THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2014

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Any unmarked paper dictionary is permitted

An unmarked Bilingual dictionary is permitted

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

1 x 6 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Rough Paper

Instructions To Students:

Please answer ALL questions. Thank you.

For Examiner Use Only

Question	Mark

Total ________

This exam has THREE (3) Sections for a total of 100 Points

Section 1: Linear Signals & Systems .... 25 %

Section 2: Signal Processing .... 30 %

Section 3: Digital Control.. 45 %

Please answer **ALL** questions.

⇨ **PLEASE RECORD ALL ANSWERS IN THE ANSWER BOOKLET** ⇦

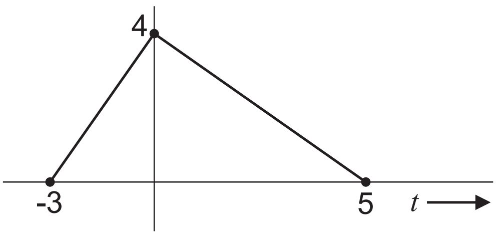
Any material not in Answer Booklet(s) **will not be seen**. In particular, the exam paper **will not be graded** or reviewed.

Section 1: Linear Signals & Systems

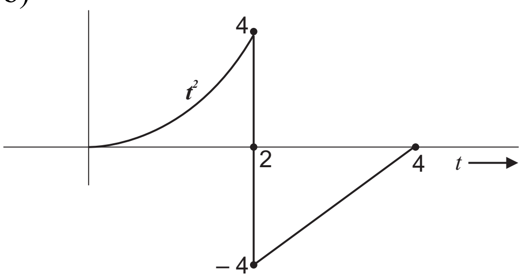
Please Record Answers in the **Answer Book** (Total: **25%**)

1. **Whatchamacallit?** (5%)
   Express the signals in the figure below by a single expression valid for all t.

a)



b)



2. **A Fast and E-Z Fourier** (5%)

Given a discrete-time **unit impulse response** with the following difference equation:

$$\boldsymbol{y[n] = \delta[n-3] - \delta[n+0] - \delta[n+0] + \delta[n+4]}$$

a) What is its Z-transform?
(i.e., what is **Y(z)**)?

b) What is its frequency (or Fourier) response?
(i.e., what is **Y(ω)**)
[hint: for partial credit, you may leave it in terms of the phasor $e^{j\omega}$]

3. **Is Digital Really Better?** (5%)

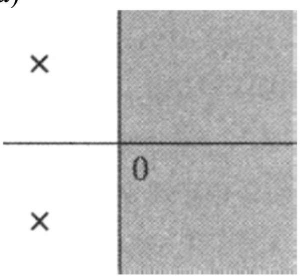
a) Briefly give TWO advantages of digital signals over analogue signals?

b) Briefly give TWO advantages of analogue signals over digital signals?

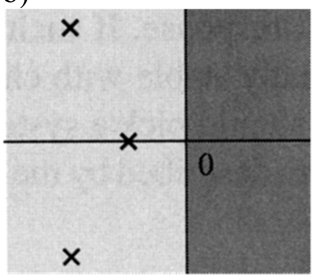
4. **Characteristic Roots and Characteristic Modes** (5%)

For systems having the following pole-zero plots (on the **s-plane**), please sketch the corresponding zero-input response.

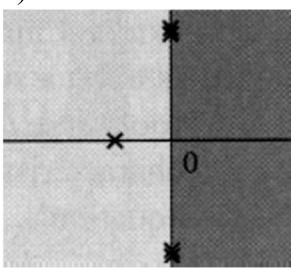
a)



b)



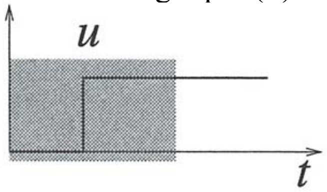
c)



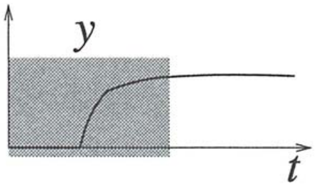
5. **I Spy LTI?** (5%)

A system has the following input (**u**) and output (**y**) relation.

Input:



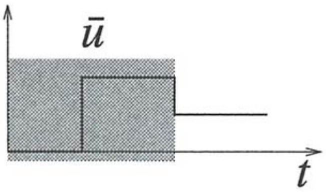
➔ Output:



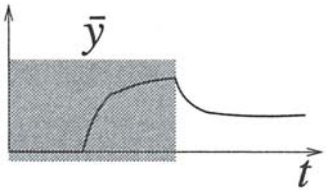
Suppose, for three different (and separate) cases the input is slightly varied as below.

