THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

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Family Name	_____________________
First Name	_____________________

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2013

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Any unmarked paper dictionary is permitted

An unmarked Bilingual dictionary is permitted

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Rough Paper

Instructions To Students:

Please answer ALL questions. Thank you.

For Examiner Use Only

Question	Mark

Total ________

This practice exam is based on the same template as the final exam paper. I have tried to ask practice questions that will help you study the material. In my opinion, the practice exam is **harder than the actual exam**, which is to say that I think that if you can handle this exam you should be able to handle the final exam paper. I have tried to include example problems from Lathi so as to allow one to one to reference associated material covered in the textbook. The problems in the exam are mostly written by me.

This practice exam has THREE (3) Sections for a total of 100 Points

Section 1: Linear Systems .... 30 %

Section 2: Signal Processing.. 30 %

Section 3: Digital Control... 40 %

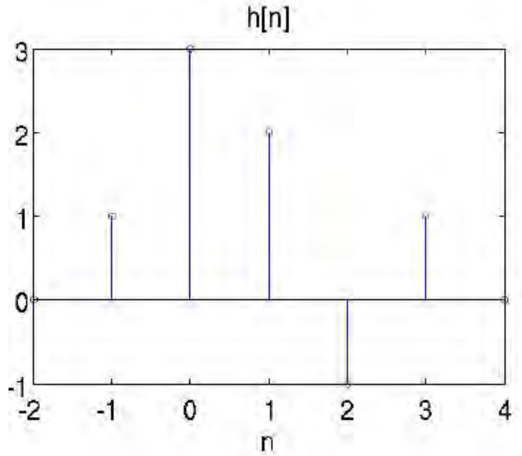
Please answer **ALL** questions

Section 1: Linear Systems

Please Record Answers in the **Answer Book** (Total: **30%**)

1. **Digital Convolution** (5%)

Suppose an LTI system has the following impulse response h[n] given below:



Suppose the input to the system is:

$$x[n] = u[n+1] + u[n-2]$$

Plot the output of the system, y[n]. Please Label your plot.

2. **Linearity** (5%)

Consider a system given with the following impulse response:

$$h[n] = 4u[1-n]$$

a) Is the system LTI?
b) Is it causal?
c) Is it stable?

3. **The Everlasting Exponential** (10%)

Show that the transfer function of an ideal integrator is

$$\boldsymbol{H(s) = \frac{1}{s}}$$

and that of an ideal differentiator is

$$\boldsymbol{H(s) = s}$$

[**Practice Exam Hint:** This is based on Lathi 2.4-4. See Eq 2.48 and Eq. 2.50. You also may need to use the result in Prob. 1.4-9.]

4. **Just in BIBO** (10%)

For the following systems, please:

(i) Determine its character roots;
(ii) Plot its characteristic roots in the complex plane;
(iii) Use this to determine whether it is asymptotically stable, marginally stable, or unstable (assuming that the equations describe its internal system); and,
(iv) Also, determine BIBO stability for each system.

(a) D(D + 2)y(t) = 3x(t)
(b) (D + 1)(D + 2)y(t) = (2D + 3)x(t)
(c) (D + 1)(D2 −4D + 9)y(t) = (D + 7)x(t)

[Practice Exam Hints:

- This is also Lathi Example E2.16, p. 214
- Also, Section 2.7 of Lathi is a good overview section.]

Section 2: Signal Processing

Please Record Answers in the **Answer Book** (Total: **30%**)

5. **Sketch the Following Signals** (5%)

a) $u[n-2]-u[n-6]$
b) $(n-2)\{u[n-2]-u[n-6]\}$
c) $(n-2)\{u[n-2]-u[n-6]\}+(-n+8)\{u[n-6]-u[n-9]\}$

[Practice Exam Hints:

- This is from Lathi Problem 3.3-3
- On the nature of Signals, Lathi **Example 3.4** (Savings Account) is a good example to look at.]

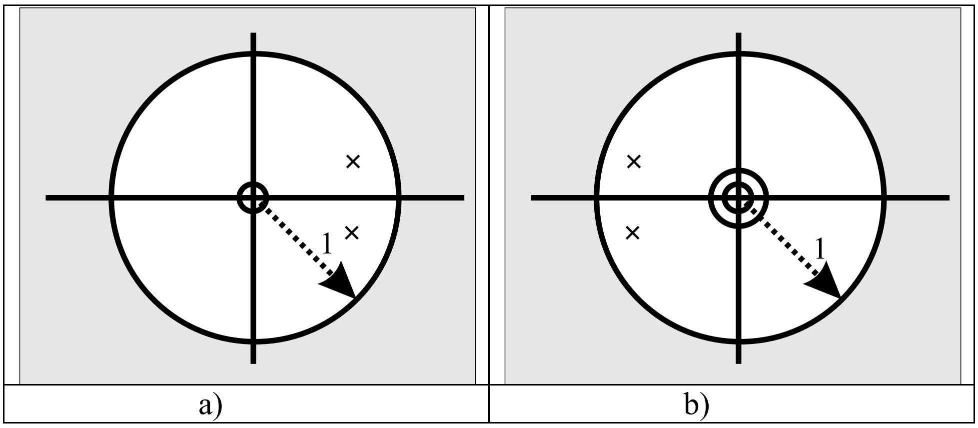
6. **Digital Processing of Analog Signals** (5%)

