Vectors, Signals & Shannon’s Information Theory

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At a frequency of 3000 Mc and a total current of 15 ma, a net gain of 46 db was obtained, even though no attempt was made to match either the input or output circuits. The lack of appropriate match is responsible for the fact that the gain curve assumes negative values when the electronic gain is not sufficient to overcome the losses due to mismatch. At the peak of the curve, it is estimated that the electronic gain is of the order of 80 db.

The curves of output voltage versus the potential of the drift tube were shown in Figs. 8 and 9. Fig. 9 shows this characteristic for the electron-wave tube of the factory solution to the problem of generation and amplification of energy at millimeter wavelengths, and thus will aid in expediting the exploitation of that portion of the electromagnetic spectrum.

ACKNOWLEDGMENT

The author wishes to express his appreciation of the enthusiastic support of all his co-workers at the Naval Research Laboratory who helped to carry out this project from the stage of conception to the production and tests of experimental electron-wave tubes. The untiring efforts of two of the author's assistants, C. B. Smith and R. S. Ware, are particularly appreciated.

Communication in the Presence of Noise*

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***Summary*—A method is developed for representing any communication system geometrically. Messages and the corresponding signals are points in two "function spaces," and the modulation process is a mapping of one space into the other. Using this representation, a number of results in communication theory are deduced concerning expansion and compression of bandwidth and the threshold effect. Formulas are found for the maximum rate of transmission of binary digits over a system when the signal is perturbed by various types of noise. Some of the properties of "ideal" systems which transmit at this maximum rate are discussed. The equivalent number of binary digits per second for certain information sources is calculated.**

* Decimal classification: 621.38. Original manuscript received by the Institute, July 23, 1940. Presented, 1948 IRE National Convention, New York, N. Y., March 24, 1948; and IRE New York Section, New York, N. Y., November 12, 1947.

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I. INTRODUCTION

A GENERAL COMMUNICATIONS system is shown schematically in Fig. 1. It consists essentially of five elements.

1. *An information source.* The source selects one message from a set of possible messages to be transmitted to the receiving terminal. The message may be of various types; for example, a sequence of letters or numbers, as in telegraphy or teletype, or a continuous function of time $f(t)$, as in radio or telephony.

2. *The transmitter.* This operates on the message in some way and produces a signal suitable for transmission to the receiving point over the channel. In teleph-

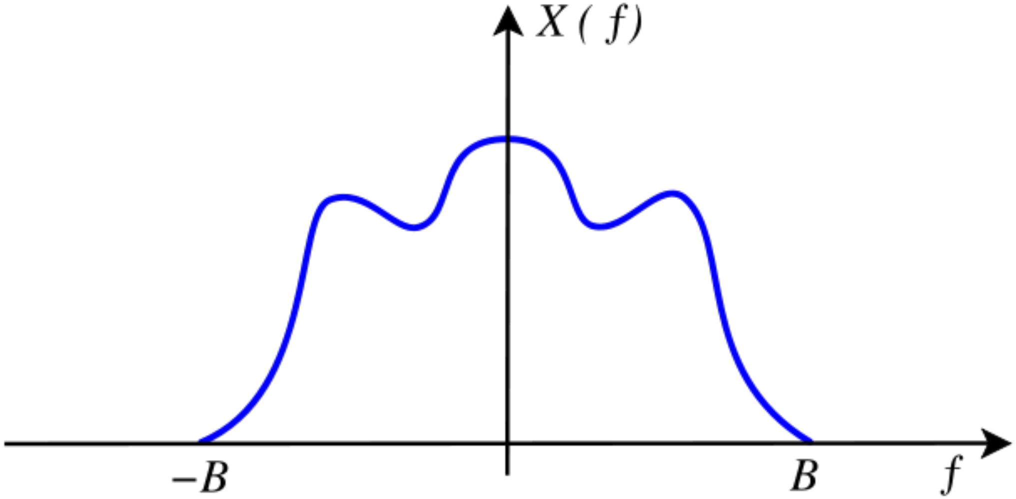
Shannon's Information Theory

In the late 1940s, Claude Shannon published a series of papers which introduced his *information theory.*

- It revolutionised communications systems.

