http://elec3004.com

An Introduction to
Digital Linear Systems: Signals & Controls

Welcome!

ELEC 3004: **Digital Linear Systems**: Signals & Controls
Dr. Surya Singh

Lecture 1

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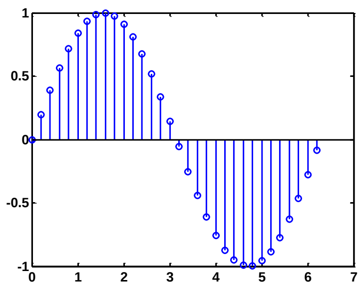
March 4, 2014

What's a Signal?

≡ *A set of data or* ***information***



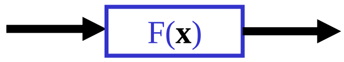
- Can be a function of in **space** and/or **time**
- Various types: electrical, economics, dating, etc.
- Data → "information" is a process of understanding its structure/ forms:

$$\sin(\omega t)$$

What is a System?

≡ A **process** (function) by which information (signals) are modified so as to extract additional information from them

- Systems modify the signal(s) to yield a new result (also a signal)
- Can be of various forms: electrical, mechanical, etc.



Systems Can Be Simpler Than You Think

- B747
  - level flight,
  - 40000 ft, 774 ft/sec …

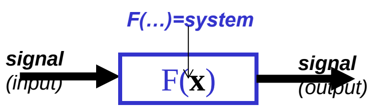
$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -.003 & .039 & 0 & -.322 \\ -.065 & -.319 & 7.74 & 0 \\ .020 & -.101 & -.429 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u - u_w \\ v - v_w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} .01 & 1 \\ -.18 & -.04 \\ -1.16 & .598 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix}$$

  - u, w: horizontal/vertical velocity
  - q, θ: orientation & pitch rate
  - δe, δt : elevator and thrust commands

Source: Boyd, Stanford EE263 Lecture 14 (Slide 14-4)

Signals and Systems Together

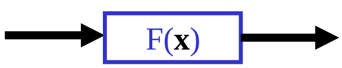
- A **signal** can be seen as that which goes in and out of a **system**



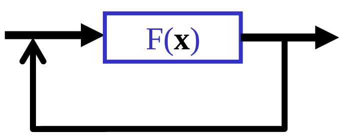
Signals and Systems Together

- A **signal** can be seen as that which goes in and out of a **system**
- **Signal Processing / "Filters" :** can be seen as a open-loop system
- **Feedback Control:** can be viewed as the case where the output signal **shapes** the input signal

Filter:

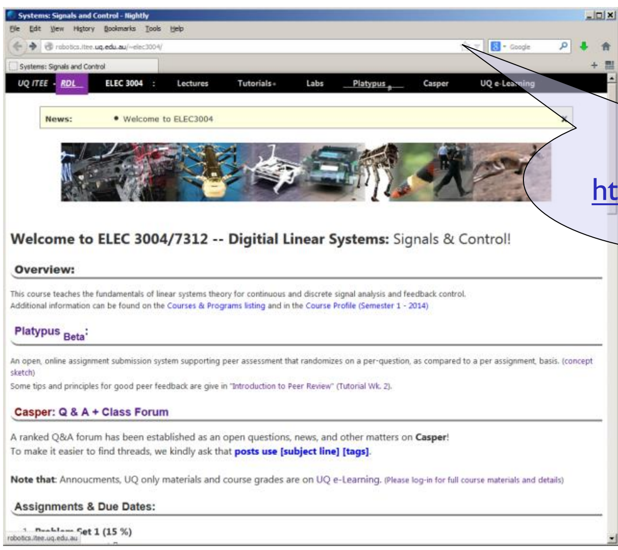


Feedback:



Introducing
ELEC3004/7312

Website: http://robotics.itee.uq.edu.au/~elec3004/



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Schedules and Locations:

- **Lectures:**
  - **Tuesdays** from **4:00 -- 6:00 pm**
  - Here! ☺

- New **"Open Interactive Lectures" :**
  - **Thursdays** from **3:00 -- 5:00 pm**
  - Physiology -- Room 358
  - Interactive: **Student Lead, Q&A, etc.**
  - ∴ **No Questions ⇒ No Answers!** ←

