	Http://elec3004.com
Information Theory + Review & Applications	
ELEC 3004: Digital Linear Systems : Signals & Controls Dr. Surya Singh	
Lecture 13	
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Week	Date	Lecture Title	
	4-Mar	Introduction & Systems Overview	
I	6-Mar	[Linear Dynamical Systems]	
2	11-Mar	Signals as Vectors & Systems as Maps	
2	13-Mar	[Signals]	
2	18-Mar	Sampling & Data Acquisition & Antialiasing Filters	
3	20-Mar	[Sampling]	
4	25-Mar	System Analysis & Convolution	
4	27-Mar	[Convolution & FT]	
5	1-Apr	Frequency Response & Filter Analysis	
3	3-Apr	[Filters]	
6	8-Apr	8-AprDiscrete Systems & Z-Transforms	
0	10-Apr	[Z-Transforms]	
7	15-Apr	Introduction to Digital Control	
	17-Apr	[Feedback]	
8	29-Apr	Digital Filters	
0	1-May	[Digital Filters]	
9	6-May	Digital Control Design	
	8-May	[Digitial Control]	
10	13-May	Stability of Digital Systems	
10	15-May	[Stability]	
11	20-May	State-Space	
	22-May	Controllability & Observability	
12	27-May	PID Control & System Identification	
	29-May	Digitial Control System Hardware	
		Applications in Industry & Information	
13	3-Jun	Theory & Communications	
	5-Jun	Summary and Course Review	











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- We collect our set of uncertain variables into a vector ... $\mathbf{x} = [x_1, x_2, ..., x_N]^T$
- The set of values that **x** might take on is termed the *state space*
- There is a *single* true value for **x**, but it is unknown

State Space Dynamics $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ $\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$ $H(s) = C(sI - A)^{-1}B$



Recovering The Truth • Numerous methods
 Termed "Estimation" because we are trying to estimate the truth from the signal A strategy discovered by Gauss Least Squares in Matrix Representation
$\begin{bmatrix} p_0 \\ p_1 \end{bmatrix} = \begin{bmatrix} n & \sum_1^n t_i \\ \sum_1^n t_i & \sum_1^n t_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_1^n z_i \\ \sum_1^n t_i z_i \end{bmatrix}$
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Duals and D	ual Terminology		
	Estimation		Control
Model:	$\mathbf{x} = \mathbf{F}\mathbf{x}$ (discrete: $\mathbf{x} = \mathbf{F}_k \mathbf{x}$)	\leftrightarrow	$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}, \mathbf{A} = \mathbf{F}^{T}$
Regulates:	P (covariance)	\leftrightarrow	M (performance matrix)
Minimized function:	Q (or GQG^{\dagger})	\leftrightarrow	V
Optimal Gain:	K	\leftrightarrow	G
Completeness law:	Observability	\leftrightarrow	Controllability
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ELEC 3004: A Review

AKA ELEC 3004: What do I need to know about *.* ???

ELEC 3004: Systems

ſ	To Review:				
E	Back to the Beginni	ng	gLecture 1 Slide	9	
•	Systems			•	Controllability and state transfer
•	Signal Abstractions	•	Discrete Time	•	Observability and state estimation
•	Signals as Vectors / Systems as Maps	•	Continuous Time		
				•	And that, of course,
•	Linear Systems and Their Properties	•	Laplace Transformation		Linear Systems are Cool!
•	LTI Systems	•	Feedback and Control		
•	Autonomous Linear Dynamical Systems	•	Additional Applications		
	Convolution	•	Linear Functions		
•	FIR & IIR Systems	•	Linear Algebra Review		
•	Frequency domain	•	Least Squares		
•	Fourier Transform (CT)	•	Least Squares Problems		
•	Fourier Transform (DT)	•	Least Squares Applications		
	Even and Odd Signals	•	Matrix Decomposition and Linear Algebra		
•	Likelihood	•	Regularized Least Squares		
•	Causality				
		•	Least-squares		
•	Impulse Response	•	Least-squares applications		
•	Root Locus	•	Orthonormal sets of vectors		
•	Bode Functions	•	Eigenvectors and diagonalization		
	Left-hand Plane	•	Linear dynamical systems with inputs and outputs		
	Frequency Response	•	Symmetric matrices, quadratic forms, matrix norm, and SVD		
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Linear algebra provides the tools/foundation for working with (linear) differential equations.

• Signals are vectors. Systems are matrices.



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Linear Systems

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Extra Material: (For Fun!)

