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First Name	_____

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2015

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Any unmarked paper dictionary is permitted

An unmarked Bilingual dictionary is permitted

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Rough Paper

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

For Examiner Use Only

Question	Mark
1	
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Total   _____

⇨　**PLEASE RECORD ALL ANSWERS**　⇦
　⇨　**IN THE ANSWER BOOKLET**　　⇦

Any material not in Answer Booklet(s) **will not be seen**. In particular, the exam paper **will not be graded** or reviewed.

(Otherwise the rest of page is intentionally left blank – feel free to use as scratch paper)

This exam has THREE (3) Sections for a total of 180 Points
(which very roughly, on the whole, corresponds to ~1 Point/Minute)

Section 1: Digital Linear Dynamical **Systems**...50 Points (28 %)

Section 2: Digital Processing/Filtering of **Signals**................................ 60 Points (33 %)

Section 3: Digital & State-Space **Control**...70 Points (39 %)

⇨ Please answer **ALL** questions + **ALL Answers MUST Be Justified** ⇦
(answers alone are not sufficient)

⇨ **PLEASE RECORD ALL ANSWERS IN THE ANSWER BOOKLET** ⇦
(Any material not in Answer Booklet(s) **will not be seen**. In particular, the
exam paper **will not be graded** or reviewed.)

Section 1: Digital Linear Dynamical Systems
Please Record Answers in the **Answer Book**   (**4 Questions | 50 Points**)
Please **Justify and Explain All Answers**

1. **Starting With a Little Sampler**   (10 Points)

A sampler may be described as a continuous time system with by the function

$$y(t) = \sum_{k=-\infty}^{\infty} x(t)\delta(t - kT)$$

   A. Is the sampling operation
      i.   Causal?
      ii.   Linear?
      iii. Time invariant?
      iv.   Invertible?
   B. If $x(t) = \cos(\pi^2 t)$, please sketch the sampled output $y(t)$ for $T = 1$.
      (**Hint:** What is the frequency of this signal in Hz?)

2. **Matrix Inversion Singled Out**   (10 Points)
   Assume $x \in \mathbb{R}^n$, can we form a matrix and invert it via $(xx^T)^{-1}$?
   Why or **Why Not**? Please explain and be specific.

3. **Convolution in Order?**   (15 Points)
   Given:

   $$x_0[k] = [8 \quad 3 \quad 4 \quad 1 \quad 5 \quad 9 \quad 6 \quad 7 \quad 2]$$
   $$h_1[k] = [0 \quad 1 \quad 0]$$
   $$h_2[k] = [1 \quad 0 \quad -1]$$
 $$y_A[k] = x_0[k] * h_1[k]$$
   $$y_B[k] = x_0[k] * h_2[k]$$

   Discuss whether following statements are surely true or surely false or cannot be
   determined given the information.

   A. The convolution $y_A[k] = x_0[k] * h_1[k]$ is
      $y_A[k] = [\, 2 \quad 7 \quad 6 \quad 9 \quad 5 \quad 1 \quad 4 \quad 3 \quad 8 \,]$
   B. The convolution $y_B[k] = x_0[k] * h_2[k]$ is
      $y_B[k] = [\, 8 \quad 3 \quad -4 \quad -2 \quad 1 \quad 8 \quad 1 \quad -2 \quad -4 \quad -7 \quad -2 \,]$
   C. That time shifting of x and h_1 by k_0 and k_1 is equivalent to time shifting the
      convolution by k_0 plus k_1 – that is $y_A[k - k_0 - k_1] = x_0[k - k_0] * h_1[k - k_1]$
   D. If $S(x[n]) \equiv \sum_{n=-\infty}^{n=\infty} x[n]$, then $S(y_A) = S(x_0)S(h_1)$ and $S(y_B) = S(x_0)S(h_2)$