Schedules and Locations: Tutorials & Practicals

➔ All in Hawken -- Room S202 ➔

- **Tutorials: EVEN Weeks (Starting on Week 2)**
  - **Please come to your assigned session**.
  - Sessions are:
    - **Wednesday 9:00--10:00**
    - **Thursday 9:00--10:00**
    - **Friday 8:00--9:00**
    - **Friday 3:00--4:00**
  - ~ 1 hour (unless extra time is needed)

Schedules and Locations: Tutorials & Practicals

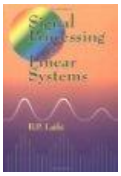
➔ All in Hawken -- Room S202 ←

- **Prac Sessions: ODD Weeks (Starting Week 3)**
  - **Please come to your assigned session.**
  - Sessions are:
    - **Wednesday 9:00--10:50**
    - **Thursday 9:00--10:50**
    - **Friday 8:00--9:50**
    - **Friday 3:00--4:50**
  - ~ 2 hours (unless less time is needed)

Lecture Schedule:

Week	Date	Lecture Title
1	4-Mar	Introduction & Systems Overview
	6-Mar	[Linear Dynamical Systems]
2	11-Mar	Signals as Vectors & Systems as Maps
	13-Mar	[Signals]
3	18-Mar	Sampling & Data Acquisition & Antialiasing Filters
	20-Mar	[Discrete Signals]
4	25-Mar	Filter Analysis & Filter Design
	27-Mar	[Filters]
5	1-Apr	Digital Filters
	3-Apr	[Digital Filters]
6	8-Apr	Discrete Systems & Z-Transforms
	10-Apr	[Z-Transforms]
7	15-Apr	Convolution & FT & DFT
	17-Apr	Frequency Response
8	29-Apr	Introduction to Control
	1-May	[Feedback]
9	6-May	Introduction to Digital Control
	8-May	[Digitial Control]
10	13-May	Stability of Digital Systems
	15-May	[Stability]
11	20-May	State-Space
	22-May	Controllability & Observability
12	27-May	PID Control & System Identification
	29-May	Digitial Control System Hardware
13	3-Jun	Applications in Industry & Information Theory & Communications
	5-Jun	Summary and Course Review

Reference Texts:



B. P. Lathi
Signal processing and linear systems
1998
[TK5102.9.L38 1998](#)

João Hespanha
Linear Systems Theory,
2009
[UQ Ebooks]

- Yes!
  You may use the Internet!!
  - Khan Academy
  - Wikipedia
  - YouTube
  - & **Google Scholar** Too!

- This field is vast & there are countless references present

E-mail

- **elec3004@itee.uq.edu.au**

- Casper!
  - https://casper.ceit.uq.edu.au/courses/elec3004/

- [That's it!]
- {Not the instructors/tutors personally}

The Point of the Course

- Introduction to terminology/semantics
- An appreciation of how to frame problems in a linear systems engineering context
- Modeling and learning assumptions/when to trust the model
- Ability to identify critical details from the problem

➔ It's a **shortcut** …
Once you see that a system is **"linear"**
you can then apply the raft of
"linear systems" tools
(time & frequency analysis) to them
without having to do all the analysis from scratch

Not the Point of the Course

- Get good grades
- Just do homework
- Memorize pointless facts
- Rote "learning" of material with no comprehension
- Ask yourself, is the wonder still there?

Lots of Stuff To Cover…

- Systems
- Signal Abstractions
- Signals as Vectors / Systems as Maps

- Linear Systems and Their Properties
- LTI Systems
- Autonomous Linear Dynamical Systems

- Convolution
- FIR & IIR Systems
- Frequency domain
- Fourier Transform (CT)
- Fourier Transform (DT)

- Even and Odd Signals
- Likelihood
- Causality

- Impulse Response
- Root Locus
- Bode Functions

- Left-hand Plane

- Frequency Response

- Discrete Time
- Continuous Time

- Laplace Transformation
- Feedback and Control
- Additional Applications

- Linear Functions
- Linear Algebra Review
- Least Squares
- Least Squares Problems
- Least Squares Applications
- Matrix Decomposition and Linear Algebra
- Regularized Least Squares

- Least-squares
- Least-squares applications
- Orthonormal sets of vectors
- Eigenvectors and diagonalization
- Linear dynamical systems with inputs and outputs
- Symmetric matrices, quadratic forms, matrix norm, and SVD

