ELEC 3004 / 7312 – Digital Linear Systems: Signals & Controls **2014**

Problem Set 1: Linear Signals

Total marks: 80 **Due Date:** March 21, 2014 (at 11:59pm, AEST)

Note: This assignment is worth **15%** of the final course mark. Please submit answers via Platypus. It is requested that solutions, including equations, should be typed please. The final grade is the median of the marks from the peer reviews and the staff (with provisions for review). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions. Thank you. :-)

Short Questions

(Please keep it simple)

Q1. A Ponderance: Two Wrongs Makes a Right? [10 points]

Briefly discuss the validity of following statements (are these true? and why?)

1. The series interconnection of two nonlinear systems is itself nonlinear
2. A difference equation with constant coefficients is always linear and time-invariant. (e.g, x[n+1]=y[n] -- 3.14y[n-1])

Q2. Linear Systems Review [10 points]

An LTIC system is specified by the equation:

$(D^2 + 9)\, y(t) = (3D + 2) f(t)$

1. Find the characteristic polynomial, characteristic equation, characteristic roots, and characteristic modes of this system.
2. Solve the system.
   Find $y_0(t)$, the zero-input component of the response y(t) for $t \geq 0$,
   for the initial conditions are $y_0(0) = 0$ and $\frac{d}{dt} y_0(0) = 5$

Q3. What's the Difference? [15 points]

We may describe a system as being:

- linear[1]
- time-invariant[2]
- causal[3]

Determine which of these three properties hold (**yes**/**no**) for:

(a) Backward differencer (`diff` in MATLAB): $y[t] = x[t] - x[t-1]$

(b) Forward differencer: $y[t] = x[t+1] - x[t]$

(c) Central differencer: $y(t) = x(t + \frac{1}{2}) - x(t - \frac{1}{2})$

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[1] Superposition holds, that is for the input/output pairs $\mathbf{x_1[n]} \rightarrow \mathbf{y_1[n]}$ and $\mathbf{x_2[n]} \rightarrow \mathbf{y_2[n]}$, the combination $\mathbf{ax_1[n] + bx_2[n]} \rightarrow \mathbf{ay_1[n] + by_2[n]}$

[2] Characteristics of the system are fixed over time, so for a system with input **x[n]** and output **y[n]**, the input **x[n-m]** will give the output **y[n-m]**

[3] The output only depends on time inputs at the present and past times. The system is nonanticipative.

Q4. Linearity: Full Speed Linearity! **[15 points]**

Dr Shilling pensively does tests on a new system F with inputs x[n] to get outputs y[n] and observes the following input/output signal pairs (for n=1 to 5):

i	Input ($x_i[n]$)	Output ($y_i[n]$)
1	[0, 0, 0, 1, 1]	[0,1,0,0,0]
2	[0, 1, 1, 1, 1]	[0,1,0,1,0]
3	[0, 1, 1, 2, 2]	[0,2,0,1,0]

(In all cases below please briefly justify your answers)

A. From this, can Dr Shilling conclude that the system F is linear?
B. Assume that F is indeed linear, then is it possible to determine the output signal associated with an input of x=[0, 0, 0, 0, 0]? If so, please compute it.
C. Assume that F is indeed linear, then is it possible to determine the input signal associated with an output of y=[0, 0, 0, 0, 0]? If so, please compute it.

Q5. Sampling: In Sync With Nyquist **[15 points]**

Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals

(a) $\text{sinc}^2(100\pi t)$
(b) $0.01\ \text{sinc}^2(100\pi t)$
(c) $\text{sinc}^2(100\pi t) + 3\ \text{sinc}^2(60\pi t)$
(d) $\text{sinc}(50\pi t)\ \text{sinc}(100\pi t)$

Q6. Sampling: Sounds Good **[15 points]**

Data at a rate of 1 million pulses per second are to be transmitted over a certain communications channel. The unit step response g(t) for this channel is shown in the figure below.

(a) Explain if this channel can transmit data at the required rate.
(b) Can an audio signal consisting of components with frequencies up to 15kHz be transmitted over this channel with reasonable fidelity?