4. **Images of the $\mathcal{Z}$-Plane**   (15 Points)
   For a first order system with a pole at the locations A, B, C, D, or E as indicated the
   following diagram of the $\mathcal{Z}$-Plane
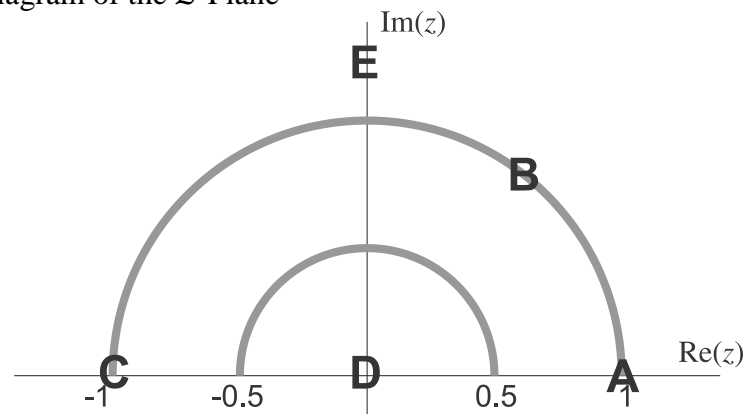
   Please briefly sketch the time response associated with each of these locations (i.e.,
   you should have 5 small sketches for the typical signal response at each location
   marked **A, B, C, D,** or **E**.

Section 2: Digital Processing & Filtering of Signals
Please Record Answers in the **Answer Book**   (**6 Questions | 60 Points**)
Please **Justify and Explain All Answers**

5. **Towards a Perfect Reconstruction**   (10 Points)
    Consider the melodious signal

$$x(t) = cos(2\pi t) + sin(5\pi t) + 4\,cos(3\pi(t + 0.5))$$

    A. What is the minimum sampling frequency ω_s (assuming $\omega_s = n\pi$) and number
        of samples N_s that will allow resolution of all the frequencies and their perfect
        reconstruction?
    B. If you were to reconstruct with an ideal lowpass filter, what cut-off frequency
        should it have?
    C. What reconstruction algorithm (e.g., "Whittaker–Shannon interpolation",
        "ZOH", "FOH", etc.) would give the best reconstruction for the least
        bandwidth (number of recorded samples)?

6. **Moving on to the Cocktail Party Problem**   (10 Points)
    Imagine we want to sample a signal of fine piano notes from the song *Under Paris
    Skies*. A section of this song is $x(t) = 5\sin(880\pi \cdot t) + \sin(522\pi \cdot t)$, but sadly we
    also have noise $n(t) = 0.1\sin(3000\pi \cdot t)$, thus recording, $y(t) = x(t) + n(t)$.

    A. Please draw the spectrum of $|Y(\omega)|$ of $y(t)$
    B. Assume that the signal is sampled with an ideal, proper anti-aliased sampler at
        1,000 Hz.   Please sketch the spectrum of the sampled signal
    C. An impatient student decides to sample some signals **without** an anti-aliasing
        filter first. Sketch the spectrum of the recorded signal in this case.

7. **An Impactful Filter**   (10 Points)
    Airbags uses an accelerometer to detect the impulse of a significant collision. To save
    costs The ATAKAT Airbag Company has decided to use noisier "smartphone grade"
    accelerometers and then filter the signal. You have been asked to design a **FIR filter**
    for this purpose. The impact acceleration is defined as a **½ millisecond impulse** of
    greater than **50g acceleration**. Assume there is non-trivial white noise. In addition,
    the engine vibrates at 42 Hz and the microcontroller has a 10,000 Hz clock.

    A. What sample rate would you choose for this problem?
    B. An engineer suggests that it might be better to use the derivative of the
        acceleration signal and measure the jolt (aka jerk). Is this a good idea?
    C. What type of FIR filter would you chose (if any)? **Why?**

8. **Tuning Out?** (10 Points)

Suppose that for any given frequency $0 < \omega_0 < \pi$, you can build a digital notch filter with transfer function $H_0(z)$, with a null in the frequency response at ω_0. Further, suppose that we have a signal, bandlimited to 500 Hz, but with an unwanted signal of 50 Hz together with its odd harmonics. Design a filter to remove the unwanted signals. Explain your reasoning.

9. **MAMMA Says** (10 Points)

The "Music and Mood Management Apparatus" robot has been upgraded and needs a sound check. It had an FIR digital filter that has a **notch at 5 kHz** when used with its original microcontroller that had a sampler operating at **11.05 kHz**. However, its new hardware now runs faster, with the result that the new sampler runs at a minimum frequency of **44.2 kHz**.

     A. At what frequency is the notch now?
   (**Note**: assuming the same "now untuned" coefficients)
     B. Where on the $\mathcal{Z}$ plane unit circle should in should the zero(s) be placed (or moved to) in a re-designed filter to move the notch back to at 5 kHz?
   (**Note:** This may be considered as what angle on the unit circle in the $\mathcal{Z}$ plane should the zero be placed in a re-designed filter?)