- Controllability and state transfer
- Observability and state estimation

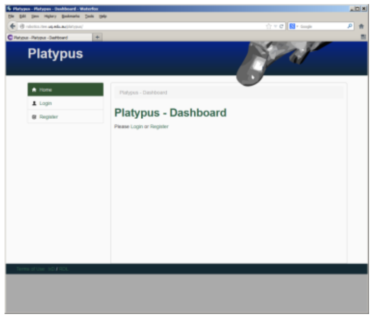
- And that, of course,
  Linear Systems are Cool! ☺

Assessment

Assessment Task	Due Date	Weighting
Laboratory Practicals Part I, II, III, IV & V	At the end of the assigned lab period	EXTRA CREDIT
In Class Quiz Participation and Pop Quizzes	TBA (Pop Quizzes)	EXTRA CREDIT
Problem Set/s Problem Set 1	21 Mar 14 23:59	15%
Problem Set/s Problem Set 2	11 Apr 14 23:59	15%
Problem Set/s Problem Set 3	9 May 14 23:59	15%
Problem Set/s Problem Set 4	30 May 14 23:59	15%
Exam - during Exam Period (Central) Final Examination	Examination Period	40%

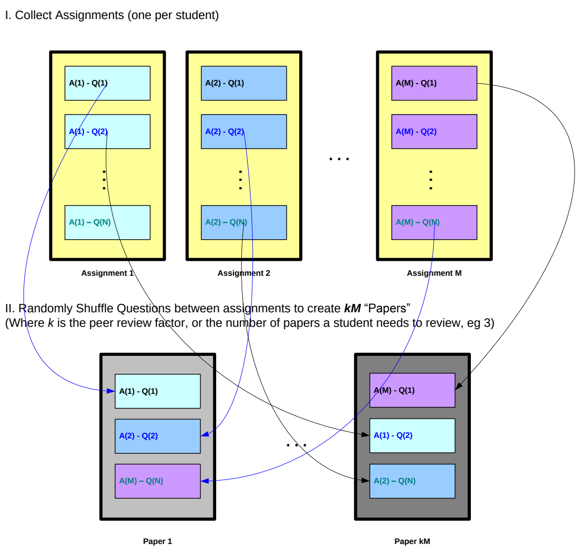
Platypus: Peer-review for Deliberate Practice/Learning

- **Peer-Review**
  - A **key** part of Engineering is being able to critically evaluate peer work (and give **good** feedback on it)
  - We **will** help teach you good habits of peer feedback



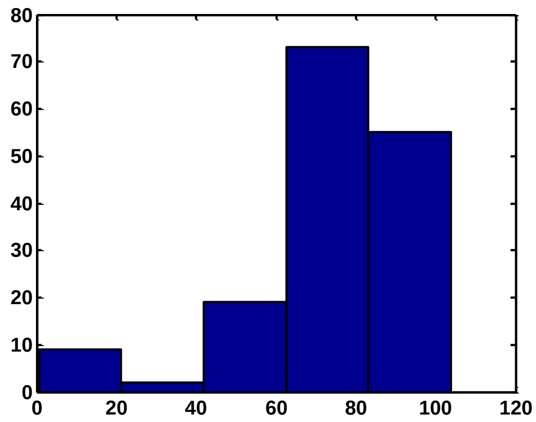
- **Question** (not Assignment) based random shuffling

Platypus: How does it work?



