http://elec3004.com

An Introduction to **Signals & Systems**

ELEC 3004: **Digital Linear Systems**: Signals & Controls
Dr. Surya Singh

Week 1 – Thursday Open Lecture 1 [WARNING: **EXPERIMENTAL!**]

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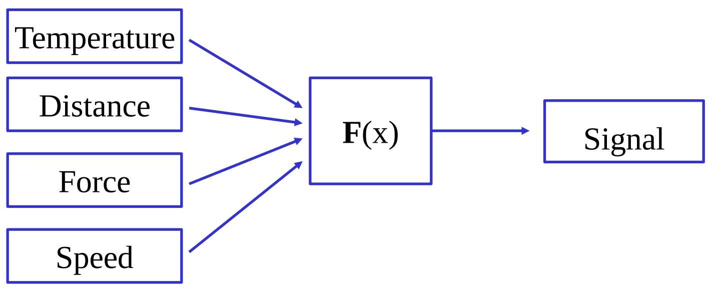
Lecture Schedule:

Week	Date	Lecture Title
1	4-Mar	Introduction & Systems Overview
	6-Mar	[Linear Dynamical Systems]
2	11-Mar	Signals as Vectors & Systems as Maps
	13-Mar	[Signals]
3	18-Mar	Sampling & Data Acquisition & Antialiasing Filters
	20-Mar	[Discrete Signals]
4	25-Mar	Filter Analysis & Filter Design
	27-Mar	[Filters]
5	1-Apr	Digital Filters
	3-Apr	[Digital Filters]
6	8-Apr	Discrete Systems & Z-Transforms
	10-Apr	[Z-Transforms]
7	15-Apr	Convolution & FT & DFT
	17-Apr	Frequency Response
8	29-Apr	Introduction to Control
	1-May	[Feedback]
9	6-May	Introduction to Digital Control
	8-May	[Digitial Control]
10	13-May	Stability of Digital Systems
	15-May	[Stability]
11	20-May	State-Space
	22-May	Controllability & Observability
12	27-May	PID Control & System Identification
	29-May	Digitial Control System Hardware
13	3-Jun	Applications in Industry & Information Theory & Communications
	5-Jun	Summary and Course Review

Signals

What is a Signal?



- Communicates information
- Varies in Time and/or Space

➔ Everywhere:

- The type
- The screen
- Your 5 senses
- Telephony
- Interest rates
- Leaf colour

Understanding & Classifying Signals

Metrics:

- Size
- Signal Energy
- Signal Power

- Frequency
- Phase

- Entropy
  - Deterministic signals
  - Random signals

Classifications:

- Continuous-Time
- Discrete-Time

- Analog
- Digital

- Even
- Odd

Signal Size

- The size of any entity is a number that indicates the largeness or strength of that entity. Generally speaking, the signal **amplitude** varies with **time**.

$$S = \int_0^T u(t)dt$$

- However, this will be a defective measure because even for a large signal $x(t)$, its positive and negative areas could cancel each other, indicating a signal of small size.

Signal Energy

- Consider the area under a signal *x*(*t*) as a possible measure of its size, because it takes account not only of the amplitude but also of the duration.
- Instead we look at it's energy or *signal energy*

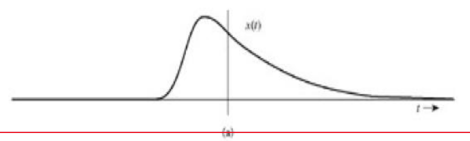
$$E = \int_{-\infty}^{\infty} u^2(t)\, dt$$

- But this can be **infinite!**
- Not to be confused with mechanical **Energy** -- **Not in Joules**

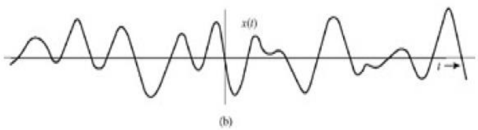
Signal Power:

- Generalize this to a finite measure via a RMS (root means square measurand)

