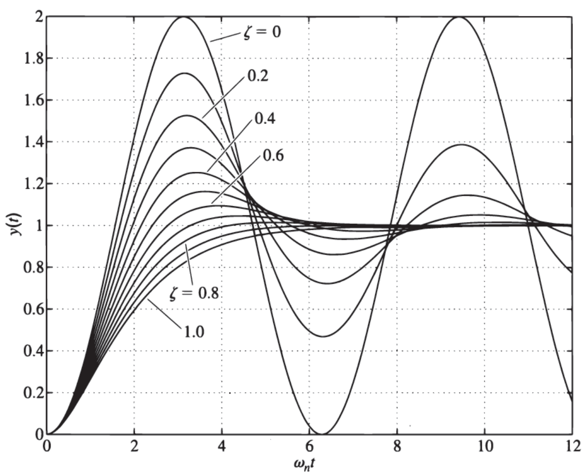
$\Longrightarrow$ unit pulse

- when $\theta = 0$ and $-1 < r < 0$:

samples of alternating signs

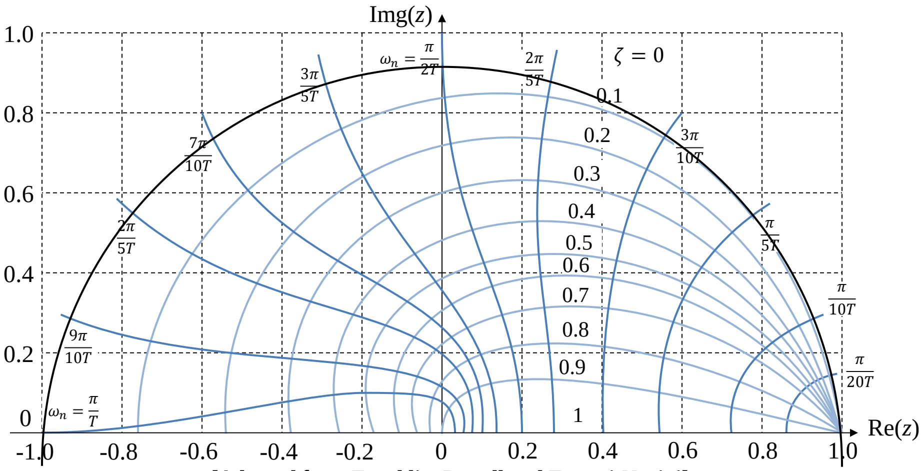
2[nd] Order System Response

- Response of a 2[nd] order system to increasing levels of damping:



Damping and natural frequency

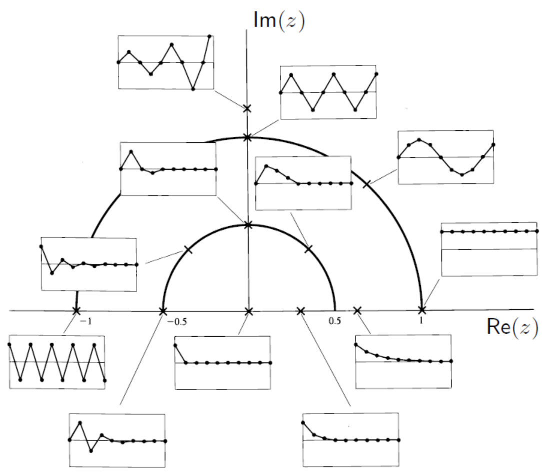
$$z = e^{sT} \text{ where } s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$



[Adapted from Franklin, Powell and Emami-Naeini]

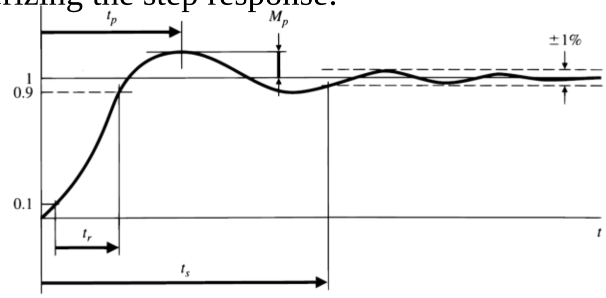
Pole positions in the z-plane

- Poles inside the unit circle are **stable**
- Poles outside the unit circle **unstable**
- Poles on the unit circle are oscillatory
- Real poles at 0 < z < 1 give exponential response
- Higher frequency of oscillation for larger
- Lower apparent damping for larer and r