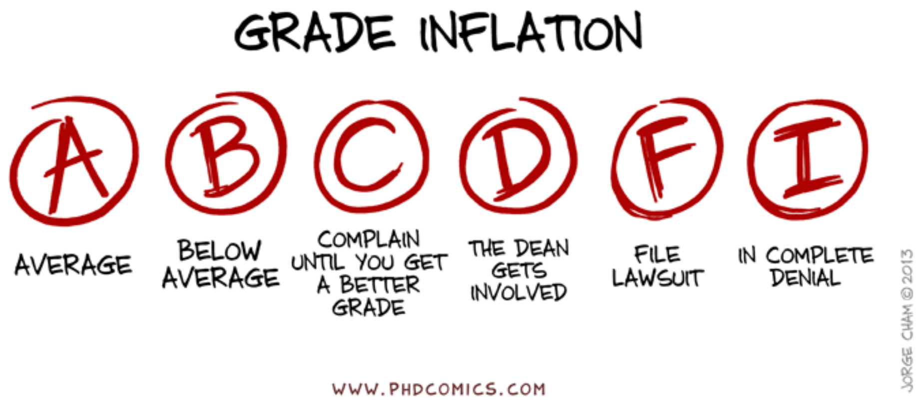
Last Year's Grade Statistics

- ~ 45 % received >5
- Worry about **learning**, not about marks



Some Philosophy

- Let's start with Why …
- To learn something is to teach it
  - The function of a teaching is not so much to cover the topics, but more to discover them
- It is actually **more** work for us!
  - We have to teach you how to reflect & then assess this as well as how to do the assignment
- It helps you understand it by giving you a different perspective
- We're a community
  - You (alone) can't do everything … that's why we work together
  - The notion of "free speech" → Trust emerges → efficiency (η)



I need a "7" for a Job!

The New York Times | http://nyti.ms/1jTJavh

SUNDAYREVIEW | OP-ED COLUMNIST

How to Get a Job at Google

FEB. 22, 2014

Thomas L. Friedman

MOUNTAIN VIEW, Calif. — LAST June, in an interview with Adam Bryant of The Times, Laszlo Bock, the senior vice president of people operations for Google — i.e., the guy in charge of hiring for one of the world's most successful companies — noted that Google had determined that "G.P.A.'s are worthless as a criteria for hiring, and test scores are worthless. ... We found that they don't predict anything." He also noted that the "proportion of people without any college education

What I expect from you

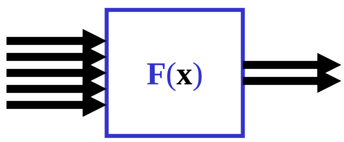
- Lectures:
  - Participate - ask questions
  - Turn up (hence the attendance marks)
  - Take an interest in the material being presented
- Tutorials:
  - Work on questions before tutorials
  - Use tutorials to clarify and enhance
  - Assignments to be submitted on time

Is the Wonder Still There?

Signals & Systems

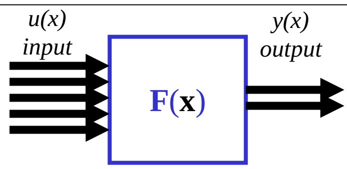
An Overview of Systems

- Today we are going to look at **F**(x)!



- F(x): System Model
  - The rules of operation that describe it's behaviour of a "system"
  - Predictive power of the responses
  - Analytic forms > Empirical ones
    - Analytic formula offer various levels of detail
    - Not everything can be experimented on *ad infinitum*
    - Also offer Design Intuition (let us devise new "systems")
    - Let's us do **analysis**! (determine the outputs for an input)
  - Various Analytic Forms
    - Constant, Polynomial, **Linear**, Nonlinear, Integral, **ODE**, PDE, Bayesian…

Linear Systems



- Model describes the relationship between the input **u**(x) and the output **y**(x)
- If it is a <u>Linear</u> System (wk 3):

$$y(t) = \int_0^t F(t-\tau)\,u(\tau)\,d\tau$$

- If it is also a (Linear and) **<u>lumped</u>**, it can be expressed **<u>algebraically</u>** as:

$$\begin{aligned}\dot{x}(t) &= A(t)\,x(t) + B(t)\,u(t)\\ y(t) &= C(t)\,x(t) + D(t)\,u(t)\end{aligned}$$

- If it is also (Linear and) **<u>time invariant</u>** the matrices can be reduced to:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

$\mathscr{L}$aplacian: $$y(s) = F(s)\,u(s)$$

Discrete Variations & Stability

$$y(s) = F(s)\,u(s)$$

- Is in <u>continuous time</u> …
- To move to <u>discrete time</u> it is more than just "sampling" at: 2 × (biggest Frequency)
- Discrete-Time Exponential

$$F(t) \to F[kT]$$

$$e^{\frac{k}{T}} = \gamma^k$$

$$\frac{1}{T} = \ln \gamma$$

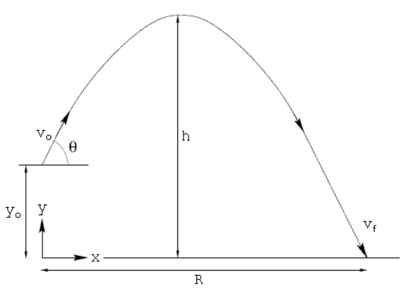
- SISO to MIMO
  - Single Input, Single Output
  - Multiple Input, Multiple Output
- BIBO:
  - Bounded Input, Bounded Output
- Lyapunov:
  - Conditions for Stability
  - ➔ Are the results of the system asymptotic or exponential