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |u(t)|^2 dt$$



➔ Finite Energy
Finite Power



➔ ∞ Energy
Finite Power

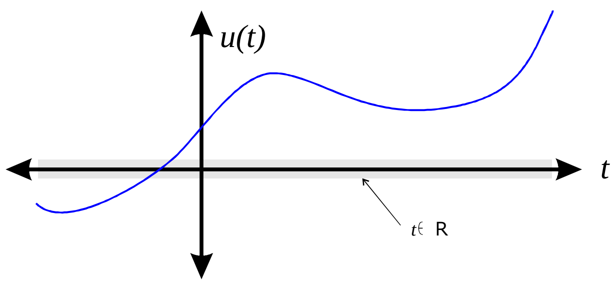
- Not to be confused with mechanical **Power** -- **Not in Watts**

Signal Classifications

- Classifications
  - A "pair-wise" way of characterizing the signal by putting it into a "descriptive bin"
  - Ex: How to describe a person – well we could measure them (weight, height, power, etc.) or we could sort by category ("North/South-side," Australian, etc.) ➔ Neither is perfect

- Some common classifications:
  1. Continuous-time and discrete-time signals
  2. Analog and digital signals
  3. Periodic and aperiodic signals
  4. Real and complex signals
  5. Deterministic and probabilistic signals

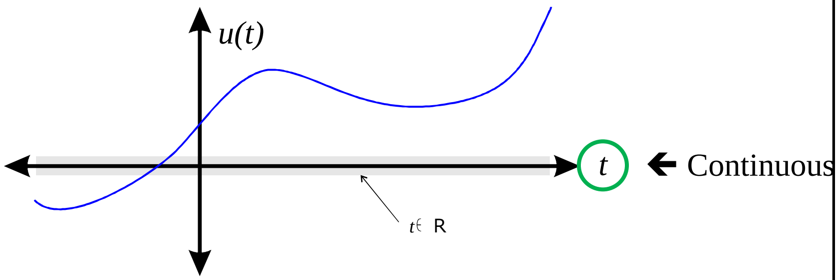
Continuous-Time



- The independent variable (x-axis) – in this case **t** – is continuous (which may be the ℝeals, but can be others)

- This does not dictate the form of u(t) -- it may be either continuous or discrete.

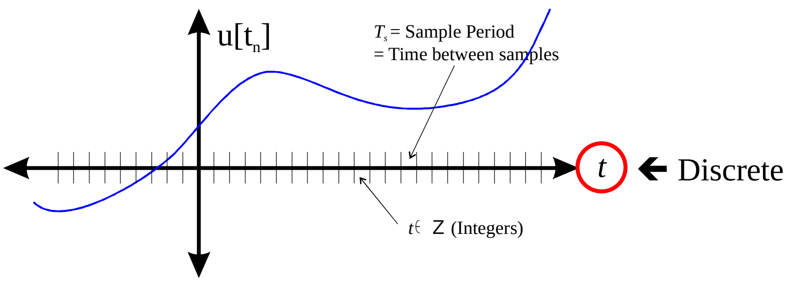
Continuous Time



- The independent variable (x-axis) – in this case **t** – is continuous (which may be the ℝeals, but can be others)

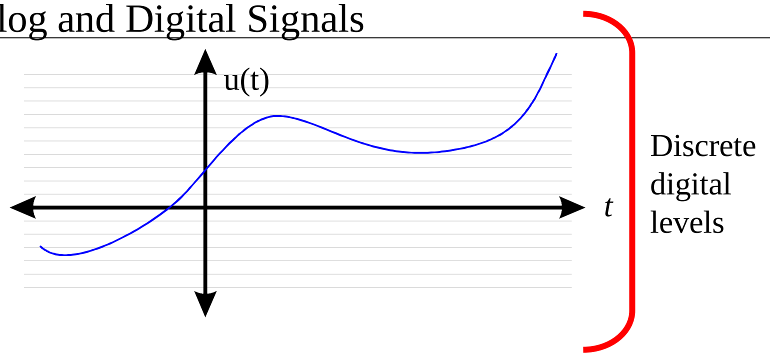
- This does not dictate the form of u(t) -- it may be either continuous or discrete.

