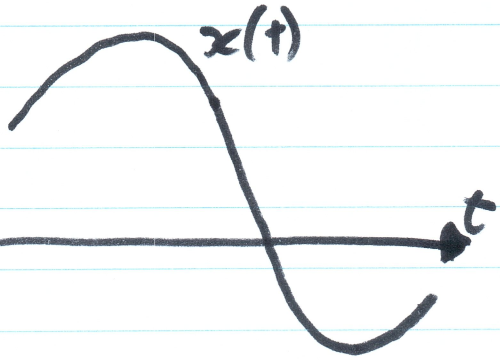
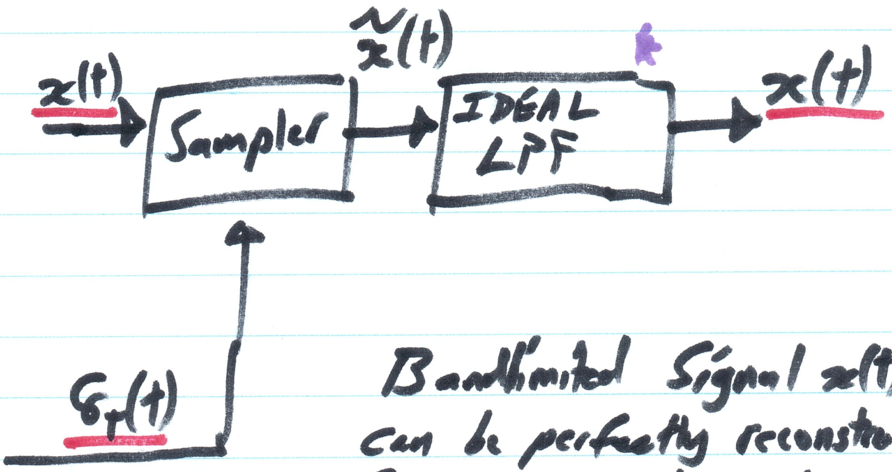
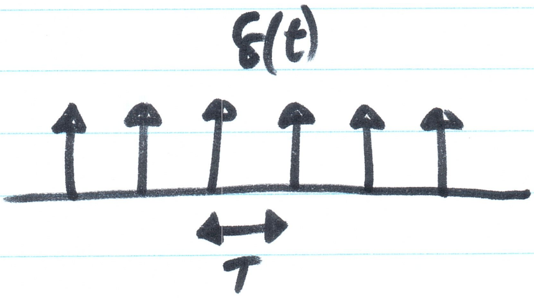
Sampling

– Bridge between continous & discrete domain.





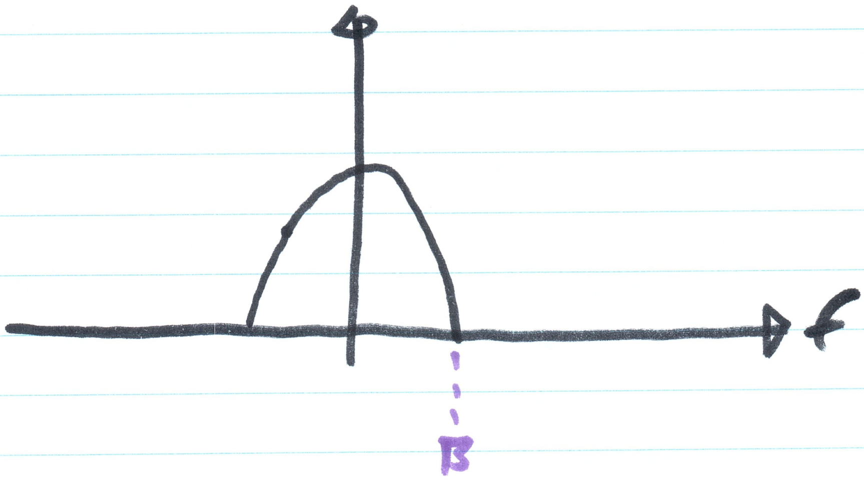


Bandlimited Signal $x(t)$ can be perfectly reconstructed from it's samples, given

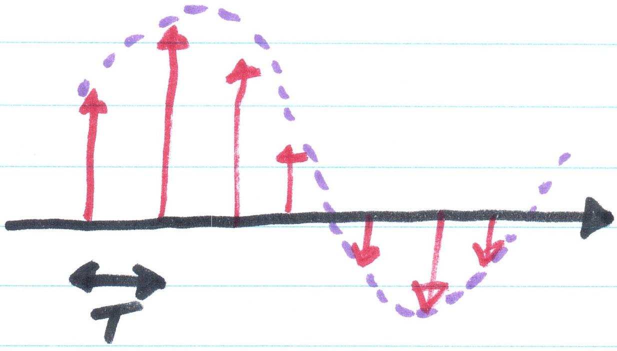
$$\underbrace{\boxed{f_s > 2B}}_{\text{Double sided Bandwidth.}}$$

Bandlimited?

