

in the fact that the control response is determined from

$$\frac{U(z)}{R(z)} = \frac{D}{1 + DG} = \frac{H(z)}{G(z)},$$

which for this example is

$$\frac{U(z)}{R(z)} = 13.06 \frac{z - 0.0793}{z^2 - 0.7859z + 0.3679} \frac{(z - 1)(z - 0.9048)}{z + 0.9672}.$$

There is a root at  $z = -0.9672!$ . This is the source of the oscillation in the control response, but it did not show up in the output response because it was exactly canceled by a zero. The control oscillation causes the “intersample ripple” in the output response, and the designer should be alert to this if poorly behaved roots arise in the control response. An actual prediction of the output intersample ripple based on linear analysis was not possible with the  $z$ -transform method described so far; rather, one would need to apply the “modified  $z$ -transform,” which is beyond the scope of this text. Alternatively, one can use a CAD simulation to find such oscillations quite easily, as was done here. To avoid this oscillation, we could introduce another term in  $H(z)$ ,  $b_3z^{-3}$ , and require that  $H(z)$  be zero at  $z = -0.9672$ , so this zero of  $G(z)$  is not canceled by  $D(z)$ . The result will be a simpler  $D(z)$  with a slightly more complicated  $H(z)$ . However, rather than pursue this method further, we will wait until the more powerful method of pole assignment by state-variable analysis is developed in the next chapter, where computer algorithms are more readily provided.

## 5.8 PID CONTROL

Just as in continuous systems, there are three basic types of control: Proportional, Integral, and Derivative, hence the name, PID. In the design examples so far, we have been using the discrete equivalent of lead compensation, which is essentially a combination of proportional and derivative control. Let us now review these three controls as they pertain to a discrete implementation. The term PID is widely used because there are commercially available modules that have knobs for the user to turn that set the values of each of the three control types.

### 5.8.1 Proportional Control

A discrete implementation of proportional control is identical to continuous; that is, where the continuous is

$$u(t) = K_p e(t) \Rightarrow D(s) = K_p,$$

the discrete is

$$u(k) = K_p e(k) \Rightarrow \boxed{D(z) = K_p}$$

where  $e(t)$  is the error signal as shown in Fig 5.2.

### 5.8.2 Derivative Control

For continuous systems, derivative or rate control has the form

$$u(t) = K_p T_D \dot{e}(t) \Rightarrow D(s) = K_p T_D s$$

where  $T_D$  is called the *derivative time*. Differentiation can be approximated in the discrete domain as the first difference, that is,

$$u(k) = K_p T_D \frac{(e(k) - e(k-1))}{T} \Rightarrow \boxed{D(z) = K_p T_D \frac{1 - z^{-1}}{T} = K_p T_D \frac{z - 1}{Tz}}$$

In many designs, the compensation is a sum of proportional and derivative control (or PD control). In this case, we have

$$D(z) = K_p \left( 1 + \frac{T_D(z-1)}{Tz} \right).$$

or, equivalently,

$$\boxed{D(z) = K \frac{z - \alpha}{z}}$$

which is similar to the lead compensations that have been used in the designs in the previous sections. The difference is that the pole is at  $z = 0$ , whereas the pole has been placed at various locations along the  $z$ -plane real axis for the previous designs. In the continuous case, pure derivative control represents the ideal situation in that there is no destabilizing phase lag from the differentiation, or, equivalently, the pole is at  $s = -\infty$ . This  $s$ -plane pole maps into  $z = 0$  for discrete rate control; however, the  $z = 0$  pole does

add some phase lag because of the necessity to wait for one cycle in order to compute the first difference. Any other stable pole location, whether on the positive or negative real axis, would also have some delay or phase lag associated with it for the same reason.

### 5.8.3 Integral Control

For continuous systems, we integrate the error to arrive at the control,

$$u(t) = \frac{K_p}{T_I} \int_{t_0}^t e(t) dt \Rightarrow D(s) = \frac{K_p}{T_I s},$$

where  $T_I$  is called the *integral*, or *reset time*. The discrete equivalent is to sum all previous errors, yielding

$$u(k) = u(k-1) + \frac{K_p T}{T_I} e(k) \Rightarrow \boxed{D(z) = \frac{K_p T}{T_I (1 - z^{-1})} = \frac{K_p T z}{T_I (z - 1)}} \quad (5.60)$$

Just as for continuous systems, the primary reason for integral control is to reduce or eliminate steady-state errors, but this typically occurs at the cost of reduced stability.