Discrete Time



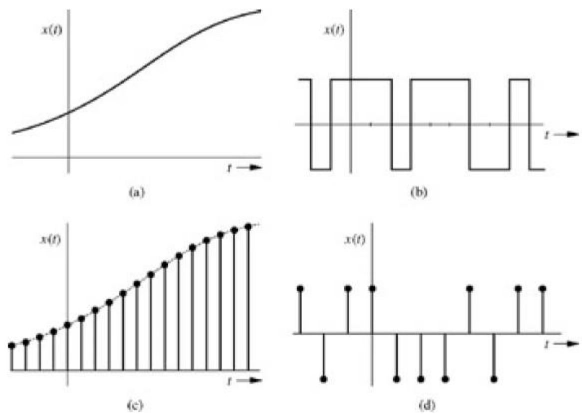
- The independent variable is only set for fixed, discrete values
- <u>Ex:</u> $\boldsymbol{t} \in$ Integers
- Written in [Square Brackets]
- $u[t_n] = u[n]$
- $t_n=\{t_0, t_1, t_2, \ldots, t_n\}$
- Relationship to continuous time: $u[n]=u(nT_s)$

Analog and Digital Signals



- Continuous and Discrete on the **y-axis**
- u(t) = Continuous ➔ Analog
- u(t) = Discrete ➔ Digital
  - Number of levels can be many
  - Simplest case is for two ➔ Binary digit (or bit) signal

Analog and Digital Signals

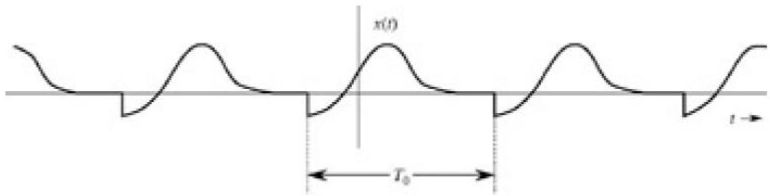


- **(a)** analog, continuous time | **(b)** digital, continuous time
- **(c)** analog, discrete time and **(d)** digital, discrete time

Periodic and Aperiodic Signals

- A signal $x(t)$ is *periodic* if for some positive constant T_0

$$x\left(t\right) = x\left(t + T_0\right)$$



- For periodic signals $x(t)$ of period T_0 :
  - the area under $x(t)$ over any interval of duration T_0 is the same

$$A = \int_{a}^{a+T_o} u\left(t\right) dt = \int_{b}^{b+T_o} u\left(t\right) dt$$

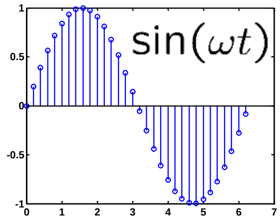
Real and Complex Signals

$$x\left(t\right) = x_1\left(t\right) + x_2\left(t\right) j$$
$$s = x_1 + x_2 j$$

$$\rightarrow e^{st} = e^{\left(x_1 + x_2 j\right)} = e^{x_1 t} e^{x_2 t j}$$
$$= e^{x_1 t}\left(\cos\left(x_2 t\right) + \sin\left(x_2 t\right) j\right)$$

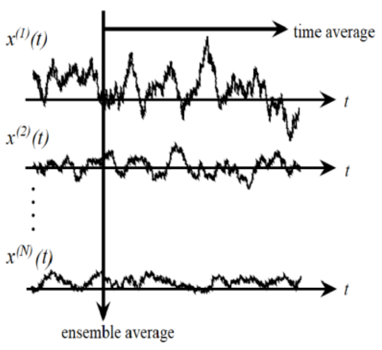
Deterministic and Random

- Deterministic
  - are those whose description is known completely



- Random
  - Those signals whose descriptions is unknown incompletely via statistical or probabilistic descriptions.

Ergodicity



1. **Over time:** multiple readings of a quantity over time
   - "stationary" or "ergodic" system
   - Sometimes called "integrating"
2. **Over space:** single measurement (summed) from <u>multiple</u> sensors each distributed in space
3. **Same Measurand**: multiple measurements take of the **<u>same observable quantity</u>** by multiple, related instruments

   e.g., measure position & velocity simultaneously

   → Basic "sensor fusion"

Classifying Signals: Even and Odd

- **Even**

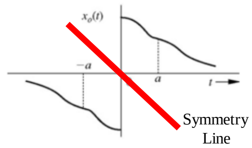
$$u_{\text{even}}(t) = u_{\text{even}}(-t)$$

