

ELEC 3004 / 7312 – Digital Linear Systems: Signals & Controls 2015

Problem Set 2: Sampling & Digital Systems

Total points: 50

Due Date: April 15, 2015 (at 11:59pm, AEST)

Note: This assignment is worth **12%** of the final course mark. Please submit answers via [Platypus](#). It is requested that solutions, including equations, should be typed please. The final grade is the median of the marks from the peer reviews and the staff (with provisions for review). Finally, the tutors will **not** assist you further unless there is real evidence you have attempted the questions. Thank you. :-)

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. For all questions, **please justify your solutions**. Answers alone are not sufficient. At all times please (note: remember to justify your solution). Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Although a rubric will be provided to guide peer marking, peers may reduce marks if they believe an answer is of poor quality, demonstrates little effort or significant misunderstanding. Furthermore, if a solution demonstrates understanding in a way not anticipated by the rubric, peers may also award higher marks than would be awarded by strict adherence to the rubric. In other words, you are free to use your judgment when marking your peers.

Q1. Friendly Signals

[15 points]

A friend of yours has been sampling sine waves in MATLAB. They've written some observations in a logbook.

Sampling Sine Waves at a Sampling Frequency of 1 kHz

Frequency 1	Frequency 2	Observation
100 Hz	1.1 kHz	Samples for both frequencies are always exactly the same
400 Hz	600 Hz	Samples from both frequencies have the same amplitude, but always the opposite sign (frequency 2 is the negative of frequency 1)
500 Hz	1 kHz	Samples from both frequencies are very small in amplitude, but don't appear to follow any pattern

Note: Frequency 1 and Frequency 2 both refer to frequencies of sine wave inputs.

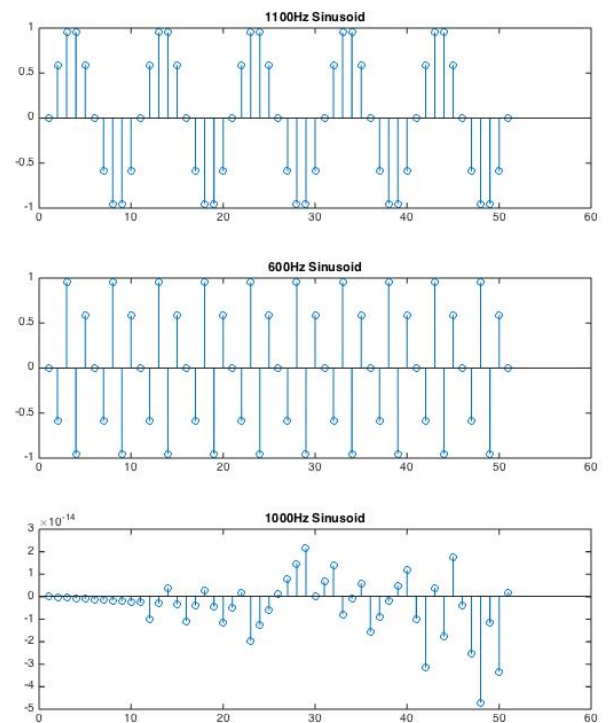
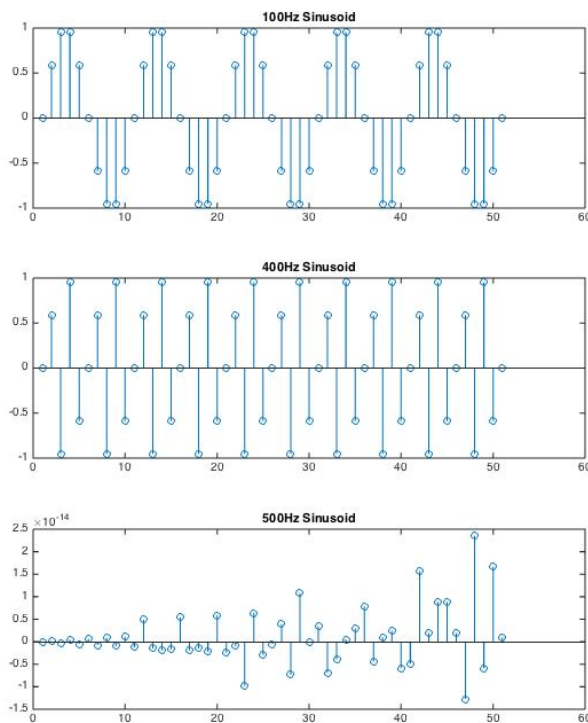
You can see the student's code and graphs as follows:.

