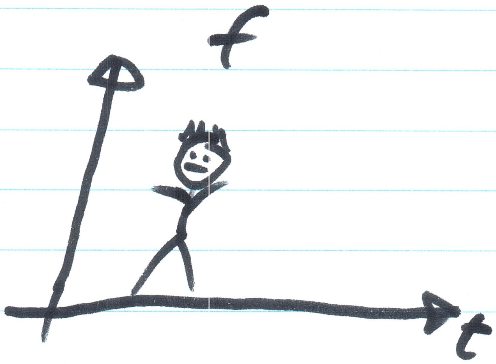
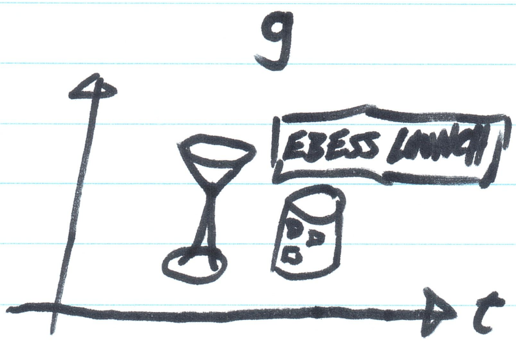
OL notes by Chris Rieger

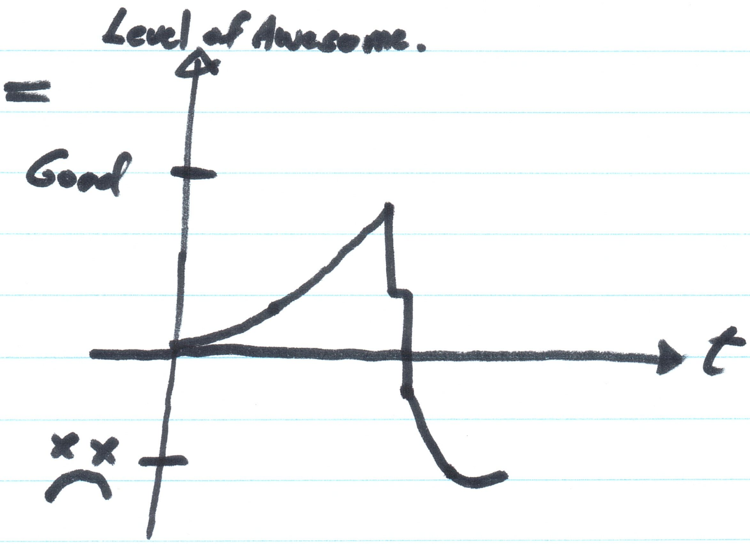
Convolution.

– There's no need to fear, convolution is here!





$f * g =$



⇒ Not this type of convolution!

②

Definition:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} f(t-\tau) g(\tau)\, d\tau$$

$\boxed{\text{Commutative Property.}}$

$f * g = g * f$

E.g. Conv. with Impulse.

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\tau = \underline{\underline{x(t)}}.$$

E.g. $f = e^{-t} u(t)$ ← note the $u(t)$ → positive time only.

$g = e^{-2t} u(t)$

$$y = f * g = \int_0^t \underbrace{f(\tau)}_{= e^{-\tau}} \underbrace{g(t-\tau)}_{= e^{-2(t-\tau)}}\, d\tau$$