10. **Good Things Come To Those Who Wait** (10 Points)

Signals have noise – in part because a channel has to be shared and one device's signal is another one's noise.

     A. **Briefly explain** (and/or show a simple sketch in the frequency domain) what is meant by the terms *white* and *pink* noise?
     B. One strategy for "beating" the noise is to wait and average (sometimes called "integration" by some sensor makers). Is this a good strategy?
     C. **Briefly describe** two other strategies for beating the noise.

Section 3: Digital & State-Space Control
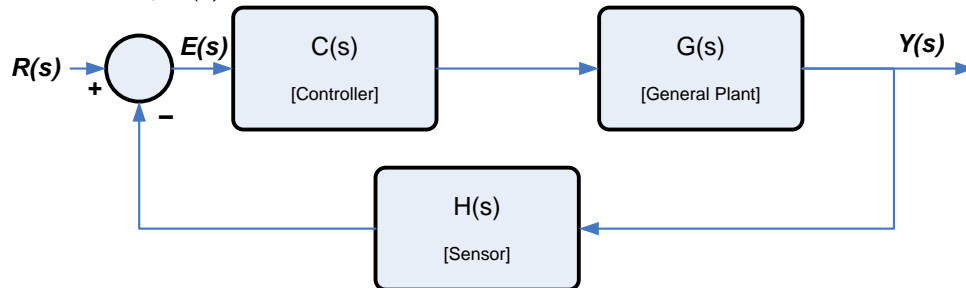Please Record Answers in the **Answer Book**   (**5 Questions | 70 Points**)
Please **Justify and Explain All Answers**

11. **Changing Perspective**   (15 Points)
    Consider a general negative feedback control system with a Plant, *G(s)*, Controller,
    C(s), and Sensor, *H(s)*.

   A. Imagine that over the course of the operation the plant changes such that it
       transforms from *G(s)* to *G⁺(s)* and the error from *E(s)* to *E⁺(s)* , where
       $G^+(s) = G(s) + \Delta G(s)$ and $E^+(s) = E(s) + \Delta E(s)$. Determine the value of
       E⁺(s).
   B. In open-loop system the components have to be selected very carefully. What
       does this suggest about the advantages/disadvantages of a closed loop system
       as it relates to component variation?
       (Please briefly and succinctly explain).

12. **Properties of a State Transition**   (15 Points)
   State politics is convoluted, but state transition can feature direct action!

    Let's explore some properties of the state transition matrix, $\mathbf{\Phi}$, for a simple SISO LTI
    system where $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$.

   A. $\mathbf{\Phi}(t)$: Please determine $\mathbf{\Phi}(t)$ for this system A.
       (**Note:** You may leave it in terms of the matrix exponential)
   B. $\mathbf{\Phi}(s)$: Kindly determine: $\mathbf{\Phi}(s)$ for a system A
   C. Characteristic Polynomial:
       Please also find System A's characteristic polynomial.
   D. Discrete Representation:
       Please express this system as a difference equation (i.e. $x(k + 1)$ and $y(k)$)
       assuming a step input at the first step ($u(k)$), ZOH sampling, **H**=**I**, and
       $\mathbf{\Gamma} = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$.

13. **Easy $P(z)$?**   (15 Points)

In a plant, it is decided to have two plants sampled simultaneously in series as:
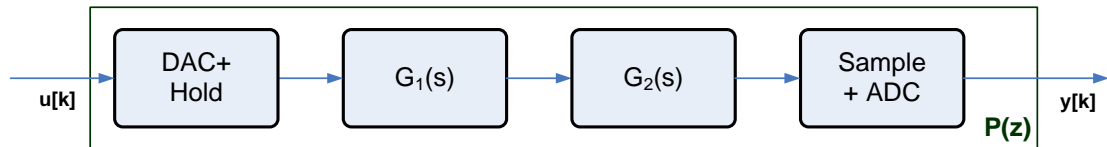
A. What is a general expression for **P(z)**, the z-transform between **u[k]** and **y[k]**?
   (Assume ZOH sampling)

B. If $G_1 = \frac{1}{(s+1)}$ and $G_2 = \frac{1}{(s+2)}$, please determine the discrete time transfer
   function, **P(z)** again assuming ZOH sampling and a known sampling time, **T**,
   of **T=0.1 seconds**.