Bandlimited Signals

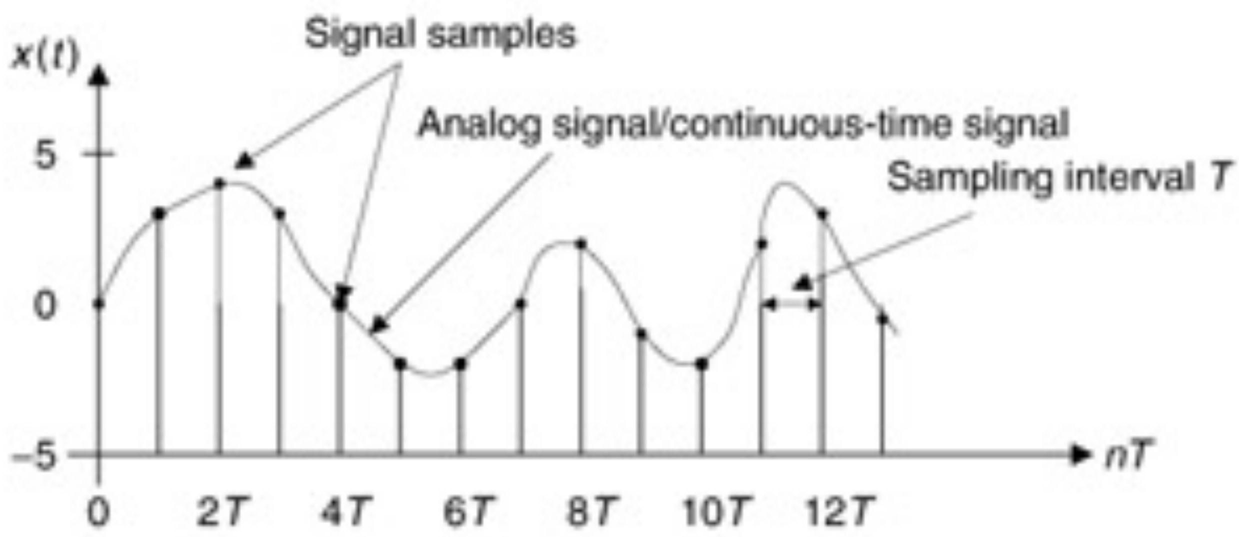
- A *signal* is just a function of time $x(t)$
  - But it has some physical significance, *e.g.*, a voltage waveform



- A *bandlimited* signal is one where the frequency components lie between a minimum and maximum frequency
  - We can force a signal to be bandlimited by passing it through a filter

Nyquist's Sampling Theorem

THEOREM 1: *If a function $f(t)$ contains no frequencies higher than W cps, it is completely determined by giving its ordinates at a series of points spaced $1/2W$ seconds apart.*



Signals $\Leftrightarrow$ Samples

- Nyquist's theorem says that every bandlimited signal can be represented by a string of numbers, its *samples*
- It's very convenient to work with just the samples
  - We can work on signals in a computer
  - Just need some device to convert between signals and samples
  - This is what an *analog-to-digital converter* and a *digital-to-analog converter* do

Signals as Vectors

- When we sample a signal, we get a string of numbers
- A snippet of the signal is a finite string of numbers
  - We can store this as a vector
  - If we take n samples, Shannon realised it's useful to think of these samples as a point in n-dimensional space