  – Area:

$$\int_{-a}^{a} u_e(t)\,dt = 2\int_{0}^{a} u_e(t)\,dt$$

- *Odd*

$$u_{\text{odd}}(t) = -u_{\text{odd}}(-t)$$

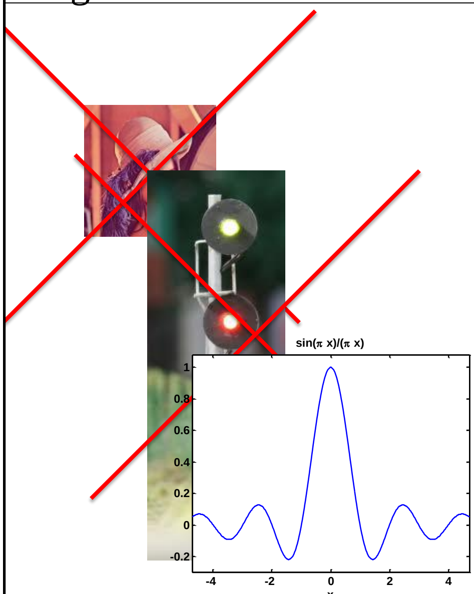


  – Area:

$$\int_{-a}^{a} u_e(t)\,dt = 0$$

➔ Every signal can be expressed as a sum of **even** and ***odd*** components

Signal Models



➔A "model" of signals

- Key functions include:
  – Step
  – Impulse
  – Exponential functions play
- Basis for representing other signals,
- Simplify many aspects of the signals and systems.

Signal Models

1] Unit Step

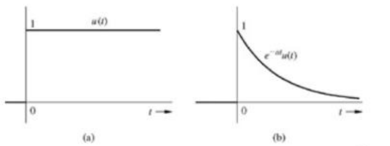
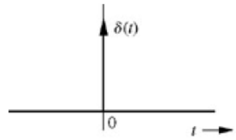


Figure 1.14: (a) Unit step function $u(t)$. **(b)** Exponential $e^{-at}u(t)$.

$$u(t) = \begin{cases} A & t > t_0 \\ 0 & t < t_0 \end{cases}$$

- if A=1, t_0=0
  - Heaviside function

2] Unit Impulse



$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

- Can be approximated as:

$$\delta_a(x) = \lim_{a \to 0} \frac{1}{a\sqrt{\pi}} e^{-x^2/a^2}$$

Signal Models: Multiplication by Impulse

- Multiply *F(t)* by δ(t)

- Is about t=0:
  - F(t) is continuous at t=0
  - δ only has value at t=0
  - ב=F(0)=F(t=0)

- Sampling Property of the Impulse:

$$F(t)\,\delta(t) = F(0)\,\delta(t)$$
$$\to F(t)\,\delta(t-T) = F(T)\,\delta(t-T)$$
$$\int_{-\infty}^{\infty} F(t)\,\delta(t)\,dt = F(0)\int_{-\infty}^{\infty}\delta(t)\,dt = \beth$$
$$\int_{-\infty}^{\infty} F(t)\,\delta(t-T)\,dt = F(T)\int_{-\infty}^{\infty}\delta(t)\,dt = F(T)$$

Signal Models

3] Sinusoidal & Exponential Signals

$$u(t) = A\cos(\omega t + \phi)$$

- A, ω : Amplitude and Frequency (ω=2πf)
- ϕ: phase angle

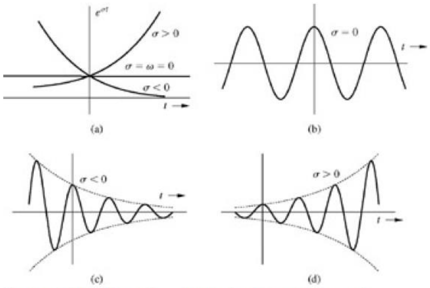


Figure 1.21: Sinusoids of complex frequency $\sigma + j\omega$.