Further Classifications of Systems

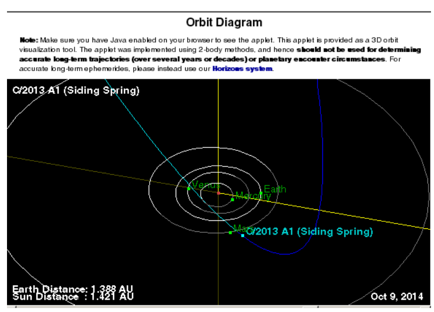
1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Modelling order

- Modelling order depends on what you are trying to achieve



The parabolic trajectory of a projectile



http://ssd.jpl.nasa.gov/

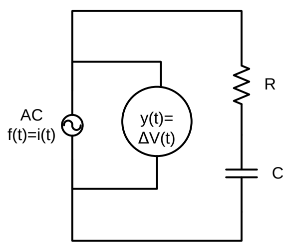
JPL Small-Body Database Browser.url

Modelling Ties Back with ELEC 2004

- Linear Circuit Theorems, Operational Amplifiers
- Operational Amplifiers
- Capacitors and Inductors, RL and RC Circuits
- AC Steady State Analysis
- AC Power, Frequency Response
- Laplace Transform
- Reduction of Multiple Sub-Systems
- Fourier Series and Transform
- Filter Circuits

➔ Modelling Tools!

Example: RC Circuits



$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{t} f(\tau)\,d\tau$$

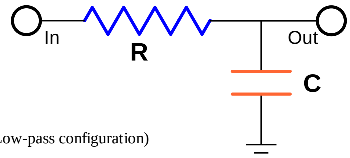
$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{0} f(\tau)\,d\tau + \frac{1}{C}\int_{0}^{t} f(\tau)\,d\tau$$

$$y(t) = v_C(0) + Rf(t) + \frac{1}{C}\int_{0}^{t} f(\tau)\,d\tau$$

$$y(t) = v_C(t_0) + Rf(t) + \frac{1}{C}\int_{t_0}^{t} f(\tau)\,d\tau$$

First Order RC Filter

- Passive, First-Order **R**esistor-**C**apacitor Design:



(Low-pass configuration)

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

- 3dB (½ Signal Power):

$$\omega = 2\pi f$$

$$f_c = \frac{1}{2\pi \mathbf{RC}}$$

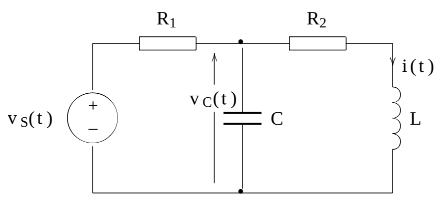
- Magnitude:

$$|V_{\text{out}}| = \sqrt{\frac{1}{(\omega RC)^2}}\,|V_{\text{in}}|$$

- Phase:

$$\phi = \tan^{-1}(-\omega RC)$$

Example of 2nd Order: RLC Circuits



- KCL:

$$\frac{V_s(t) - V_c(t)}{R_1} = C\frac{d}{dt}V_c(t) + i(t)$$

- KVL:

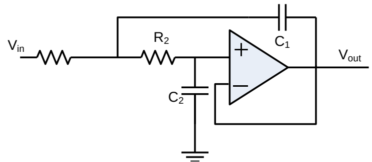
$$V_c(t) = L\frac{d}{dt}i(t) + R_2 i(t)$$

- Combining:

$$V_s(t) = R_1 LC\frac{d^2}{dt^2}i(t) + (L + R_1R_2C)\frac{d}{dt}i(t) + (R_1 + R_2)\,i(t)$$

2^{nd} Order Active RC Filter (Sallen–Key)

- 2^{nd} Order System Sallen–Key Low-Pass Topology:



Build this for Real in **ELEC 4403**

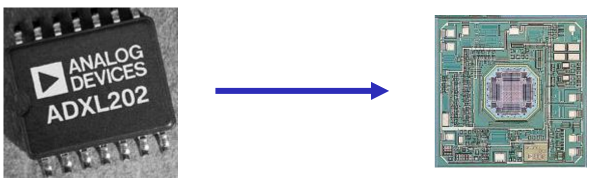
- KCL: $\frac{v_{in}-v_x}{R_1} = C_1 s\,(v_x - v_{out}) + \frac{v_x - v_{out}}{R_2}$
- Combined with Op-Amp Law:

$$\frac{v_{in}-v_{out}(C_2 s R_2+1)}{R_1} = C_1 s v_{out}\,(C_2 s R_2+1) - v_{out} + \frac{v_{out}(C_2 s R_2+1)-v_{out}}{R_2}$$