~~X(w)~~ $\mathcal{F}(x(t)) \rightarrow X(w)$



$\tilde{x}(t)$

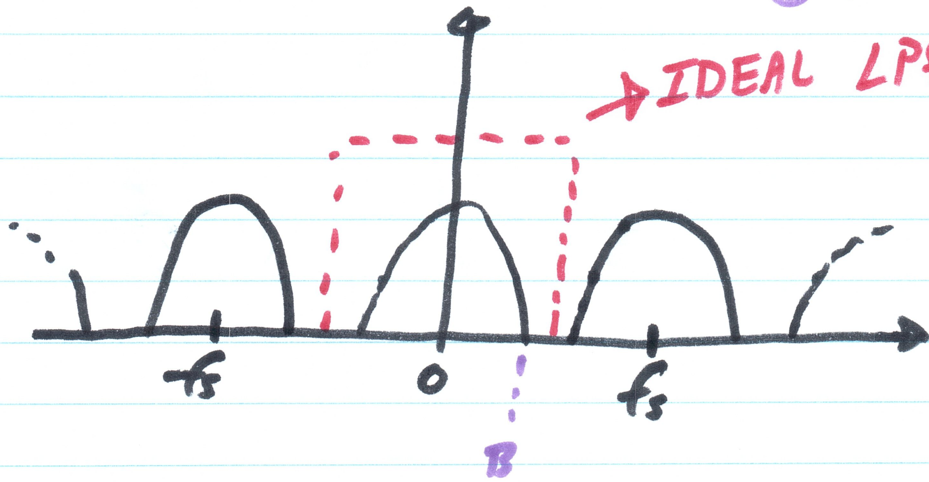


impulse train.

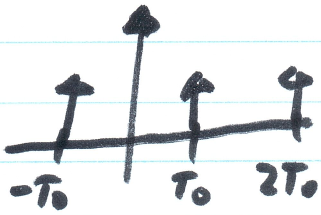
$$\tilde{x}(t) = x(t)\,\delta_T(t)$$
$$= \sum_n x(nT)\,\delta(t-nT)$$

Spectral Replication: → Why?

→ IDEAL LPF. (*)



→ Impulse train → $\sum_{n=-\infty}^{\infty} \delta(t-nT_0)$



$$= \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \quad ; \quad D_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta_{T_0}(t)\, e^{-jn\omega_0 t}\, dt.$$
$$= \frac{1}{T_0}.$$

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$$\therefore \delta_T(t) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$$

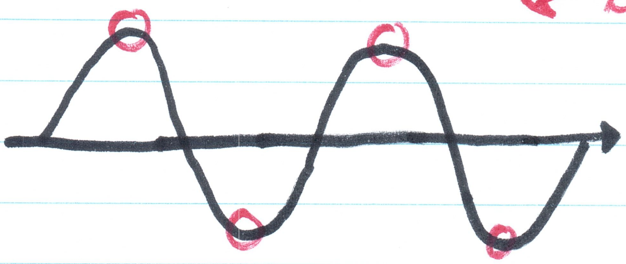
↳ causes repeating spectra! Need <u>LPF</u> (*)

Nyquist?

$$\boxed{f_s \geq 2f_c}$$

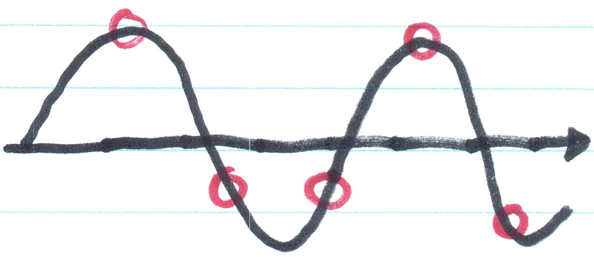
Freq = 1 Hz F_s = 2Hz.

← Bare minimum # samples.