### 5.8.4 PID Control

Combining all the above yields the PID controller

$$\boxed{D(z) = K_p \left( 1 + \frac{Tz}{T_I(z-1)} + \frac{T_D(z-1)}{Tz} \right)} \quad (5.61)$$

This form of control law is able satisfactorily to meet the specifications for a large portion of control problems and is therefore packaged commercially and sold for general use. The user simply has to determine the best values of  $K_p$ ,  $T_D$ , and  $T_I$ .

### 5.8.5 Ziegler-Nichols PID Tuning

The parameters in the PID controller could be selected by any of the design methods previously discussed. However, these methods require a dynamic model of the process which is not always readily available. Ziegler-Nichols tuning is a method for picking the parameters based on fairly simple experiments on the process and thus bypasses the need to determine a complete dynamic model.

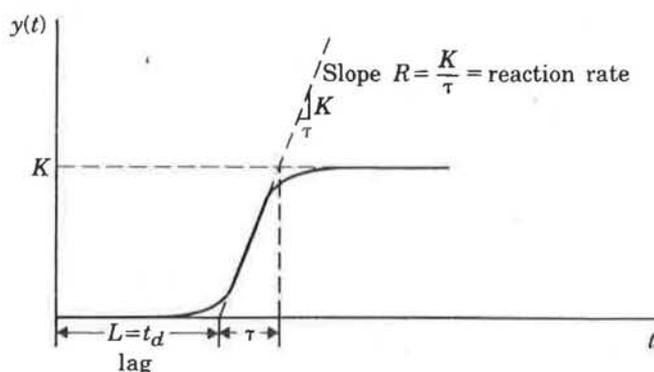


Figure 5.30 Process open-loop step response.

There are two methods. The first, called the *transient-response method*, requires that a step response of the open-loop system is obtained which looks something like that in Fig. 5.30. The response is reduced to two parameters, the time delay,  $L$ , and the steepest slope,  $R$ , which are defined in the figure. In order to achieve a damping of about  $\zeta = 0.2$ , the parameters are selected according to those in Table 5.2.

The second method is called the *stability-limit method*. The system is first controlled using proportional control only. The gain,  $K_p$ , is slowly increased until continuous oscillations result, at which point the gain and oscillation period are recorded and called  $K_u$  and  $P_u$ . The PID gains are then determined from Table 5.3.

The rules are based on continuous systems and will apply to the discrete case for very fast sampling (more than 20 times the bandwidth) provided the designer uses the value of  $T$  in (5.61) that reflects the actual sample period being used by the controller. For slower sampling, a response degradation similar to that in Example 5.3 should be expected, and additional rate control (higher  $T_D$ ) would likely be required to make up for the sampling lag.

Table 5.2 Ziegler-Nichols tuning parameters using transient response.

	$K_p$	$T_I$	$T_D$
$P$	$1/RL$		
$PI$	$0.9/RL$	$3L$	
$PID$	$1.2/RL$	$2L$	$0.5L$

**Table 5.3** Ziegler-Nichols tuning parameters using stability limit.

	$K_p$	$T_I$	$T_D$
<i>P</i>	$0.5K_u$		
<i>PI</i>	$0.45K_u$	$P_u/1.2$	
<i>PID</i>	$0.6K_u$	$P_u/2$	$P_u/8$

**Example 5.12:** Let us apply integral control to the system controlling temperature through mixing described in Appendix A. The transfer function is

$$G_3(s) = \frac{e^{-s\tau_d}}{s/a + 1}$$

For the digital implementation, we assume a zero-order hold, sampling period  $T = 1$  sec, system time constant  $a = 1$  sec, and a  $1\frac{1}{2}$  period delay ( $\tau_d = 1.5$  sec). The transfer function for this example was determined in (2.42) as