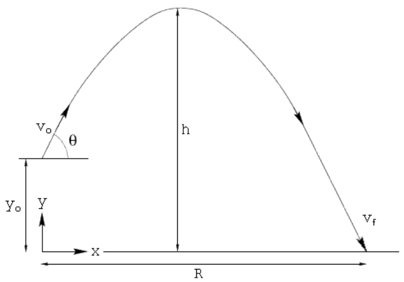
Systems

Further Classifications of Systems

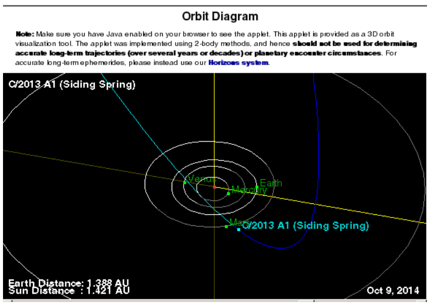
1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Modelling order

- Modelling order depends on what you are trying to achieve



The parabolic trajectory of a projectile



http://ssd.jpl.nasa.gov/

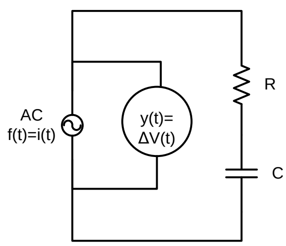
JPL Small-Body Database Browser.url

Modelling Ties Back with ELEC 2004

- Linear Circuit Theorems, Operational Amplifiers
- Operational Amplifiers
- Capacitors and Inductors, RL and RC Circuits
- AC Steady State Analysis
- AC Power, Frequency Response
- Laplace Transform
- Reduction of Multiple Sub-Systems
- Fourier Series and Transform
- Filter Circuits

➔ Modelling Tools!

Example: RC Circuits



$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{t} f(\tau)\,d\tau$$

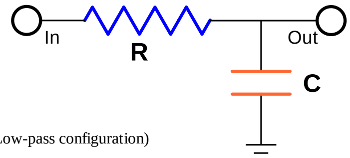
$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{0} f(\tau)\,d\tau + \frac{1}{C}\int_{0}^{t} f(\tau)\,d\tau$$

$$y(t) = v_C(0) + Rf(t) + \frac{1}{C}\int_{0}^{t} f(\tau)\,d\tau$$

$$y(t) = v_C(t_0) + Rf(t) + \frac{1}{C}\int_{t_0}^{t} f(\tau)\,d\tau$$

First Order RC Filter

- Passive, First-Order **R**esistor-**C**apacitor Design:



(Low-pass configuration)

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

- 3dB (½ Signal Power):

$$\omega = 2\pi f$$

$$f_c = \frac{1}{2\pi \mathbf{RC}}$$

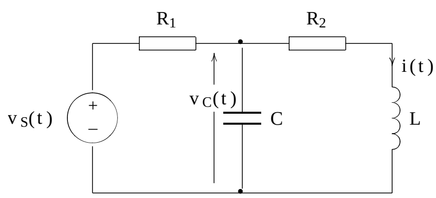
- Magnitude:

$$|V_{\text{out}}| = \sqrt{\frac{1}{(\omega RC)^2}}\,|V_{\text{in}}|$$

- Phase:

$$\phi = \tan^{-1}(-\omega RC)$$

Example of 2nd Order: RLC Circuits



- KCL: $$\frac{V_s(t) - V_c(t)}{R_1} = C\frac{d}{dt}V_c(t) + i(t)$$

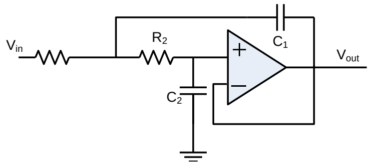
- KVL: $$V_c(t) = L\frac{d}{dt}i(t) + R_2 i(t)$$

- Combining:

$$V_s(t) = R_1 LC\frac{d^2}{dt^2}i(t) + (L + R_1 R_2 C)\frac{d}{dt}i(t) + (R_1 + R_2)\,i(t)$$

2nd Order Active RC Filter (Sallen–Key)

- 2nd Order System Sallen–Key Low-Pass Topology:



Build this for Real in **ELEC 4403**

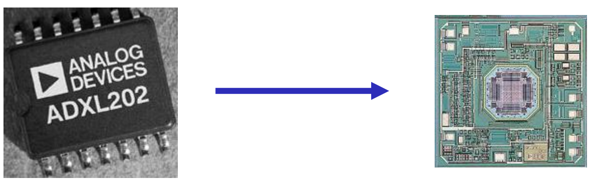
- KCL: $\frac{v_{\text{in}}-v_x}{R_1} = C_1 s\,(v_x - v_{\text{out}}) + \frac{v_x - v_{\text{out}}}{R_2}$