Shannon's Model of a Communications System

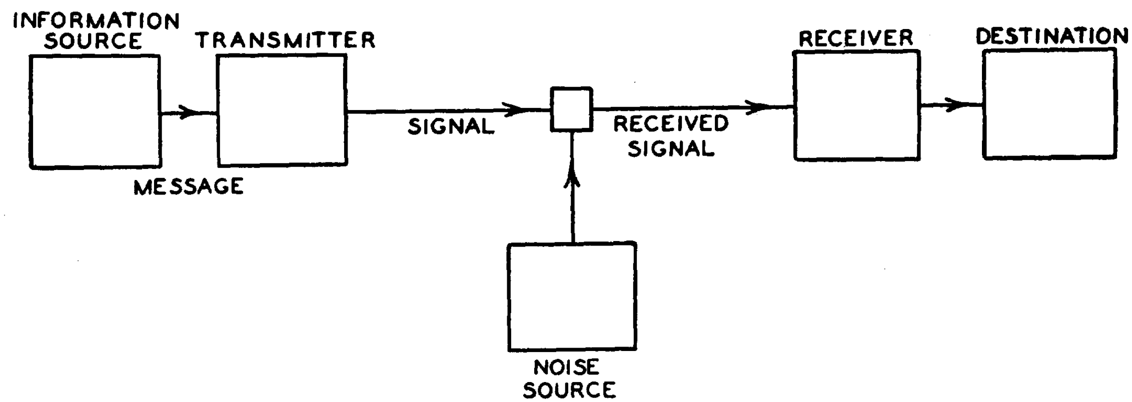
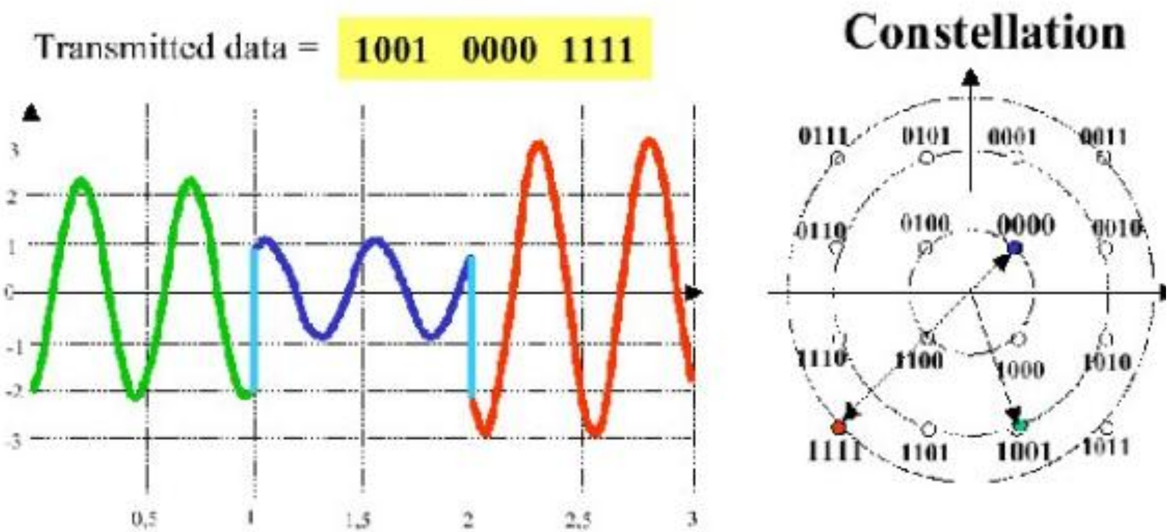


Fig. 1—General communications system.

- The *message* is something we'd like to transmit, *e.g.*, some text, our voice, a picture, usually represented as a *bitstream*
- The *signal* is the way we convey the message as, say, an EM waveform
- In between transmitter and receiver is the *channel* which modifies the signal, *e.g.*, attenuation, 'ghosting'
- *Noise* (and *interference*) corrupts the signal

Quadrature Amplitude Modulation

- A typical transmitter takes a few bits at a time and maps it to a phasor voltage at a particular carrier frequency
  - This is both amplitude and phase modulation
  - A particular format known as *quadrature amplitude modulation* (*QAM*) is commonly used
- The codebook mapping bit strings to phasors is known as the *constellation*



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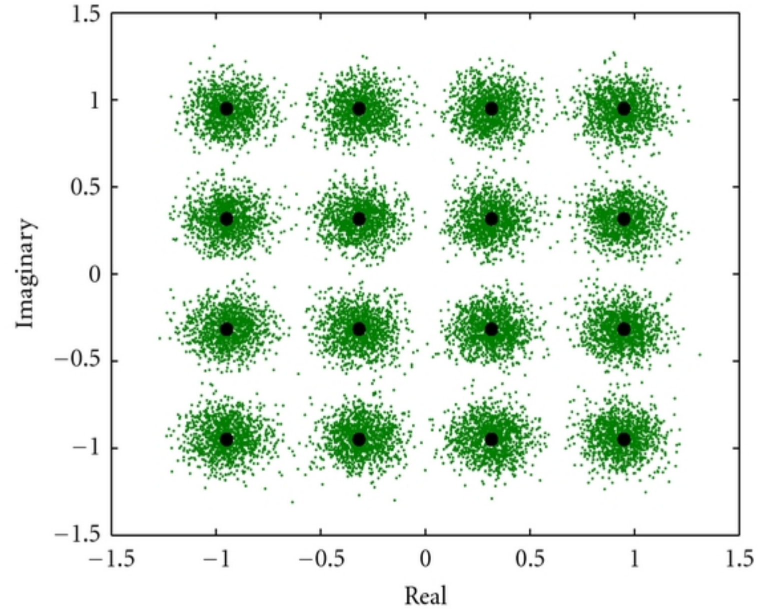
Multi-Dimensional Constellations

- Note the regular arrangement of points in space for QAM
  - This is a *lattice*
- The points are spread out to make them less susceptible to noise
- They can't be spread out too far because length of the vectors corresponds to power usage
- Shannon realised that groups of samples, as vectors, could make multi-dimensional constellations
  - Excellent (in fact complete!) noise immunity



Lattices for Communications

- It turns out that *there exist* lattices (regular arrangements of points in space) that make the best possible constellations
- But which lattices precisely? Unsolved