Design a digital filter to realize an analog transfer function given by

$$\boldsymbol{H(s)=\frac{20}{s+20}}$$

[Practice Exam Hint: See also Lathi Exercise E5.22]

7. **Frequency Response from Pole-Zero Location** (5%)
Pole-zero configurations of certain filters are shown in Fig. P5.6-1. Sketch roughly the amplitude response of these filters.



[Practice Exam Hints:
- Review Section 5.6 (Frequency Response from Pole-Zero Location)
- See also Lathi Problem 5.6-1]

8. **Filter. The Truth** (5%)
For each statement, please and briefly justify if it is **TRUE** or **FALSE**.

a) FIR filters have numerical stability issues
b) IIR filters have numerical stability issues
c) IIR filters do not contain feedback
d) FIR filter order is the same as filter length
e) IIR filters are all poles.

9. **A Lone Filter AM I** (10%)
Help design an analog bandpass filter for tuning into an AM radio station at 792 kHz. The passband is +/— 15 KHz. This is used to send signal to diode across which a crystal earpiece is connected in parallel.

a) What order is needed to have -20 dB of attenuation at +/— 50 KHz?

b) Imagine we are lost in the desert, but happen to have a collection of passive circuit kit (do not leave home without it!). If we wish to construct the tuner using as two first-order stages, what values of R, L, and/or C will we need for both sides (i.e., the low and high bands) of a passband filter to tune into this station?

[**note**: As a thought exercise, what is the performance of this filter? And, where should the cutoff frequencies be so that the signal at the station carrier (i.e., 792 kHz) is not attenuated too much (i.e., more than 3 dB)? Being a second-order filter (comprised of two first order stages), the performance of this filter may be (well) less than in part (a)].

Section 3: Digital Control

Please Record Answers in the **Answer Book** (Total: **40%**)

10. **A Canonical State** (10%)

Determine the canonical state space form of a system described by the following transfer function:

$$H(s) = \frac{2s + 10}{s^3 + 8s^2 + 19s + 12}$$
$$= \left(\frac{2}{s+1}\right)\left(\frac{s+5}{s+3}\right)\left(\frac{1}{s+4}\right) = \left(\frac{1\frac{1}{3}}{s+1}\right)\left(\frac{2}{s+3}\right)\left(\frac{\frac{2}{3}}{s+4}\right)$$

a) What are the components of the state vector (typically)?
b) What are dimensions of the state matrices (typically **A**, **B**, **C**, **D**)?
c) What are the values of the state matrices
(i.e., the matrices dimensioned in part (b))?

[Practice Exam Hints:

- Review Section 10.2 (A Systematic Procedure for Determining State Equations)
- See also Lathi Example10.4, pp. 904-911]

11. **Burn-Out** (15%)

"An electric car should burn-out tyres not battery packs" says Uncle Robert. Employing his electrifying and harmonious talents, he has designed a water-cooled battery pack that can store 85 kWhr of energy for an electric car, the Nikola.

However, it needs a controller. Prof. F. Pe, an old engineering friend, suggests the following controller with k_1 as a tuning factor.

$$D(s) = \frac{2s + 1}{s + k_1}$$

You would like to digitize this controller for better performance and to make it less sensitive to tuning values.

a) Using the Euler controller emulation approach, what is the resulting transfer function D(z) for a yet to be determined sampling time of T_s ?

b) For what range of tuning values for the factor k_1 will the discrete-time controller, D(z) (as found in part (a) using the Euler controller emulation approach) be asymptotically stable?

c) Will this same range of k_1 values be stable for the continuous controller D(s)? If not, what range of k_1 values will be stable for both the continuous D(s) and the discrete D(z)?

12. **A Smart Response** (15%)

Dr. Tjohi, a rather caring, but overworked, engineer, decides to join Uncle Robert and wants to design a smart motor controller for the Nikola car.

Consider an electric motor with shaft angular velocity ω. The motor is controlled by input voltage u(t). The equations of motion for the system are:

$$\tau = I\frac{d\omega}{dt}$$
$$\tau = \lambda i$$
$$u = Ri + \lambda\omega + L\frac{di}{dt}$$

where **I=0.01 Nm/rad/s^2** is the rotational inertia of the rotor, **τ** is the output torque of the motor in Nm, **λ=0.01 V/rad/s** is the flux-linkage coefficient of the motor, **R = 0.02 Ω** is the motor internal resistance and **L=15 mH** (unless otherwise stated) is the inductance of the motor windings.

a) Derive a discrete transfer function from **u(t)** to **ω** for this plant, using the matched pole-zero method.
b) If the inductance is equal to 15 mH, what is the slowest sampling rate that will not destabilise the system under unity-gain proportional negative feedback?
c) Under what conditions can the inductance be ignored, and the motor treated as a single pole system?