2^{nd} Order System Specifications

Characterizing the step response:



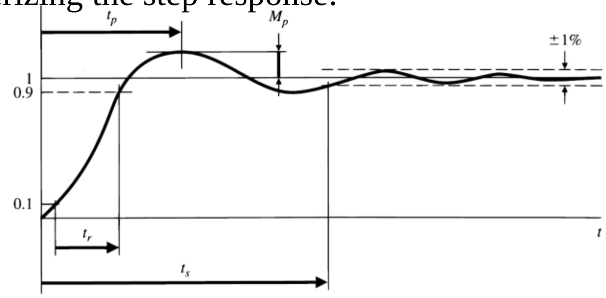
- Rise time (10% → 90%): $t_r \approx \frac{1.8}{\omega_0}$
- Overshoot: $M_p \approx \frac{e^{-\pi\zeta}}{\sqrt{1-\zeta^2}}$
- Settling time (**<u>to 1%</u>**): $t_s = \frac{4.6}{\zeta\omega_0}$
- Steady state error to unit step: e_{ss}
- Phase margin:

$$\phi_{PM} \approx 100\zeta$$

Why 4.6? It's **-ln(1%)**
$\rightarrow e^{-\zeta\omega_0} = 0.01 \rightarrow \zeta\omega_0 = 4.6 \rightarrow t_s = \frac{4.6}{\zeta\omega_0}$

2^{nd} Order System Specifications

Characterizing the step response:



- Rise time (10% → 90%) & Overshoot:
  t_r, M_p → ζ, ω_0 : Locations of dominant poles
- Settling time (to 1%):
  t_s → radius of poles: $|z| < 0.01^{\frac{T}{t_s}}$
- Steady state error to unit step:
  e_{ss} → final value theorem $e_{ss} = \lim_{z \to 1} \{(z-1)\,F(z)\}$

Ex: System Specifications → Control Design [1/4]

Design a controller for a system with:

- A continuous transfer function: $G(s) = \dfrac{0.1}{s(s+0.1)}$
- A discrete ZOH sampler
- Sampling time (T_s): T_s= 1s
- Controller:

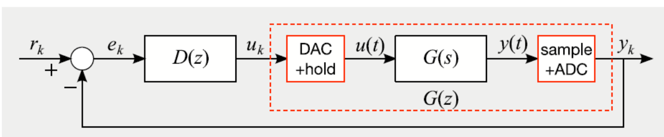
$$u_k = -0.5u_{k-1} + 13\left(e_k - 0.88e_{k-1}\right)$$

The closed loop system is required to have:

- $M_p < 16\%$
- $t_s < 10$ s
- $e_{ss} < 1$

Ex: System Specifications → Control Design [2/4]

1. (a) Find the pulse transfer function of $G(s)$ plus the ZOH



$$G(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{G(s)}{s}\right\} = \frac{(z-1)}{z}\mathcal{Z}\left\{\frac{0.1}{s^2(s+0.1)}\right\}$$

e.g. look up $\mathcal{Z}\{a/s^2(s+a)\}$ in tables:

$$\begin{aligned} G(z) &= \frac{(z-1)}{z}\,\frac{z\Big((0.1 - 1 + e^{-0.1})z + (1 - e^{-0.1} - 0.1e^{-0.1})\Big)}{0.1(z-1)^2(z - e^{-0.1})} \\ &= \frac{0.0484(z + 0.9672)}{(z-1)(z-0.9048)} \end{aligned}$$