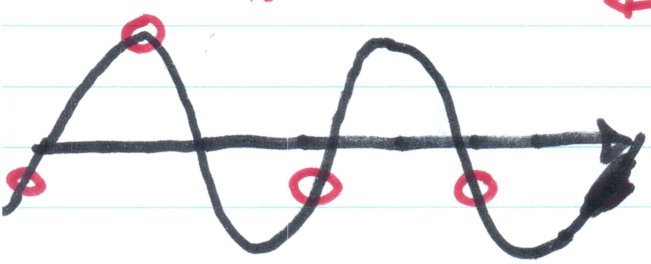
F_s = 3Hz

← more than enough to capture peaks+troughs.

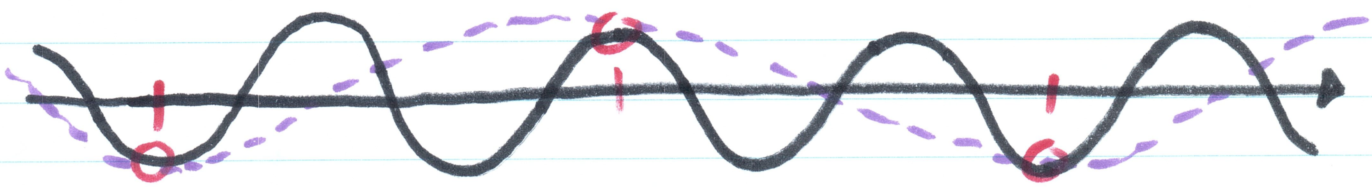


F_s = 1.5 Hz.

← Not enough.



Extreme Case:

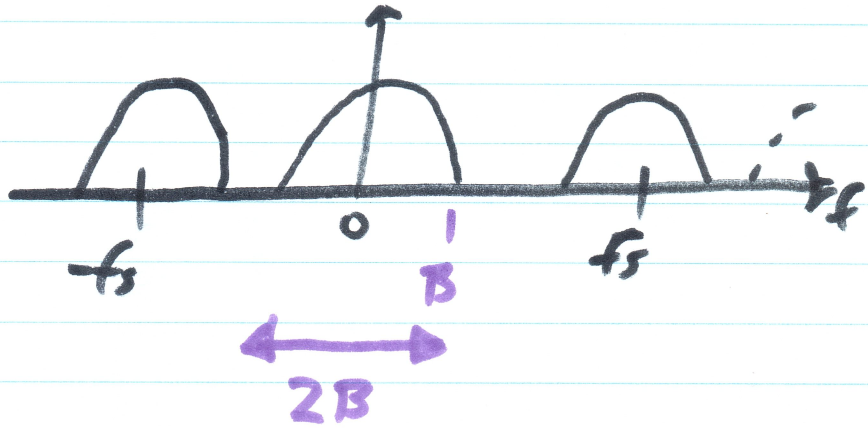


"Aliasing" – two frequencies un-distinguishable by our sampler.

Aliasing & Anti-aliasing Filtering.

Sampled Signal Spectrum →

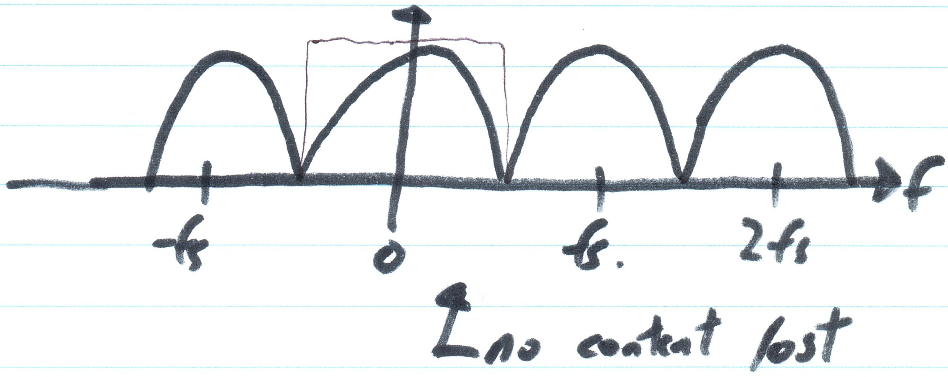
$\boxed{f_s \geq 2B}$



Okay,

Reduce sampling Speed?

$\boxed{F_s = 2B}$



Keep Going,

$\boxed{F_s < 2B}$

Anti-Aliasing Filter?

↳ LPF used to restrict Bandwidth to satisfy the sampling theorem.