C. Assess the stability of the system **P(z)**

D. In a rather desperate "efficiency enhancing" measure at this plant, its chief
   administrator PH (a rather acidic character), decides that it's more efficient to
   have one sampler per plant. Thus leading to:
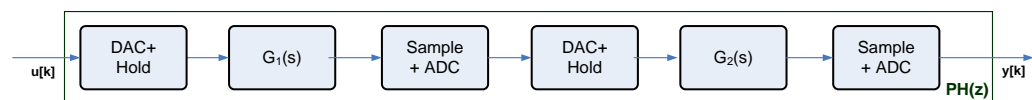
   Briefly comment on how this system compares to the original.
   Is it the same, better, or worse? (i.e. Is PH's decision wise or foolish?) **Why?**

14. **Fad Control**   (15 Points)

Vice Chancellor Richard Moby loves hype and fads and is always chasing them. The VC, who was once featured in a fashion photoshoot, is tired of "the trough of disillusionment" and visits Professor Ahab for a formula to get a pretty high ranking. The professor suggests that instead of chasing yet another institute on rat tickling; that control might determine the timing of the next popular research.

Let's apply some control theory and find out. We start with the Gartner Hype Cycle (shown below) to which we have added some markers (letters A through J).
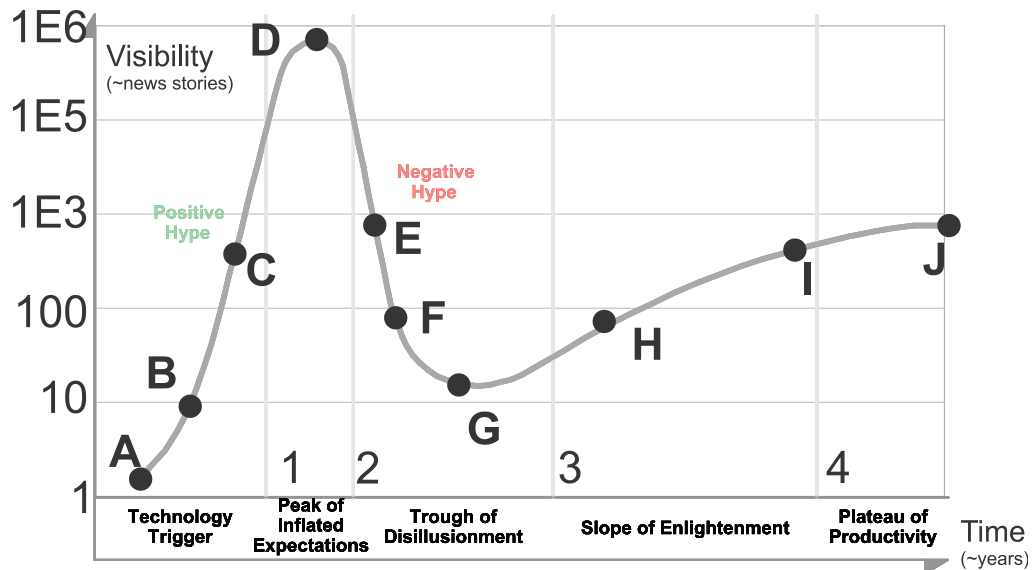
It is proposed to design a fast news controller to stay ahead of the buzz and dampen negative feedback on the university's research. The initial plan to use a PID control architecture where $C(s) = K_P \left(1 + \frac{1}{T_I s} + T_D s \right)$
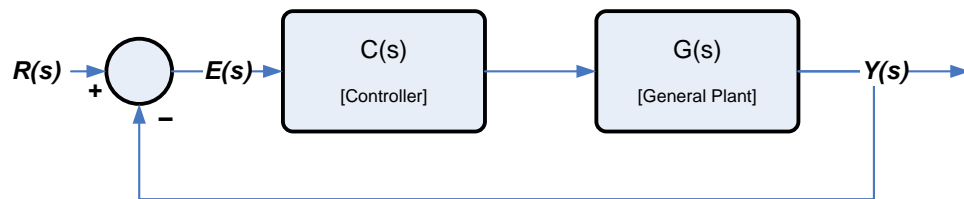
A. What points on the graph do we need to determine τ, rise (**K**), rise slope (**K/τ**), and transport delay (**L**). (Please justify your answer)
B. Given that news is very sensitive to delay, which control type (P, PI, or PID) should we use in this case?
C. Given this, please try to estimate the "control gains" for the controller you selected in Part B.
D. Briefly comment on whether this is a good decision?
   (**Hint:** Does the hype curve follow the transport delay process assumed. What would be the prescribed decay and does this intuit for this application, etc.)