- Combined with Op-Amp Law:
$$\frac{v_{\text{in}}-v_{\text{out}}(C_2 s R_2+1)}{R_1} = C_1 s v_{\text{out}}\,(C_2 s R_2 + 1) - v_{\text{out}} + \frac{v_{\text{out}}(C_2 s R_2+1)-v_{\text{out}}}{R_2}$$

- Solving for Gives a 2nd order System:

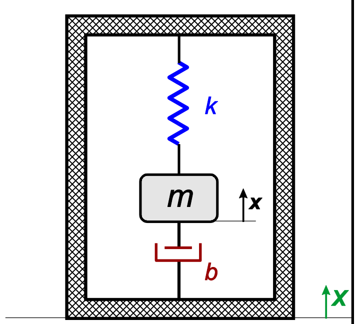
$$\frac{v_{out}}{v_{in}} = \frac{1}{C_1 C_2 R_1 R_2 s^2 + C_2(R_1+R_2)s + 1}$$

Another 2nd Order System: Accelerometer or Mass Spring Damper (MSD)



- General accelerometer:
  - Linear spring (*k*) (0th order w/r/t o)
  - Viscous damper (*b*) (1st order)
  - Proof mass (*m*) (2nd order)

➔ Electrical system analogy:
  - resistor (R) : damper (b)
  - inductance (L) : spring (k)
  - capacitance (C) : mass (m)



Measuring Acceleration:
Sense ***a*** by measuring spring motion ***Z***

- Start with Newton's 2nd Law:

$$ma = F$$

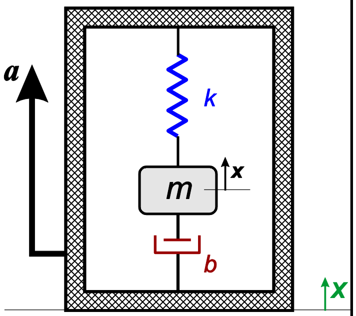
- Substitute:

$$m\frac{d^2x}{dt^2} = k\left(X-x\right) + b\frac{d\left(X-x\right)}{dt}$$

$$\mathbf{Z} \equiv \left(X-x\right) \rightarrow x = X - \mathbf{Z}$$

$$\Rightarrow m\frac{d^2X}{dt^2} = m\frac{d^2\mathbf{Z}}{dt^2} + k\mathbf{Z} + b\frac{d\mathbf{Z}}{dt}$$

The "displacement" measured by the unit (the motion of *m* relative the accelerometer frame)



- Solve ODE:

$$X\left(t\right) = X_0 e^{i\omega t} \qquad \mathbf{Z}\left(t\right) = Z_0 e^{i\omega t}$$

Measuring Acceleration [2]

- Substitute candidate solutions:

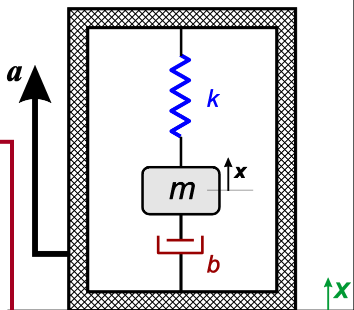
$$m\frac{d^2\left(X_0 e^{i\omega t}\right)}{dt^2} = m\frac{d^2\left(Z_0 e^{i\omega t}\right)}{dt^2} + k\left(Z_0 e^{i\omega t}\right) + b\frac{d\left(Z_0 e^{i\omega t}\right)}{dt}$$

$$-m\omega^2 X_0 e^{i\omega t} = -m\omega^2 Z_0 e^{i\omega t} + kZ_0 e^{i\omega t} + \left(i\omega\right) bZ_0 e^{i\omega t}$$

- Define Natural Frequency (ω_0)
  & Simplify for Z_0
  (the spring displacement "magnitude"):