CASE I:

Input (I):

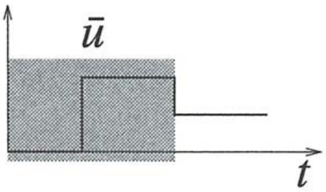


➔ Output (I):

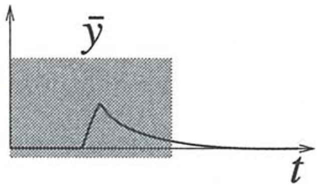


CASE II:

Input (II):

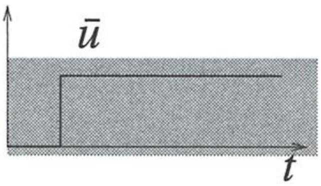


➔ Output (II):

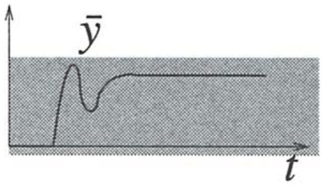


CASE III:

Input (III):



➔ Output (III):



Please determine if the conditions can or cannot be determined. If it can be determined, then please state if it is or is not the case (please mark a × in the table [**in the booklet!!**])
[Note: Please treat each case separately. That is Case I is independent of Case II]

For **CASE I**, is the system:

	Yes	**No**	**Indeterminate**
Linear			
Time-Invariant			
Causal			

For **CASE II**, is the system:

	Yes	**No**	**Indeterminate**
Linear			
Time-Invariant			
Causal			

For **CASE III**, is the system:

	Yes	**No**	**Indeterminate**
Linear			
Time-Invariant			
Causal			

Section 2: Signal Processing

Please Record Answers in the **Answer Book** (Total: **30%**)

6. **Battle of the Band(limited) Signals** (5%)

A signal $f(t)$ is bandlimited to **B** Hz.
Show that the signal $f(t)^n$ is bandlimited to **nB** Hz.
[**Hint**: Start with n=2. Use frequency convolution property and the width property of convolution.]

7. **Cogito Ergo Sum** (5%)

a) **Briefly explain** (and/or show a simple sketch) what is meant by the terms ***ergodic*** and ***ensemble*** in the context of multiple stochastic digital samples or signals?
b) Does the noise have to be "white" for the system to remain unbiased? (please **briefly explain**)
[**Hint**: what happens when "pink" noise is averaged?]

8. **An Analogue to Filtering** (5%)

Consider a filter given by the following transfer function

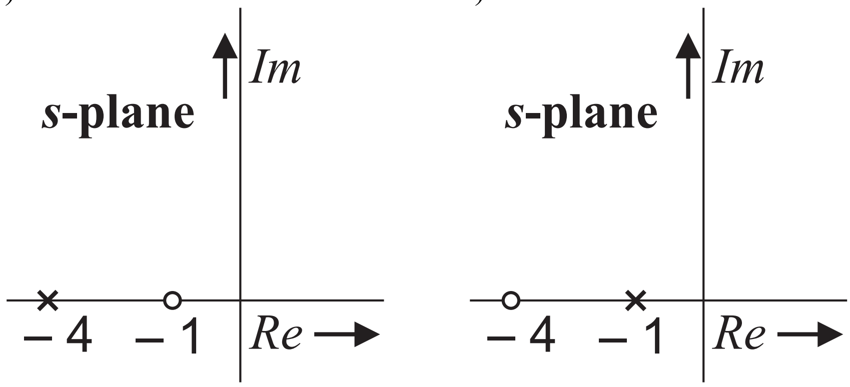
$$H(s) = \frac{4\pi}{s + 4\pi}$$

a) What is the order of *H(s)*?
b) What is the cut-off frequency of *H(s)*?
c) Design a low-pass digital filter, ***H(z)***, with a sampling frequency of 100 Hz, that has the same cut-off frequency as *H(s)*.

9. **A Filtered Whatchamacallit!** (5%)

What kind of compensator is described by pole-zero plots shown below. Please **briefly justify** the conclusion using a transfer function **and** a rough sketch of the amplitude and phase response of the filter.

a) b)



10. **A-Great Filter** (10%)

Design a high-pass filter for removing the **50 Hz** mains flicker from an audio signal from a guitar. The filter should have minimum transmission ≤ **–120 dB** (i.e., an approximate limit to human hearing). Also, the filter should not attenuate music signals from the note A-Great or higher (or more technically **110 Hz**, A_2 or "A") by more –3 dB. **The filtered audio is then played to a concert musician live on stage.**

[**Note**: if you need to you may assume 24-bit, 44.1-kHz sampled signals if needed]

a) What type of filter should we use (analogue or IIR or FIR)? (Please **justify**)
b) What order does this filter need to be?
c) Please sketch the frequency response of this filter.