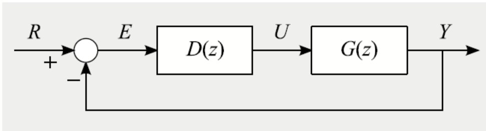
(b) Find the controller transfer function (using z = shift operator):

$$\frac{U(z)}{E(z)} = D(z) = 13\frac{(1 - 0.88z^{-1})}{(1 + 0.5z^{-1})} = 13\frac{(z - 0.88)}{(z + 0.5)}$$

Ex: System Specifications → Control Design [3/4]

2. Check the steady state error e_{ss} when $r_k =$ unit ramp

$$e_{ss} = \lim_{k\to\infty} e_k = \lim_{z\to 1}(z-1)E(z)$$



$$\frac{E(z)}{R(z)} = \frac{1}{1+D(z)G(z)}$$

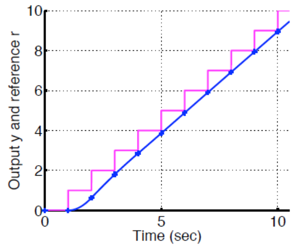
$$R(z) = \frac{Tz}{(z-1)^2}$$

so $$e_{ss} = \lim_{z\to 1}\left\{(z-1)\frac{Tz}{(z-1)^2}\frac{1}{1+D(z)G(z)}\right\} = \lim_{z\to 1}\frac{T}{(z-1)D(z)G(z)}$$

$$= \lim_{z\to 1}\frac{T}{(z-1)\dfrac{0.0484(z+0.9672)}{(z-1)(z-0.9048)}D(1)}$$

$$= \frac{1-0.9048}{0.0484(1+0.9672)D(1)} = 0.96$$

$\Longrightarrow$ $e_{ss} < 1$ (as required)



Ex: System Specifications → Control Design [4/4]

3. Step response: overshoot $M_p < 16\%$ $\Longrightarrow$ $\zeta > 0.5$

settling time $t_s < 10$ $\Longrightarrow$ $|z| < 0.01^{1/10} = 0.63$

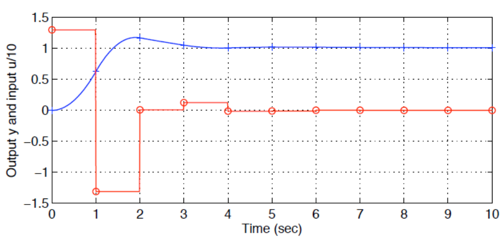
The closed loop poles are the roots of $1 + D(z)G(z) = 0$, i.e.

$$1 + 13\frac{(z-0.88)}{(z+0.5)}\frac{0.0484(z+0.9672)}{(z-1)(z-0.9048)} = 0$$

$$\Longrightarrow z = 0.88,\ -0.050 \pm j0.304$$

But the pole at $z = 0.88$ is cancelled by controller zero at $z = 0.88$, and

$$z = -0.050 \pm j0.304 = re^{\pm j\theta} \quad \Longrightarrow \begin{cases} r = 0.31,\ \theta = 1.73 \\ \zeta = 0.56 \end{cases}$$



all specs satisfied!

PID – the Good Stuff

- Proportional-Integral-Derivative control is the control engineer's hammer*
  - For P,PI,PD, etc. just remove one or more terms

$$C(s) = k\left(\underbrace{1}_{\text{Proportional}} + \underbrace{\frac{1}{\tau_i s}}_{\text{Integral}} + \underbrace{\tau_d s}_{\text{Derivative}}\right)$$

*Everything is a nail. That's why it's called "Bang-Bang" Control ☺

PID

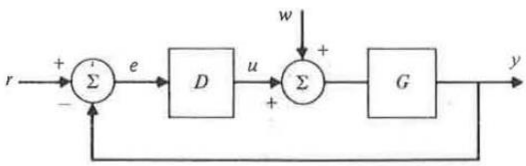
- Collectively, PID provides two zeros plus a pole at the origin
  - Zeros provide phase lead
  - Pole provides steady-state tracking
  - Easy to implement in microprocessors

- Many tools exist for optimally tuning PID
  - Zeigler-Nichols
  - Cohen-Coon
  - Automatic software processes