- Solving for Gives a 2^{nd} order System:

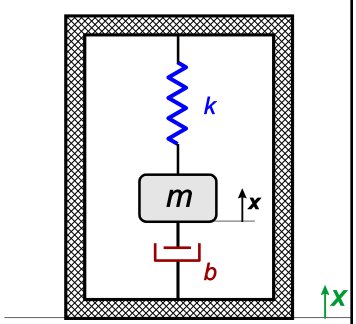
$$\frac{v_{out}}{v_{in}} = \frac{1}{C_1 C_2 R_1 R_2 s^2 + C_2 (R_1 + R_2) s + 1}$$

Another 2^{nd} Order System: Accelerometer or Mass Spring Damper (MSD)



- General accelerometer:
  - Linear spring (k) (0^{th} order w/r/t o)
  - Viscous damper (b) (1^{st} order)
  - Proof mass (m) (2^{nd} order)

➔ Electrical system analogy:
  - resistor (R) : damper (b)
  - inductance (L) : spring (k)
  - capacitance (C) : mass (m)



Measuring Acceleration: Sense *a* by measuring spring motion *Z*

- Start with Newton's 2nd Law:

$$ma = F$$

The "displacement" measured by the unit (the motion of *m* relative the accelerometer frame)

- Substitute:

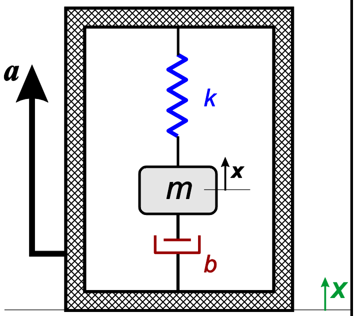
$$m\frac{d^2x}{dt^2} = k\left(X - x\right) + b\frac{d\left(X - x\right)}{dt}$$

$$\mathbf{Z} \equiv \left(X - x\right) \rightarrow x = X - \mathbf{Z}$$

$$\Rightarrow m\frac{d^2X}{dt^2} = m\frac{d^2\mathbf{Z}}{dt^2} + k\mathbf{Z} + b\frac{d\mathbf{Z}}{dt}$$

- Solve ODE:

$$X\left(t\right) = X_0 e^{i\omega t} \qquad \mathbf{Z}\left(t\right) = Z_0 e^{i\omega t}$$



Measuring Acceleration [2]

- Substitute candidate solutions:

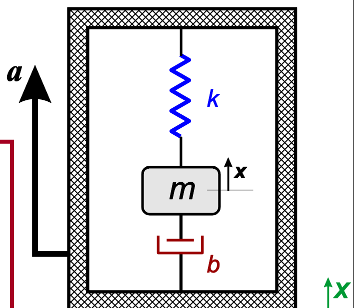
$$m\frac{d^2\left(X_0 e^{i\omega t}\right)}{dt^2} = m\frac{d^2\left(Z_0 e^{i\omega t}\right)}{dt^2} + k\left(Z_0 e^{i\omega t}\right) + b\frac{d\left(Z_0 e^{i\omega t}\right)}{dt}$$

$$-m\omega^2 X_0 e^{i\omega t} = -m\omega^2 Z_0 e^{i\omega t} + kZ_0 e^{i\omega t} + \left(i\omega\right) bZ_0 e^{i\omega t}$$

- Define Natural Frequency (ω_0)
  & Simplify for Z_0
  (the spring displacement "magnitude"):