Section 3: Digital Control

Please Record Answers in the **Answer Book** (Total: **45%**)

11. **A Discrete Convolution** (5%)

What is the convolution ($y_1 \circledast y_2$)[n] between these discrete signals?

a) $y_1[n]=\delta[n-3]$, $y_2[n]=\delta[n+4]$
b) $y_1[n]=\cos[2\pi n]$, $y_2[n]=\delta[n-3]$

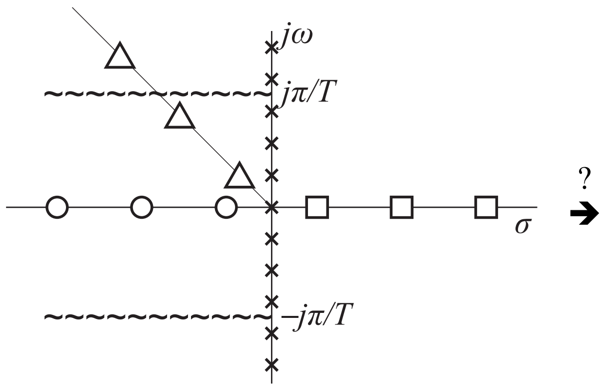
12. **An E-Z Correspondence** (10%)

In mapping from the ***s-plane*** to the ***z-plane***, recall that the duration of a time signal is related to the radius (of the pole location) and the sample rate is related to the angle by $z = e^{sT}$. From this we can sketch major features of the ***s-plane*** to the ***z-plane*** such that they have the same features.

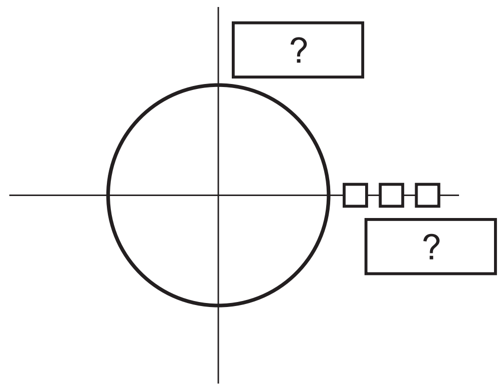
For the following poles marked on the ***s-plane***:

a) "○"
b) "∆"
c) "×",
d) "~", and
e) The axis labels (σ and $j\omega$)

please draw their corresponding locations (and/or terms) on the ***z-plane***.
Please also **briefly justify** your mapping/answer.



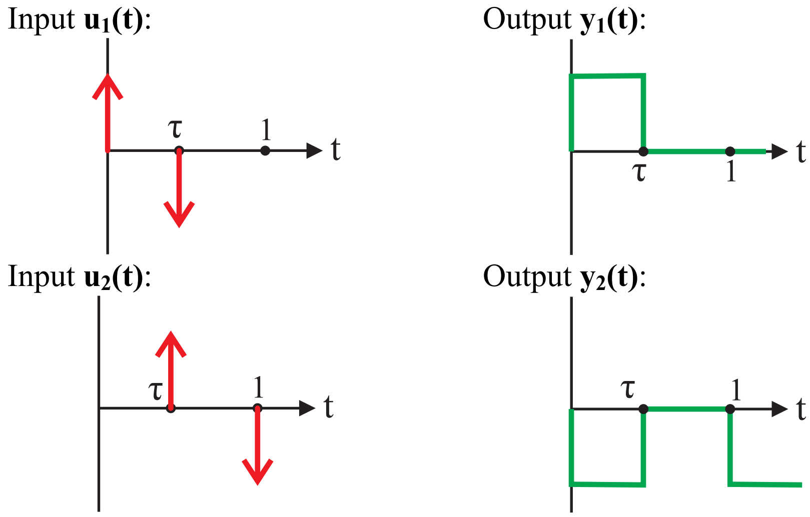
s-plane



Corresponding points on the ***z-plane***
(e.g. the ☐ on the *σ-jω* maps to points outside the unit circle in the **z-plane** as marked)

13. **Got LTI?** (10%)

A system consists of two blocks, with the following input **u(t)** and output **y(t)** pairs:



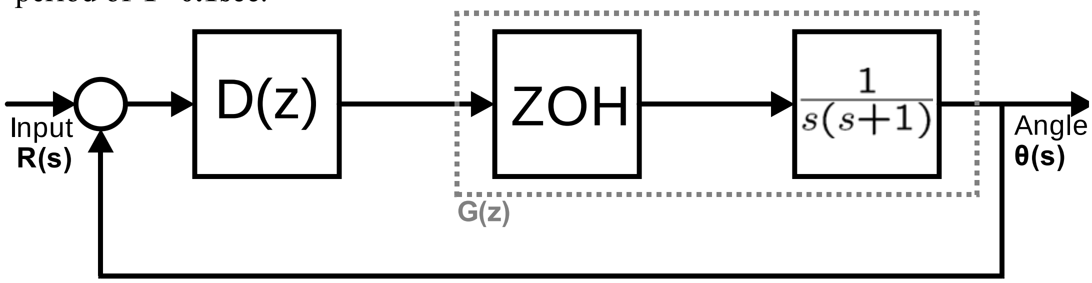
a) Please provide a transfer function, $\mathbf{H_1}$(s) and $\mathbf{H_1(z)}$ for y_1 given u_1.
b) Is the **entire** system ($\mathbf{H_1H_2}$) LTI? (please **briefly explain**)
c) If it is LTI, what is the order of the system?
If it is not LTI, what could be done to easily make it LTI?

14. **Steer-by-Wire** (10%)

A steer-by-wire system is proposed in which a DC motor regulates the hydraulic flow of a power steering system[1]. It has the following continuous time plant

$$P(s) = \frac{1}{s(s+1)}$$

The system is connected to a digital controller D(z) by a ZOH process[2] having a period of **T=0.1sec**.



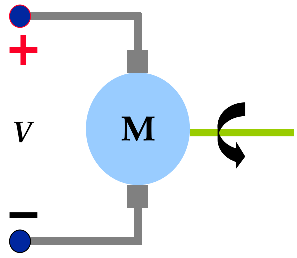
a) Determine G(z)
[**Hint**: For this part you may leave it in terms of the z-Transform, **Z{•}** (i.e. G(z)=Z{G(s)}).]
b) Sketch the impulse response of G(z)

[1] Such systems are commercially available (often in very high-end vehicles) and should not be confused with all electric steer-by-wire systems (e.g. Nissan's Q50).

[2] Zero Order Hold. Recall that a ZOH is modelled by $G_{ZOH} = \frac{1-e^{-sT}}{s}$

15. One Last Stop on the ELEC3004 Express! (10%)

As part of an electric train, you need to implement a control system that stops a DC electric motor as fast as possible.



Recall that a DC motor is characterized by

$$v = k\omega + Ri + Li'$$

Where:

- v : the voltage at its electrical terminals
- i : the current
- i': the time derivative of i (i.e. $\dot{i}$)
- ω : shaft rotational speed (in rad/sec)
- R : resistance of the motor winding
- L : inductance of the motor winding
- k : motor constant

Recall that the torque is given by $\tau = ki$. The mechanical torque model is given by

$$J\omega' = -b\omega + \tau$$

To simplify exam calculations, assume unity values for all constants (with, of course, the appropriate physical units). Thus, $R=1$, $L=1$, $k=1$, $J=1$ and $b=1$.

Given that the motor has some initial speed when the brake is applied at t=0, **ω(0)**, the task is to stop the motor. To do this, we throw a switch that disconnects the driving voltage and connects the terminals of the motor to a "stopping circuit". One basic design for this is a big resistor, $\boldsymbol{R_{Big}}$, resulting in

$$v_{stopping} = -R_{Big}i$$

a) Derive a discrete transfer function from **ω(0)** to $\boldsymbol{v_{stopping}(z)}$ for this plant, using the Tustin's method.

b) What is the slowest sampling rate that will not destabilise the system under unity-gain proportional negative feedback?

c) Under what conditions can the inductance be ignored, and the motor treated as a single pole system?

d) What value of $\boldsymbol{R_{Big}}$ results in the motor velocity stopping the fastest?

e) Prof Gordian DuKnot suggests that the best thing to do is to set $\boldsymbol{R_{Big}}= 0$. That is, short circuit the motor. DuKnot's argument is simple: "if more voltage makes it go faster, then less voltage makes it slower. Thus, zero voltage stops it." Is this wise? (please **explain** your reasoning)

END OF EXAMINATION — Thank you !!!