PID

- Three basic types of control:
  - Proportional
  - Integral, and
  - Derivative

- The next step up from lead compensation
  - Essentially a combination of proportional and derivative control



Proportional Control

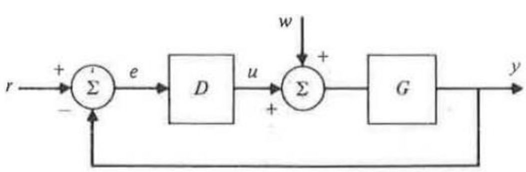
A discrete implementation of proportional control is identical to continuous; that is, where the continuous is

$$u(t) = K_p e(t) \quad \Rightarrow \quad D(s) = K_p,$$

the discrete is

$$u(k) = K_p e(k) \quad \Rightarrow \quad \boxed{D(z) = K_p}$$

where $e(t)$ is the error signal as shown in Fig 5.2.



Derivative Control

For continuous systems, derivative or rate control has the form

$$u(t) = K_p T_D \dot{e}(t) \quad \Rightarrow \quad D(s) = K_p T_D s$$

where T_D is called the *derivative time*. Differentiation can be approximated in the discrete domain as the first difference, that is,

$$u(k) = K_p T_D \frac{(e(k) - e(k-1))}{T} \quad \Rightarrow \quad \boxed{D(z) = K_p T_D \frac{1 - z^{-1}}{T} = K_p T_D \frac{z-1}{Tz}.}$$

In many designs, the compensation is a sum of proportional and derivative control (or PD control). In this case, we have

$$D(z) = K_p \left(1 + \frac{T_D(z-1)}{Tz}\right).$$

or, equivalently,

$$\boxed{D(z) = K \frac{z - \alpha}{z}}$$

Derivative Control [2]

- Similar to the lead compensators
  - The difference is that the pole is at $z = 0$

    [Whereas the pole has been placed at various locations along the z-plane real axis for the previous designs.]

- In the continuous case:
  - pure derivative control represents the ideal situation in that there is no destabilizing phase lag from the differentiation
  - the pole is at $s = -\infty$

- In the discrete case:
  - z=0
  - However this has phase lag because of the necessity to wait for one cycle in order to compute the first difference

Integral Control

For continuous systems, we integrate the error to arrive at the control,

$$u(t) = \frac{K_p}{T_I}\int_{t_o}^{t} e(t)dt \;\Rightarrow\; D(s) = \frac{K_p}{T_I s},$$

where T_I is called the *integral*, or *reset time*. The discrete equivalent is to sum all previous errors, yielding

$$u(k) = u(k-1) + \frac{K_p T}{T_I} e(k) \;\Rightarrow\; \boxed{D(z) = \frac{K_p T}{T_I(1-z^{-1})} = \frac{K_p T z}{T_I(z-1)}.} \quad (5.60)$$

Just as for continuous systems, the primary reason for integral control is to reduce or eliminate steady-state errors, but this typically occurs at the cost of reduced stability.

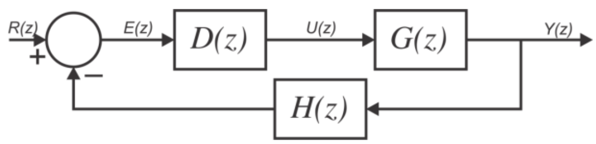
PID Control

$$\boxed{D(z) = K_p\left(1 + \frac{Tz}{T_I(z-1)} + \frac{T_D(z-1)}{Tz}\right).}$$

The user simply has to determine the best values of

- K_p
- T_D and
- T_I

PID as Difference Equation



$$\frac{U(z)}{E(z)} = D(z) = K_p + K_i\left(\frac{Tz}{z-1}\right) + K_d\left(\frac{z-1}{Tz}\right)$$

$$\boxed{u(k) = \left[K_p + K_i T + \left(\frac{K_d}{T}\right)\right] \cdot e(k) - [K_d T] \cdot e(k-1) + [K_i] \cdot u(k-1)}$$

PID Intuition & Tuning

- Tuning – How to get the “magic” values:
  - Dominant Pole Design
  - Ziegler Nichols Methods
  - Pole Placement
  - Auto Tuning

- Although PID is common it is often poorly tuned
  - The derivative action is frequently switched off!
  (Why ∵ it’s sensitive to noise)
  - Also lots of “I” will make the system more transitory & leads to integrator wind-up.