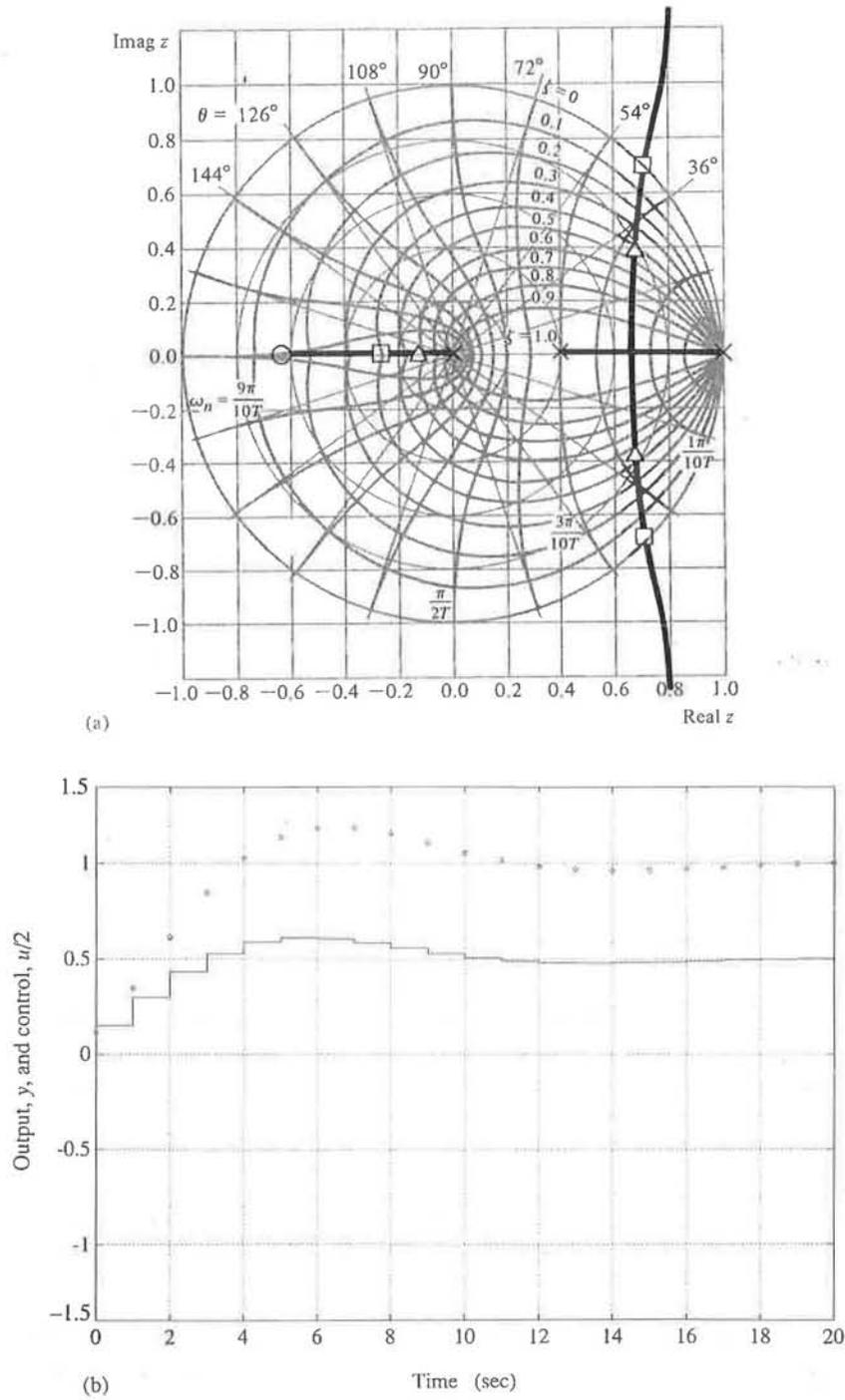
$$G_3(z) = 0.3935 \frac{z + 0.6065}{z^2(z - 0.3679)} \quad (5.62)$$

As it stands, this transfer function has unity gain to a constant control and will have a steady-state error to a constant command or disturbance. If we assume that such behavior in the steady state is unacceptable, we can correct the problem by including integral control by using (5.60) to arrive at the effective system transfer function of