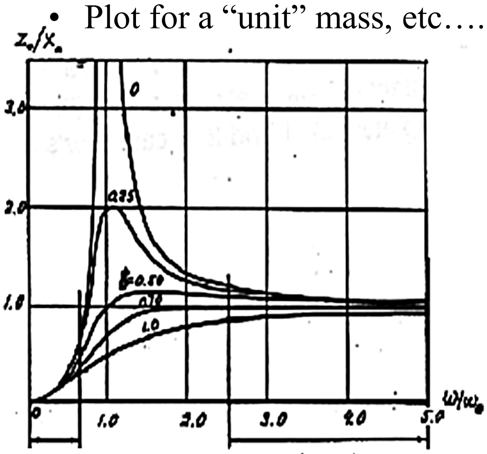
$$\omega_0 \equiv \sqrt{\frac{k}{m}}$$

$$Z_0 = \frac{m\omega^2 X_0}{m\omega^2 - k - i\omega b} = \boxed{\frac{X_0}{\sqrt{1 - \frac{\omega_0{}^2}{\omega^2} - \frac{b^2}{m^2\omega^2}}}}$$



Acceleration: 2nd Order System

- Plot for a "unit" mass, etc….



Accelerometer Seismometer

- For $\omega \ll \omega_0$:

$$Z_0 \approx \frac{\omega^2 X_0}{\omega_0^2} = \frac{a}{\omega_0^2}$$

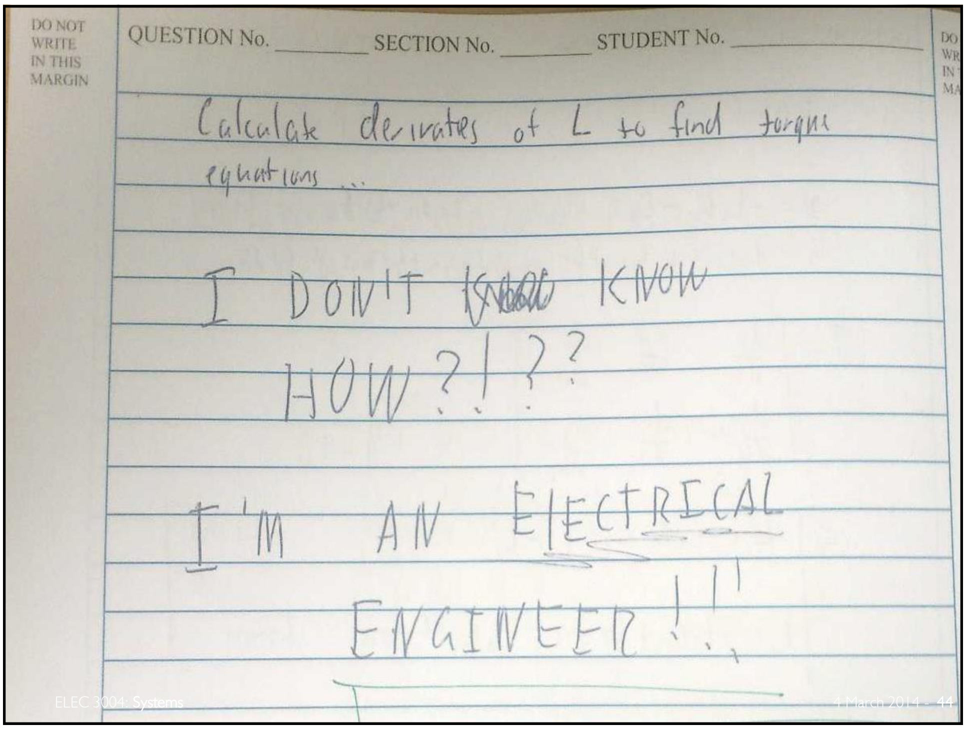
$$\rightarrow a = Z_0 \omega_0^2$$

➔ **it's an Accelerometer**

- For $\omega \sim \omega_0$
  - As: b➔ 0, Z ➔ ∞
  - Sensitivity ↑
- For $\omega \gg \omega_0$:

$$Z_0 \approx X_0$$

➔ **it's a Seismometer**



Equivalence Across Domains

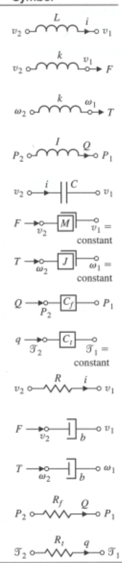
Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}{}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2{}^2$	
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2{}^2$	
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}{}^2$	
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}{}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}{}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}{}^2$	
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}{}^2$	
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	



Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 74

Next Time…

- We'll talk about System Models
- Review:
  - Phasers, complex numbers, polar to rectangular, and general functional forms.
  - Chapter 1 of Lathi
    (particularly the first sections on signals & classification thereof)
- Register on Platypus
- Try the practise assignment (will be posted soon)