15. **Time to Unwind**   (10 Points)

Of course, all things have a limit – even this exam! When it comes to actuators in plants, a common problem is that they saturate[1]. When they are paired with an integral controller, this leads to an effect known as integrator windup[2].

The issue is that output of the integral controller (u_{ic}) keeps increasing even though the actuator has reached its saturation limit ($a_{min/max}$), thus $u_{ic} > a_{max}$ or $u_{ic} < a_{min}$. Let's consider the first case ($u_{ic} > a_{max}$), the problem this causes is that the increasing control action of u_{ic} falls on "deaf ears" and does not reduce the system tracking error (**e**). Indeed, it will require significant (**e**) to empty (or discharge) the integrator to a reasonable value.

A solution to this is to stop integrating as soon as u_{ic} reaches the actuator limit that is an "anti-windup" mechanism. This can be implemented easily in a digital controller as a simple limiter (or saturation function/block).

Control Engineer Ishmael (Prof Ahab's chief assistant) proposes the following design with a limiter
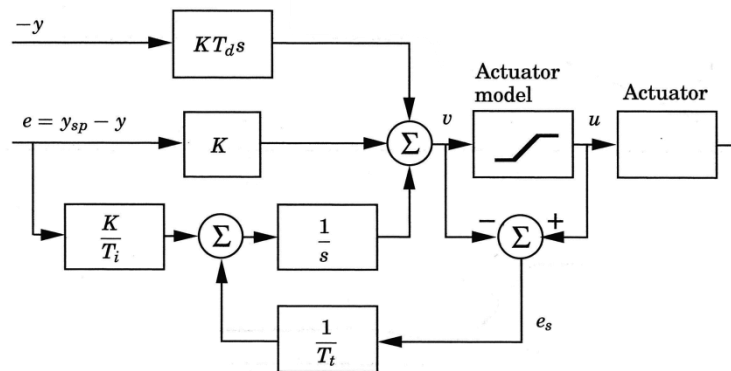
A. What is the integrator input (the input to the $\frac{1}{s}$ block) as a function of T_t, e_s, K, T_i, and **e** ?
B. What is the signal **v** going to the actuator model?
C. What if the actuator model is not tuned correctly and underrepresents what the actuator can do. What will happen then?
D. It is conjectured that this will exhibit smaller overshoot and settling time, even though the control signal to the motors is limited. Is this true? **Please Explain**.

END OF EXAMINATION   —   Thank you !!!

Is the wonder still there? ☺

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[1] Here saturation means reach the limits (in this case of the actuator)
[2] It also leads to a loss of controllability because one cannot drive a motor past its limits; however, when states are coupled it may still be possible to affect control action, which motivates an area of control/robotics known as "underactuated robotics"

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}\, dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^\infty f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t}\tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1 - z^{-1})}{\Delta t(1 + z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi}\int_0^{\pi/\Delta t} H_d(\omega)\cos(n\omega\Delta t)\, d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	Periodic	Non-periodic	Freq. Domain
Discrete	**Discrete Fourier Transform** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$	**Discrete-Time Fourier Transform** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega n}\, d\omega$	**Periodic**
Continuous	**Complex Fourier Series** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\, dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty}X[k]e^{j2\pi kt/T}$	**Fourier Transform** $X(j\omega) = \int_{-\infty}^{\infty}x(t)e^{-j\omega t}\, dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}\, d\omega$	**Non-periodic**
	Discrete	**Continuous**	