PID Intuition

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int e(s)\,ds + T_d\frac{de(t)}{dt}\right]$$

- P:
  - Control action is proportional to control error
  - It is necessary to have an error to have a non-zero control signal

- I:
  - The main function of the integral action is to make sure that the process output agrees with the set point in steady state

PID Intuition

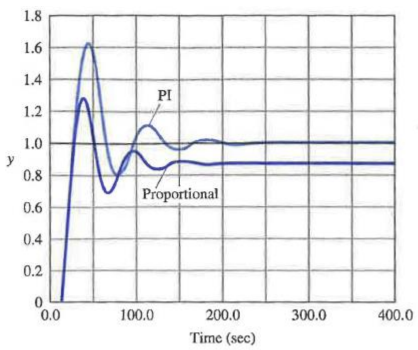
$$u(t) = K\left[e(t) + \frac{1}{T_i}\int e(s)\,ds + T_d\frac{de(t)}{dt}\right]$$

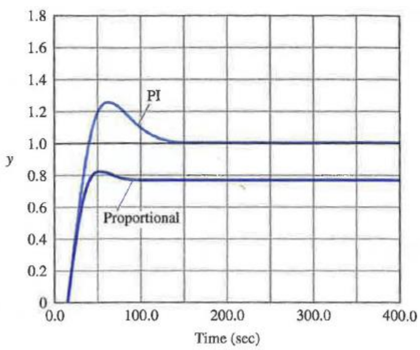
- P:
- I:
- D:
  - The purpose of the derivative action is to improve the closed loop stability.
  - The instability "mechanism" "controlled" here is that because of the process dynamics it will take some time before a change in the control variable is noticeable in the process output.
  - The action of a controller with proportional and derivative action may e interpreted as if the control is made proportional to the *predicted* process output, where the prediction is made by extrapolating the error by the tangent to the error curve.

PID Intuition

Effects of increasing a parameter independently					
Parameter	Rise time	Overshoot	Settling time	Steady-state error	Stability
K_p	↓	⇑	Minimal change	↓	↓
K_I	↓	⇑	⇑	Eliminate	↓
K_D	Minor change	↓	↓	No effect / minimal change	Improve (if K_D small)

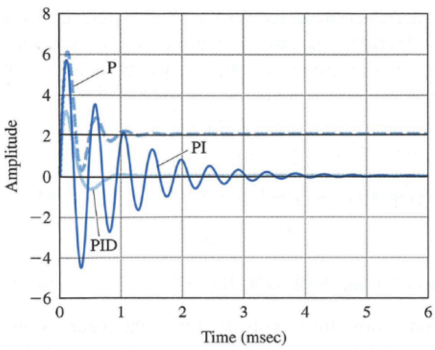
PID Intuition: P and PI



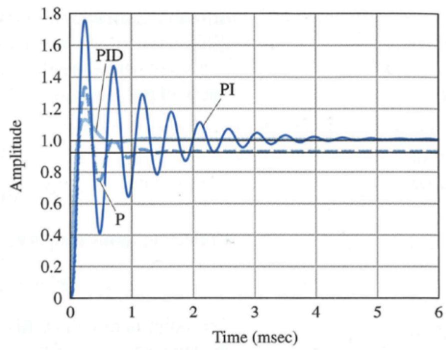


PID Intuition: P and PI and PID

- Responses of P, PI, and PID control to



(a) step disturbance input



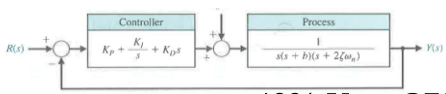
(b) step reference input

PID Example

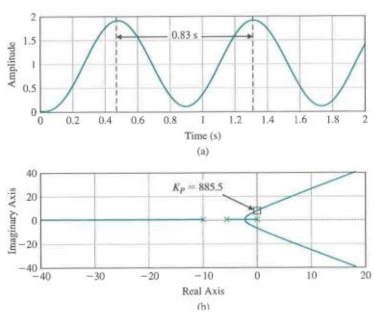
- A 3rd order plant: b=10, ζ=0.707, ω_n=4