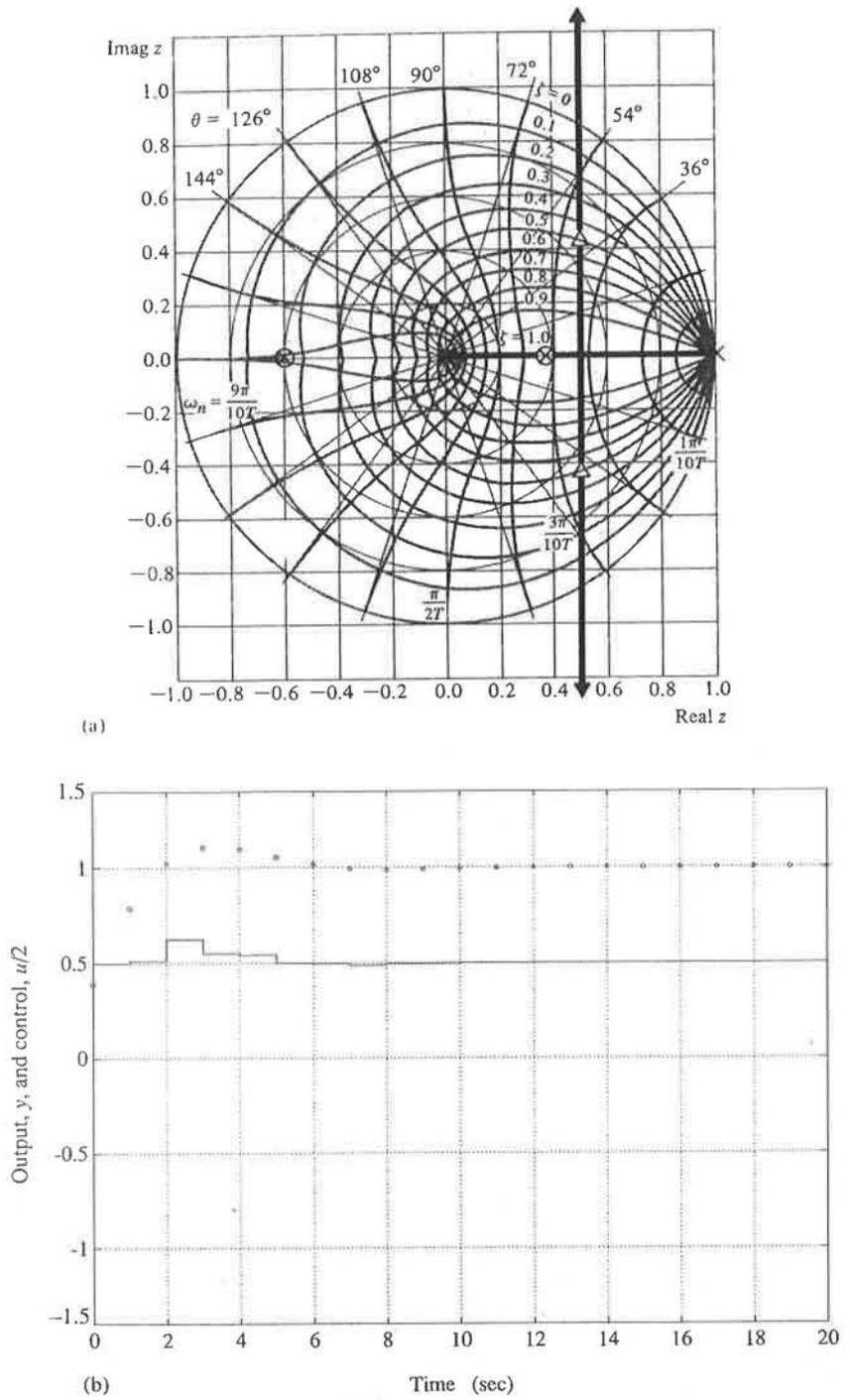
$$DG_3 = 0.3935 K \frac{z + 0.6065}{z(z - 1)(z - 0.3679)}, \quad (5.63)$$

where  $K = (K_p T)/T_I$  following (5.60).