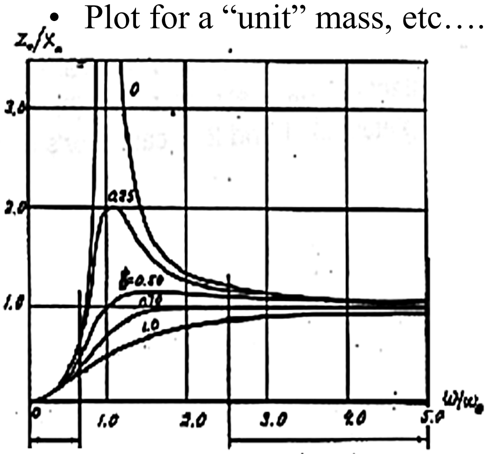
$$\omega_0 \equiv \sqrt{\frac{k}{m}}$$

$$Z_0 = \frac{m\omega^2 X_0}{m\omega^2 - k - i\omega b} = \boxed{\frac{X_0}{\sqrt{1 - \frac{\omega_0{}^2}{\omega^2} - \frac{b^2}{m^2\omega^2}}}}$$



Acceleration: 2nd Order System

- Plot for a "unit" mass, etc….



Accelerometer Seismometer

- For $\omega \ll \omega_0$:

$$Z_0 \approx \frac{\omega^2 X_0}{\omega_0^2} = \frac{a}{\omega_0^2}$$

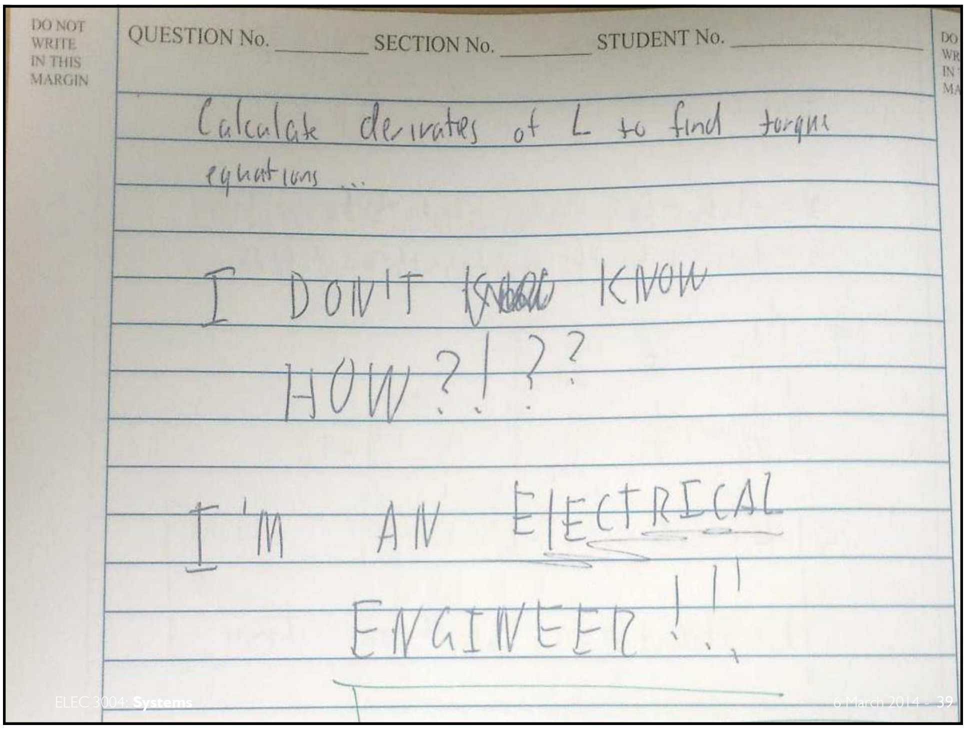
$$\rightarrow a = Z_0 \omega_0^2$$

→ **it's an Accelerometer**

- For $\omega \sim \omega_0$
  - As: b→ 0, Z → ∞
  - Sensitivity ↑
- For $\omega \gg \omega_0$:

$$Z_0 \approx X_0$$

→ **it's a Seismometer**



Equivalence Across Domains

Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2^2$	
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2^2$	
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}^2$	
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}^2$	
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}^2$	
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 74

Next Time…

- We'll talk about System Models

- Review:
  - Phasers, complex numbers, polar to rectangular, and general functional forms.
  - Chapter 1 of Lathi
    (particularly the first sections on signals & classification thereof)

- Register on Platypus

- Try the practise assignment (will be posted soon)