Poles are Eigenvalues

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The Direct Method of Digital Controls –

NOT to be confused with **Controller Emulation** (e.g., Tustin's Method)

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Implementation of Digital PID Controllers

We will consider the PID controller with an s-domain transfer function

$$\frac{U(s)}{X(s)} = G_c(s) = K_P + \frac{K_I}{s} + K_D s.$$
 (13.54)

We can determine a digital implementation of this controller by using a discrete approximation for the derivative and integration. For the time derivative, we use the **backward difference rule**

$$u(kT) = \frac{dx}{dt}\Big|_{t=kT} = \frac{1}{T}(x(kT) - x[(k-1)T]).$$
(13.55)

The z-transform of Equation (13.55) is then

$$U(z) = \frac{1 - z^{-1}}{T} X(z) = \frac{z - 1}{Tz} X(z).$$

The integration of x(t) can be represented by the **forward-rectangular integration** at t = kT as

$$u(kT) = u[(k-1)T] + Tx(kT), \qquad (13.56)$$

Source: Dorf & Bishop, Modern Control Systems, §13.9, pp. 1030-1

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Implementation of Digital PID Controllers (2) where u(kT) is the output of the integrator at t = kT. The z-transform of Equation (13.56) is $U(z) = z^{-1}U(z) + TX(z),$ and the transfer function is then $\frac{U(z)}{X(z)} = \frac{Tz}{z-1}.$ Hence, the z-domain transfer function of the PID controller is $G_c(z) = K_P + \frac{K_I T z}{z-1} + K_D \frac{z-1}{T z}.$ (13.57)The complete difference equation algorithm that provides the PID controller is obtained by adding the three terms to obtain [we use x(kT) = x(k)] $u(k) = K_P x(k) + K_I [u(k-1) + T x(k)] + (K_D/T) [x(k) - x(k-1)]$ $= [K_P + K_I T + (K_D/T)]x(k) - K_D T x(k-1) + K_I u(k-1).$ (13.58)Equation (13.58) can be implemented using a digital computer or microprocessor. Of course, we can obtain a PI or PD controller by setting an appropriate gain equal to zero. Source: Dorf & Bishop, Modern Control Systems, §13.9, pp. 1030-1

Now, What's Next?

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What's Next? Research: Courses • Thesis Projects: • ELEC4620: **Digital Signal Processing** - Signal Processing (Eulerian Video • ELEC4630: Magnification) Image Processing & **Computer Vision** - Digital Control (OpenROVs) **METR 4202:** - Robotics • Advanced Control & Robotics - More! • CSSE4010: Digital System Design ELEC 3004: Systems

Economic Signals!

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UA Funding Note

• Old

Funding cluster	Part of funding cluster	Maximum student contribution amounts (\$)	Australian Government contribution (\$)	Total resourcing (\$)
Funding cluster 1 Law, accounting, commerce, economics, administration		10,085	1,951	12,036
Funding cluster 2 Humanities		6,044	5,419	11,463
Funding cluster 3 Mathematics, statistics, behavioural science, social	Mathematics, statistics, computing, built environment or other health	8,613	9,587	18,200
environment, other health	Behavioural science or social studies	6,044		15,631
Funding cluster 4 Education		6,044*	9,974	16,018
Funding cluster 5 Clinical psychology, allied health, foreign languages, visual and performing arts	Clinical psychology, foreign languages, or visual and performing arts	6,044	11,790	17,834
Funding cluster 6	Allied nealth	8,613	12 162	20,403
Nursing Funding cluster 7 Engineering, science, surveying	Engineering, science, surveying	8,613	16,762	25,375
Funding cluster 8 Dentistry, medicine, veterinary science,	Dentistry, medicine or veterinary science	10,085	21,273	31,358
agriculture	Agriculture	8.613		29.886

Funding tier	Discipline(s) within funding tier	Australian Government contribution
Funding tier 1	Law, Accounting, Administration, Economics, Commerce	\$1,805
Funding tier 2	Humanities, Social Studies, Communications (excluding Audio- Visual)	\$6,021
Funding tier 3	Computing, Behavioura Science, Welfare Studies, Education, Visual And Performing Arts, Built Environment, Other Health	\$9,033
Funding tier 4	Mathematics, Clinical Psychology, Allied Health, Nursing, Engineering, Science, Surveying, Environmental Studies, Foreign Languages	\$12,045
Funding tier 5	Dentistry, Medicine, Veterinary Science, Agriculture	\$18,067

Economic Signals

New

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- The funding cut is a 28% (\$16,762 -> \$12,045).
- To keep funding at par would result in a student fee increase of 55% (\$8613 to \$13330).
- Students would now be paying the majority of the cost (52% compared to the 33% currently).

USA

- Georgia Tech (A\$11,450)
- UCSB (A\$12,040).
- MIT!
 - \$43k/year
 - 90% of students will get financial aid
 - average net price of an MIT education being \$38k
 - Median debt at graduation of only \$11k.