Is the wonder still there? ☻

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t}\tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1-z^{-1})}{\Delta t(1+z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi}\int_0^{\pi/\Delta t} H_d(\omega)\cos(n\omega\Delta t)\,d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	***Periodic***	***Non-periodic***	
Discrete	***Discrete Fourier Transform*** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$	***Discrete-Time Fourier Transform*** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega$	***Periodic***
Continuous	***Complex Fourier Series*** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\,dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	***Fourier Transform*** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	***Non-periodic***
	Discrete	***Continuous***	***Freq. Domain***

Table 3: Selected Fourier, Laplace and z-transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$\overset{DFT}{\longleftrightarrow}$	$\tilde{X}[k] = \frac{1}{N}$	
$x[n] = \delta[n]$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FS}{\longleftrightarrow}$	$X[k] = \frac{1}{T}$	
$\delta_T[t] = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
$\sin(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } \lvert t \rvert \leqslant T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \frac{1}{\pi t}\sin(\omega_c t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \begin{cases} 1 & \text{when } \lvert\omega\rvert \leqslant \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = 1$	
$x(t) = \delta(t - t_0)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(w) + \frac{1}{jw}$	
$x[n] = \frac{\omega_c}{\pi} \operatorname{sinc} \omega_c n$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } \lvert\omega\rvert < \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s}$	
(unit ramp) $x(t) = t$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s_0}{(s^2 + {s_0}^2)}$	
$x(t) = \cos(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s}{(s^2 + {s_0}^2)}$	
$x(t) = e^{s_0 t}u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s - s_0}$	$\Re\mathfrak{e}\{s\} > \Re\mathfrak{e}\{s_0\}$
$x[n] = \delta[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = 1$	all z
$x[n] = \delta[n - m]$	$\overset{z}{\longleftrightarrow}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - 1}$	
$x[n] = z_0^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert > \lvert z_0\rvert$
$x[n] = -z_0^n u[-n - 1]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert < \lvert z_0\rvert$
$x[n] = a^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - a}$	$\lvert z\rvert < \lvert a\rvert$

Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\lvert X(e^{j\omega})\right\rvert^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\dfrac{d\tilde{x}(t)}{dt}$	$\dfrac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t-t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k-k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\displaystyle\frac{1}{T}\int_{-T/2}^{T/2} \lvert\tilde{x}(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$	

Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\dfrac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\dfrac{1}{\lvert a\rvert}X\left(\dfrac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\, d\omega$	

Table 7: Properties of the z-transform.

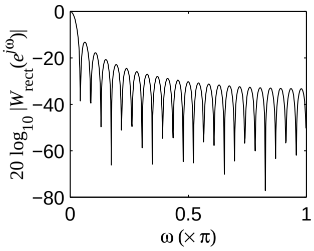
Property	Time domain	z-domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n-n_0]$	$z^{-n_0}X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0\rvert R_x$
Differentiation in z	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\mathfrak{Re}\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim_{z\to\infty} X(z)$		

† $z = 0$ or $z = \infty$ may have been added or removed from the ROC.

Table 8: Commonly used window functions.

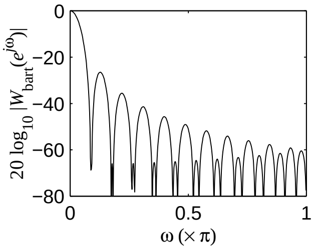
Rectangular:

$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



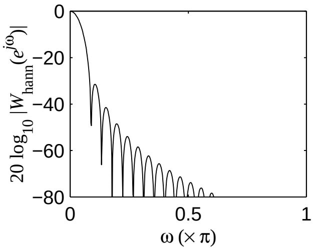
Bartlett (triangular):

$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leqslant n \leqslant M/2, \\ 2 - 2n/M & \text{when } M/2 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



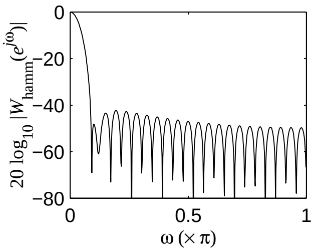
Hanning:

$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos\left(2\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



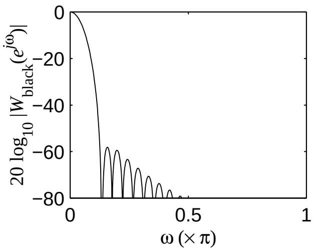
Hamming:

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46\cos\left(2\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Blackman:

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5\cos\left(2\pi n/M\right) + 0.08\cos\left(4\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74