The root locus of this system versus  $K$  is sketched in Fig. 5.31(a) with the roots corresponding to  $K = 1$  marked with a square and those corresponding to  $K = 0.3$  marked with a triangle. The  $K = 0.3$  roots have  $\zeta = 0.5$  and, therefore, should have about a 15% overshoot. Fig. 5.31(b) shows the step response, which verifies the overshoot and indicates that the system has a settling time of



**Figure 5.31** Mixing-flow plant with pure discrete integral control: (a) root locus, (b) step response.



**Figure 5.32** Mixing flow plant with lead compensation and discrete integral control: (a) root locus, (b) step response.

$t_s = 18$  sec. A system with an open-loop time constant of 1 sec is capable of a much faster response than this; however, the delay will cause stability problems if we ask for too much. Let's simply add a lead compensation in order to investigate how to speed up the system. One that cancels the plant pole at  $z = 0.3679$  and the plant zero at  $z = -0.6065$  is

$$D_L = K \frac{z - 0.3679}{z + 0.6065}.$$

With this addition, the complete compensation becomes

$$D(z) = K \frac{z(z - 0.3679)}{(z - 1)(z + 0.6065)};$$

and the system open-loop transfer function reduces to

$$DG_3(z) = 0.3935K \frac{1}{z(z - 1)},$$

whose root locus versus  $K$  is sketched in Fig. 5.32(a). The triangle marks the location of  $\zeta = 0.5$ , which occurs for  $K = 1$  and yields a step response as shown in Fig. 5.32(b). Note that the overshoot has slightly improved and the settling time has been cut in half to 9 sec.

In this example, the transfer function was available for design purposes; therefore, it was possible to determine the integral control using the design methods discussed in Section 5.4. Had we not had a model, the Zeigler-Nichols method could have been applied to help determine the gains.

## 5.9 SUMMARY

In this chapter we have reviewed the philosophy and specifications of the design of control systems by transform techniques and discussed four such methods. First we developed the relations between the time-domain specifications of overshoot, rise time, and settling time and poles in the  $z$ -plane. Using the theory and techniques of discrete equivalents, we then showed how a continuous design can be converted into a discrete design. This design process was called *emulation*. With a sample rate of six times the bandwidth, we found that the approximation was quite coarse and would require substantial adjustment to meet the design specifications. As a second design

approach, we discussed the root locus in the  $z$ -plane. We saw that the root locus is the same as for  $s$ -plane designs, but the relations to time-domain response must refer to the  $z$ -plane. Our third design method was based on frequency-response techniques. The process was carried out in the  $z$ -plane, where extensive reliance on a computer is required, and in the  $w$ -plane, where much of the experience from continuous design can be used more readily. Our final method was a direct transfer-function calculation wherein we found causality and stability constraints on an overall transfer function so that an acceptable compensator can be derived. Here we found that canceling poles near the unit circle can have undesirable effects. In the final section we presented a design by root locus methods for a plant which required the introduction of discrete integral control.

## PROBLEMS AND EXERCISES

**5.1** Use the  $z = e^{sT}$  mapping function and prove that the curve of constant  $\zeta$  in  $s$  is a logarithmic spiral in  $z$ .

**5.2** Sketch the acceptable region in the  $s$ -plane for the specification on the antenna given before (5.15) and sketch the  $s$ -plane root locus corresponding to the controller of (5.15).

**5.3** *Root locus review.* The following root loci illustrate important features of the root locus technique. All are capable of being done by hand, and it is recommended that they be done that way in order to develop skills in verifying a computer's output. Once sketched roughly by hand, it is useful to fill in the details with a computer.

a) The locus for

$$1 + K \frac{s + 1}{s^2(s + P_1)}$$

is typical of the behavior near  $s = 0$  of a double integrator with lead compensation or a single integration with a lag network and one additional real pole. Sketch the locus for values of  $P_1$  of 5, 9, and 20. Pay close attention to the real axis break-in and break-away points.

b) The locus for

$$1 + K \frac{1}{s(s + 1)((s + a)^2 + 4)}$$

illustrates the possibility of complex multiple roots and shows the value of departure angles. Plot the locus for  $a = 0, -1,$  and  $-2$ . Be sure to note the departure angles from the complex poles in each case.