ELEC 3004/7312: Signals Systems & Controls
Assignment 3[i], Due: 25th of May 2012

Note: This assignment is worth **10%** of the final course mark. You should spend approximately 3 hours preparing for the tutorial. The tutors will *not* assist you further unless there is real evidence you have attempted questions prior to the tutorial. Beyond the lecture and tutorial sessions, it is estimated that you will need 4 to 5 hours to complete the assignment (**7-8 hours total**).

Total marks: 100

1. [15] Digitise the following systems, assuming 10 Hz sample rate
   - i. As difference equations, using Euler's method
   - ii. As z-domain transfer functions between x and u

   a) $\dot{x} = -3x + u$
   b) $\ddot{x} = -2\dot{x} - 4x + \dot{u} + u$
   c) $x = 10\left(u + 0.1\int u\,dt + 2\frac{du}{dt}\right)$
   d) $\ddot{x} = \dot{y} - 2\dot{x},\ \ddot{y} = -y - \dot{x} + u$
   e) $\dot{x} = -\sin(t) - x - \dot{u}$

2. [15] Find the steady-state value of x at $t = \infty$ (if any) for each system in part 1

3. [15] Sketch the root locus of the system $\ddot{x} = -10x + u$ in feedback with a 5 Hz controller implementing a
   - i. lead compensator
   - ii. lag compensator
   - iii. PID compensator

   Which is more robust to arbitrarily large system gains?

4. [15] For what range of proportional feedback gains are the following systems stable?
   - i. $H(z) = \frac{0.1052}{z-1.105}$
   - ii. $H(s) = \frac{2}{s^2+4s+4}$, 20 Hz (approximate using Tustin's method)
   - iii. $H(s) = \frac{2}{s^2+4s+4}$, 20 Hz (exact z-transform)

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5. [10] Determine analytically the lowest frequency that does not destabilise $H(s) = \frac{1}{s^2+s+1}$, given proportional feedback gain of 48.

6. [10] What design parameter governs the slowest allowable controller sample rate, and why? What design assumptions must be made in order to achieve this slowest rate?

7. [20] A missile guidance system has a control computer that operates at 50 Hz. The missile's heading is governed by the following dynamic equations:

$$\mathrm{I}\ddot{\theta} = -c\dot{\theta} + k\theta + \tau$$

Where I = 0.1 is the rotational inertia of the missile, $c = 1.5$ is the aerodynamic damping applied by the stabiliser fins, k = 10 is the torque due to nose drag, τ is the control torque generated by the guide fins, as commanded by the controller. The guide fin actuators are driven by a servomechanism with a two sample communication delay.

Design a controller to regulate the missile's heading, and prove its stability analytically. Write a program that implements your controller in pseudo-code, noting all stored values and constants.

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[i] Revised May 16:
1c) x = 10(u + 0.1*int(u) + 2*du/dt) from x = 10(1 + 0.1*int(x) + 2*du/dt)
1e) sin(x) changed to **<u>sin(t)</u>**
7) *k* changed to *k*=10