$$G(s) = \frac{1}{s(s+b)(s+2\zeta\omega_n)}$$

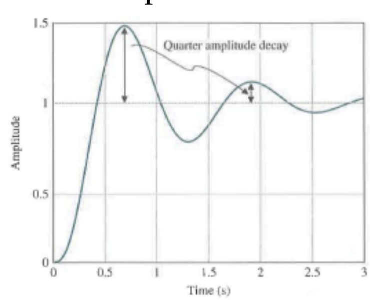
- PID:



- Kp=855:



- 40% Kp = 370



PID Algorithm (in various domains):

FPW § 5.8.4 [p.224]

- PID Algorithm (in Z-Domain):

$$D(z) = K_p\left(1 + \frac{Tz}{T_I(z-1)} + \frac{T_D(z-1)}{Tz}\right)$$

- As Difference equation:

$$u(t_k) = u(t_{k-1}) + K_p\left[\left(1 + \frac{\Delta t}{T_i} + \frac{T_d}{\Delta t}\right)e(t_k) + \left(-1 - \frac{2T_d}{\Delta t}\right)e(t_{k-1}) + \frac{T_d}{\Delta t}e(t_{k-2})\right]$$

- Pseudocode [Source: Wikipedia]:

```
previous_error = 0, integral = 0
start:
  error = setpoint - measured_value
  integral = integral + error*dt
  derivative = (error - previous_error)/dt
  output = Kp*error + Ki*integral + Kd*derivative
  previous_error = error
  wait(dt)
  goto start
```


Ziegler-Nichols Tuning – Reaction Rate

FPW § 5.8.5 [p.224]

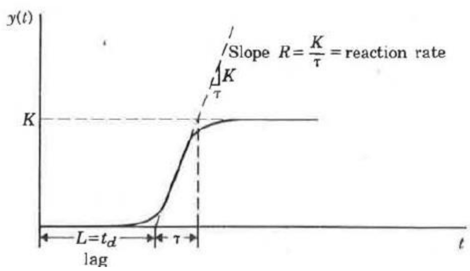
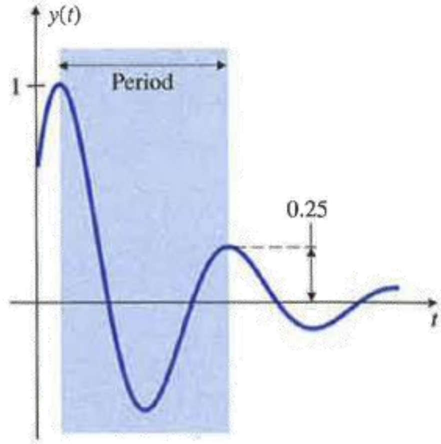


Table 5.2 Ziegler-Nichols tuning parameters using transient response.

	K_p	T_I	T_D
P	$1/RL$		
PI	$0.9/RL$	$3L$	
PID	$1.2/RL$	$2L$	$0.5L$

Quarter decay ratio



Ziegler-Nichols Tuning – Stability Limit Method

FPW § 5.8.5 [p.226]

- Increase K_P until the system has continuous oscillations
  - $\equiv K_U$: Oscillation Gain for "Ultimate stability"
  - $\equiv P_U$: Oscillation Period for "Ultimate stability"

Table 5.3 Ziegler-Nichols tuning parameters using stability limit.

	K_p	T_I	T_D
P	$0.5K_u$		
PI	$0.45K_u$	$P_u/1.2$	
PID	$0.6K_u$	$P_u/2$	$P_u/8$

That's it for Today!
See you tomorrow! ☺