$$= \int_0^t e^{-\tau} e^{-2(t-\tau)}\, d\tau$$

$$= e^{-2t} \int_0^t e^{\tau}\, d\tau$$

$$= e^{-2t}\left[e^{t} - 1\right]$$

$$\boxed{= e^{-t} - e^{-2t}} \qquad (t \geq 0)$$

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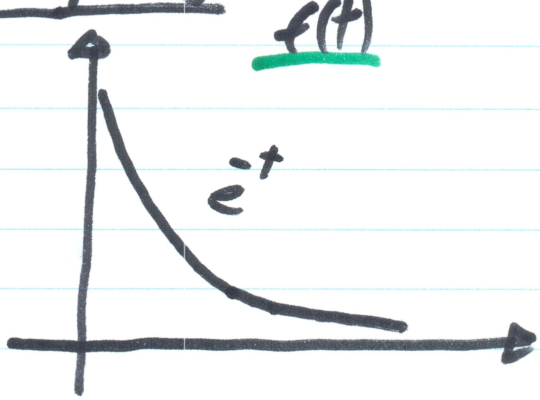
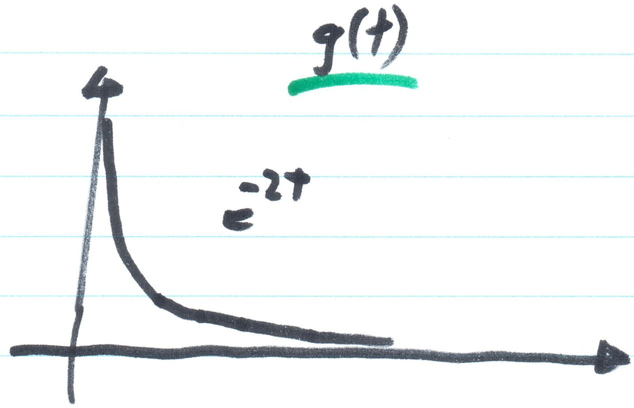
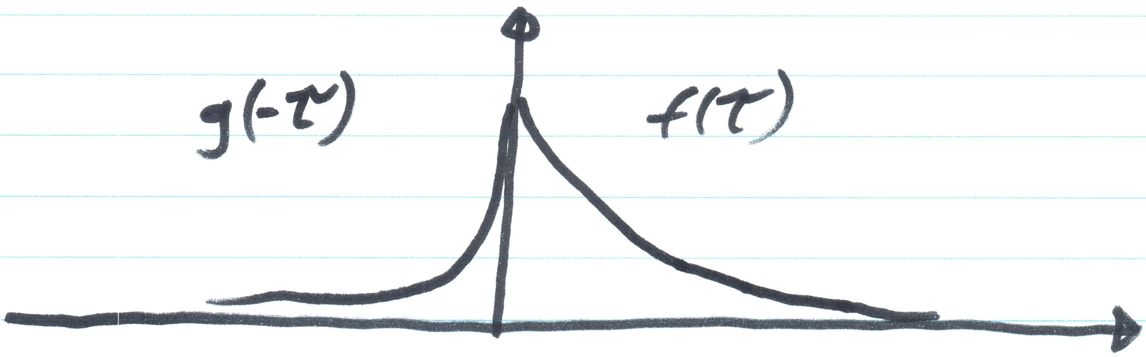
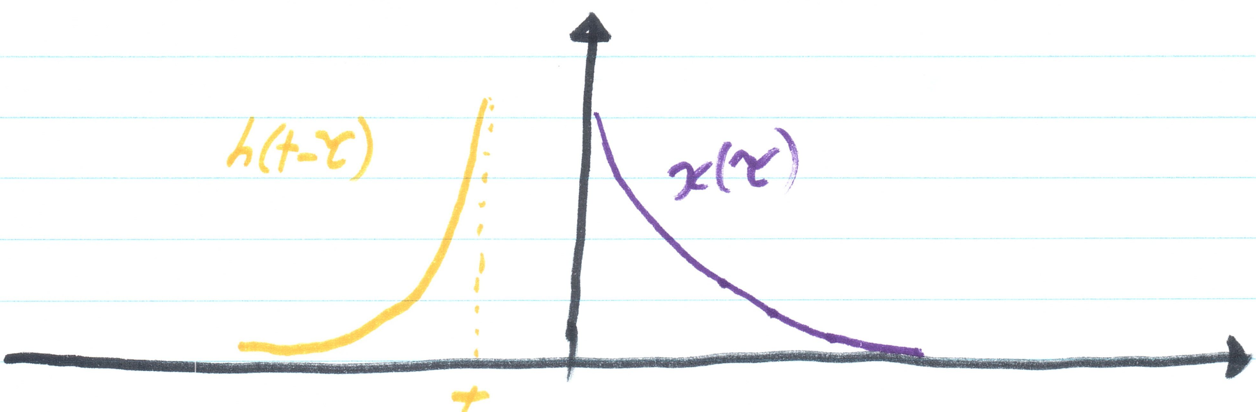
Flip 'N Shift.