Table 3: Selected Fourier, Laplace and z-transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum\limits_{p=-\infty}^{\infty} \delta[n-pN]$	$\xleftrightarrow{DFT}$	$\tilde{X}[k] = \dfrac{1}{N}$	
$x[n] = \delta[n]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum\limits_{p=-\infty}^{\infty} \delta(t-pT)$	$\xleftrightarrow{FS}$	$X[k] = \dfrac{1}{T}$	
$\delta_T[t] = \sum\limits_{p=-\infty}^{\infty} \delta(t-pT)$	$\xleftrightarrow{FT}$	$X(j\omega) = \dfrac{2\pi}{T} \sum\limits_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
$\sin(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } |t| \leqslant T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\xleftrightarrow{FT}$	$X(j\omega) = \dfrac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \dfrac{1}{\pi t}\sin(\omega_c t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \begin{cases} 1 & \text{when } |\omega| \leqslant |\omega_c|, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = 1$	
$x(t) = \delta(t - t_0)$	$\xleftrightarrow{FT}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(w) + \dfrac{1}{jw}$	
$x[n] = \dfrac{\omega_c}{\pi}\operatorname{sinc}\omega_c n$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } |\omega| < |\omega_c|, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \dfrac{1}{s}$	
(unit ramp) $x(t) = t$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \dfrac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \dfrac{s_0}{(s^2 + s_0{}^2)}$	
$x(t) = \cos(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \dfrac{s}{(s^2 + s_0{}^2)}$	
$x(t) = e^{s_0 t} u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \dfrac{1}{s - s_0}$	$\Re\{s\} > \Re\{s_0\}$
$x[n] = \delta[n]$	$\xleftrightarrow{z}$	$X(z) = 1$	all z
$x[n] = \delta[n - m]$	$\xleftrightarrow{z}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\xleftrightarrow{z}$	$X(z) = \dfrac{z}{z - 1}$	
$x[n] = z_0^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \dfrac{1}{1 - z_0 z^{-1}}$	$|z| > |z_0|$
$x[n] = -z_0^n u[-n - 1]$	$\xleftrightarrow{z}$	$X(z) = \dfrac{1}{1 - z_0 z^{-1}}$	$|z| < |z_0|$
$x[n] = a^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \dfrac{z}{z - a}$	$|z| < |a|$

Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi} X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\displaystyle\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\dfrac{d\tilde{x}(t)}{dt}$	$\dfrac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t - t_0)$	$e^{-j2\pi kt_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k - k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\dfrac{1}{T}\displaystyle\int_{-T/2}^{T/2} |\tilde{x}(t)|^2 dt = \sum_{k=-\infty}^{\infty} |X[k]|^2$	

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Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\dfrac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\displaystyle\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\dfrac{1}{\|a\|}X\left(\dfrac{j\omega}{a}\right)$
Parseval	$\displaystyle\int_{-\infty}^{\infty} \|x(t)\|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \|X(j\omega)\|^2\,d\omega$	

Table 7: Properties of the z-transform.

Property	Time domain	z-domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n - n_0]$	$z^{-n_0}X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$\|z_0\|R_x$
Differentiation in z	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\mathfrak{Re}\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim\limits_{z\to\infty} X(z)$		

† $z = 0$ or $z = \infty$ may have been added or removed from the ROC.
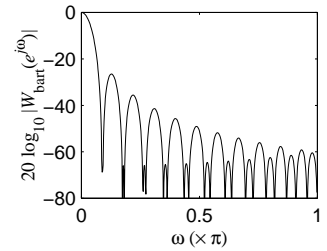
TURN OVER

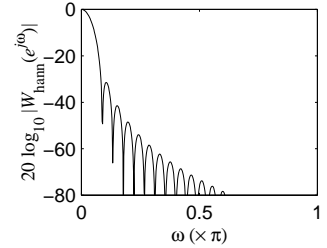Table 8: Commonly used window functions.

Rectangular:

$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$
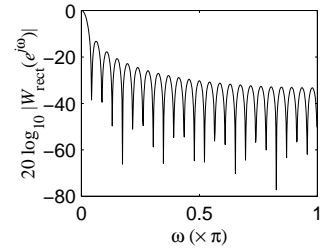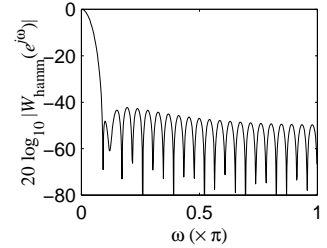
Bartlett (triangular):

$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leqslant n \leqslant M/2, \\ 2 - 2n/M & \text{when } M/2 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$
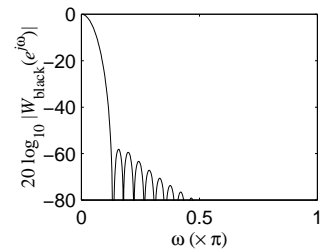
Hanning:

$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos\left(2\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$

Hamming:

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46\cos\left(2\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$

Blackman:

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5\cos\left(2\pi n/M\right) \\ \quad + 0.08\cos\left(4\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$

Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74

END