END OF EXAMINATION — Thank you ☺

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t}\tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1-z^{-1})}{\Delta t(1+z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi}\int_0^{\pi/\Delta t} H_d(\omega)\cos(n\omega\Delta t)\,d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	***Periodic***	***Non-periodic***	
Discrete	***Discrete Fourier Transform*** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$	***Discrete-Time Fourier Transform*** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega$	***Periodic***
Continuous	***Complex Fourier Series*** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\,dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	***Fourier Transform*** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	***Non-periodic***
	Discrete	***Continuous***	***Freq. Domain***

Table 3: Selected Fourier, Laplace and z-transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$\overset{DFT}{\longleftrightarrow}$	$\tilde{X}[k] = \frac{1}{N}$	
$x[n] = \delta[n]$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FS}{\longleftrightarrow}$	$X[k] = \frac{1}{T}$	
$\delta_T[t] = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
$\sin(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } \lvert t\rvert \leqslant T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \frac{1}{\pi t}\sin(\omega_c t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \begin{cases} 1 & \text{when } \lvert\omega\rvert \leqslant \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = 1$	
$x(t) = \delta(t - t_0)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(w) + \frac{1}{jw}$	
$x[n] = \frac{\omega_c}{\pi} \operatorname{sinc} \omega_c n$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } \lvert\omega\rvert < \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s}$	
(unit ramp) $x(t) = t$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s_0}{(s^2 + {s_0}^2)}$	
$x(t) = \cos(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s}{(s^2 + {s_0}^2)}$	
$x(t) = e^{s_0 t}u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s - s_0}$	$\mathfrak{Re}\{s\} > \mathfrak{Re}\{s_0\}$
$x[n] = \delta[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = 1$	all z
$x[n] = \delta[n - m]$	$\overset{z}{\longleftrightarrow}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - 1}$	
$x[n] = z_0^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert > \lvert z_0\rvert$
$x[n] = -z_0^n u[-n - 1]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert < \lvert z_0\rvert$
$x[n] = a^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - a}$	$\lvert z\rvert < \lvert a\rvert$

Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\lvert X(e^{j\omega})\right\rvert^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\frac{d\tilde{x}(t)}{dt}$	$\frac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t-t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k-k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\frac{1}{T}\int_{-T/2}^{T/2} \lvert\tilde{x}(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$	

Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \|x(t)\|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \|X(j\omega)\|^2\, d\omega$	

Table 7: Properties of the z-transform.

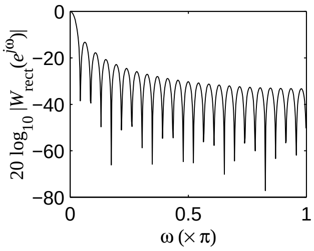
Property	Time domain	z-domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n - n_0]$	$z^{-n_0}X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$\|z_0\|R_x$
Differentiation in z	$nx[n]$	$-z\frac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\mathfrak{Re}\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim_{z\to\infty} X(z)$		

† $z = 0$ or $z = \infty$ may have been added or removed from the ROC.

Table 8: Commonly used window functions.

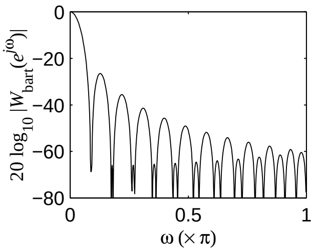
Rectangular:

$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



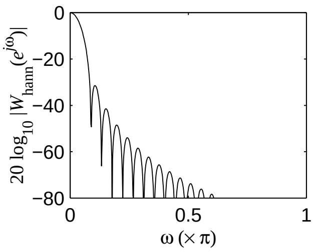
Bartlett (triangular):

$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leqslant n \leqslant M/2, \\ 2 - 2n/M & \text{when } M/2 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



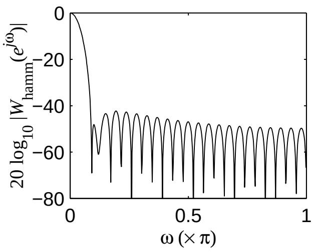
Hanning:

$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



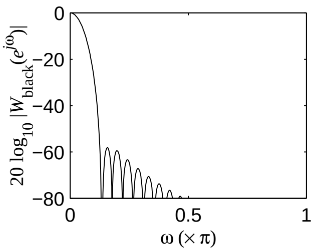
Hamming:

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Blackman:

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) \\ \quad + 0.08\cos(4\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74

END