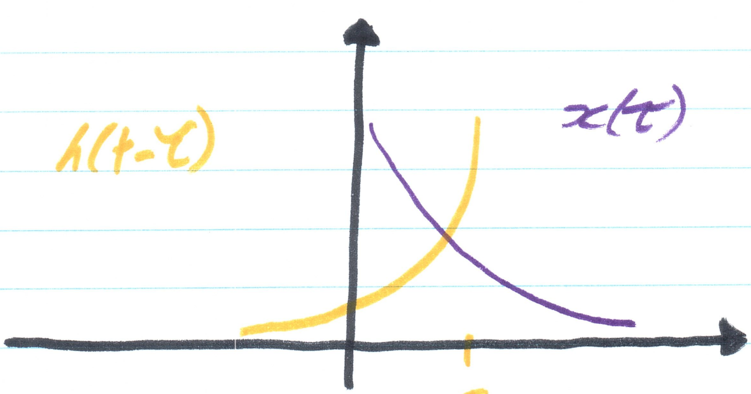
Graphically

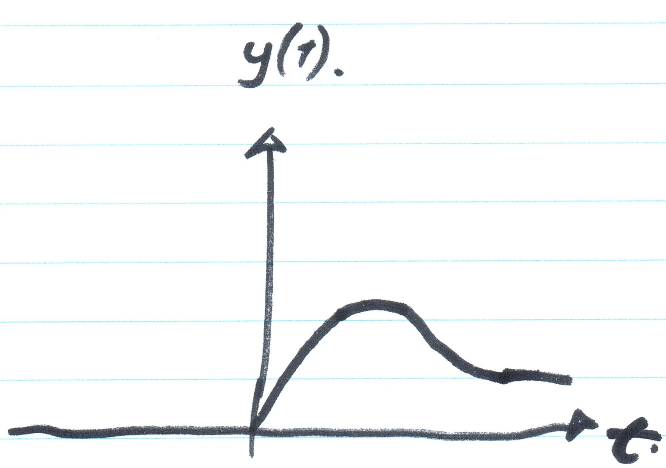












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Laplace Domain.

$$f(t) * g(t) \Longleftrightarrow F(s) \cdot G(s)$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a} \qquad \therefore \mathcal{L}(e^{-t}) = \frac{1}{s+1}$$

$$\mathcal{L}(e^{-2t}) = \frac{1}{s+2}.$$

$$\frac{1}{s+1} \cdot \frac{1}{s+2} = \frac{A}{s+1} + \frac{B}{s+2}. \Longrightarrow 1 = A(s+2) + B(s+1)$$

$$\therefore A = -B.$$

$$1 = 2A + B. \rightarrow A = 1.$$

$$= \frac{1}{s+1} - \frac{1}{s+2}.$$

$$\mathcal{L}^{-1}\left(\frac{1}{s+1} - \frac{1}{s+2}\right) = e^{-t} - e^{-2t}.$$

[Same result.]

Convolution Dual:

$$f(t) \cdot g(t) \Longleftrightarrow F(s) * G(s)$$

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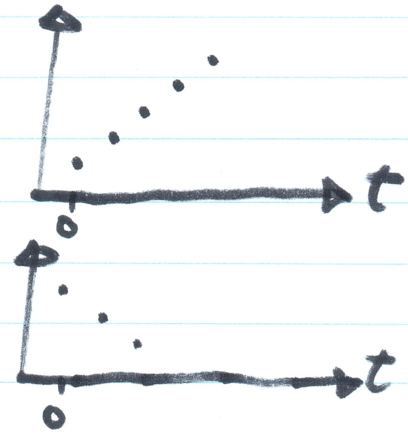
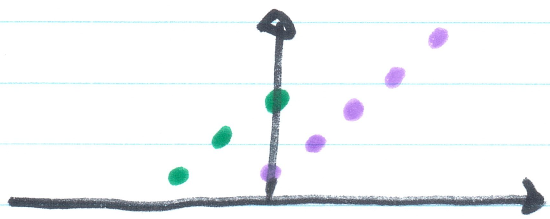
$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Discrete Time:

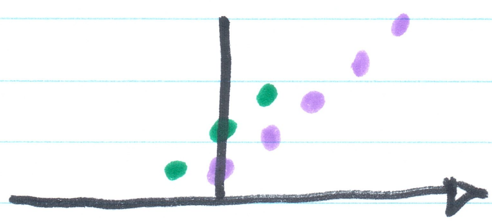
$x[n] = [1 \quad 2 \quad 3 \quad 4 \quad 5]$

$h[n] = [3 \quad 2 \quad 1]$

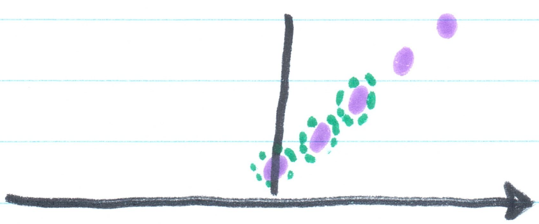




$3 \times 1 = 3.$



$2 \times 1 + 3 \times 2 = 8$



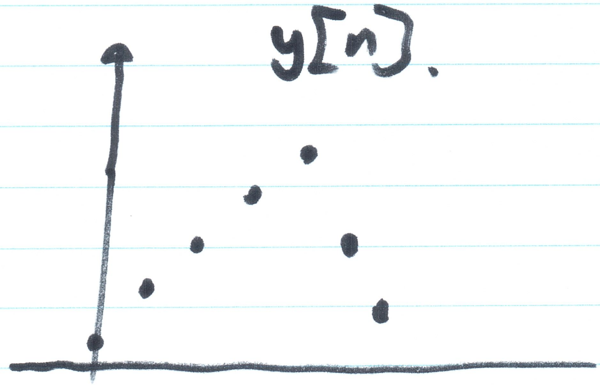
$1 \times 1 + 2 \times 2 + 3 \times 3 = 14.$

⋮

etc.

		[1	2	3	4	5]
				[3	2	1]
		1	2	3	4	5
	2	4	6	8	10	
3	6	9	12	15		
3	8	14	20	26	14	5

$y[n].$



Note Length: $len(x) + len(h) - 1$