Matlab Code (You are welcome to put this code in an m-file and run it for a closer look at the graphs and data) :

```
discreteTime = @(startTime, endTime, sampFreq)(startTime:1/sampFreq:endTime);
sampledTime = discreteTime(0, 50e-3, 1e3);

sinSampled = @(freqHz, sampledTime)(sin(2*pi*freqHz * sampledTime));
sinFreqs = [100, 1100, 400, 600, 500, 1e3];
for i = 1:numel(sinFreqs)
    curFreq = sinFreqs(i);
    subplot(3, 2, i), stem(sinSampled(curFreq, sampledTime));
    title(sprintf('%d Hz Sinusoid', curFreq));
end
```

Graphs:



1. Explain mathematically why sine of 100 Hz and 1.1 kHz appear identical to a 1 kHz sampler.
2. Explain the change in sign between samples of 400 Hz and 600 Hz sinusoid.
3. Does the theory from your answer to (1) & (2) predict the observations of the 500 Hz - 1 kHz sinusoids? Discuss.

Q2. Doppelganger Signals

[10 points]

Consider the discrete-time sequence

$$x[n] = \sin\left(\frac{n\pi}{k}\right)$$

1. For $k=10$, find two signals that produce the same sequence when sampled at $f_s = 1$ kHz (Note: you may assume that signal has **not** been properly anti-aliased filtered before sampling.)
2. Determine a formula for this in general – that is for a positive integer constant k and f_s . (Hint: The sequence $x[n]$ from sampling a sinusoid $x(t) = \sin(2\pi f t)$ at frequency f_s is $x[n] = \sin(2\pi \frac{f}{f_s} n)$)

Q3. A Reconstruction Effort

[15 points]

The Nyquist principle¹ is for bandlimited signals, and as such assumes an ideal (“rect”) low-pass filter; however, such an ideal filter not possible to implement.

One approach for treating this is the *zero-order hold* (ZOH). Its impulse and frequency response are^{2,3}:

$$h_{ZOH} = \begin{cases} 1 & 0 \leq t \leq T_s \\ 0 & \text{else} \end{cases}$$
$$\text{ie, } h_{ZOH} = \begin{cases} 1 & 0 \leq t \leq T_s \\ 0 & \text{else} \end{cases}$$
$$H_{ZOH} = \frac{1-e^{sT}}{sT} = e^{-j(\frac{\omega}{2})T} \text{sinc}((\frac{\omega}{2\pi})T) = e^{-j\pi f T} \text{sinc}(fT)$$

1. Plot the normalized frequency response of the (a) **ideal low-pass filter** and the (b) ZOH. That is, this should be on the **f-axis or Ω axis** and should span at least the sampling rate (ie. **-2B to 2B Hz or $-\frac{2\pi}{T_s}$ to $\frac{2\pi}{T_s}$ rad/s**)
2. One way to improve this is to design a **Reconstruction Compensation Filter (RCF)** such that this filter cascaded with a ZOH would give an ideal low-pass filter response. Please **determine the frequency response of a simple, ideal version of this filter** (ie. H_{RCF}).
3. Please graph your solution (to Part 2).

Q4. An e-Z Unit

[10 points]

The z-transform is the discrete-time systems counterpart of the Laplace transform (\mathcal{L}). The z-transform represents difference equations as algebraic ones, just as the \mathcal{L} allows ODEs to be represented and manipulated algebraically. This transformation simplifies systems analysis by allowing cascades of systems (and their associated time convolutions) to be treated as multiplications (ie. $x_1[n]*x_2[n] \rightarrow X_1[z]X_2[z]$).

Let's explore some properties of the z-transform⁴ as applied to key system functions:

1. Show that the transfer function of a unit delay (in the z-domain) is $1/z$
2. Show that the transfer function of an accumulator is $Y(z) = X(z)\frac{1}{1-z^{-1}}$.
Note: Accumulation is the discrete time counterpart to integration in the time domain.
It is defined as: $y[n] = \sum_{k=-\infty}^n x[k]$

¹ **Recall:** The Nyquist principal gives that for a signal bandlimited to **B** Hz, that the minimum sampling frequency is **2B** Hz (In radians, $\omega=2\pi f$, thus saying a signal bandlimited to $\omega=2\pi B$ rad/s has $\omega_s=4\pi B$ rad/s).

² **Note:** for simplicity we may use sampling rate normalized frequencies, Ω , where $\Omega=\omega T_s$, such that the Nyquist rate is $\Omega_N=\pi T_s$

³ **Normalized Sinc Function:** $\text{sinc}(x)=\sin(\pi x)/\pi x$

⁴ **Note:** a z-transform table, such as those [widely available online](#) or in the textbook (Lathi) Tables 11.1 [p. 674, described in §11.1] and 11.2 [p. 686, described in §11.2] may be helpful.