

Name: Quick Solution

Student ID: _____

Peer Reviewer Name (& Student ID): _____

Final Mark: _____

1. Linearity**[2 points]**

Peer Feedback

Is the system described by the following linear or non-linear?
(Please **briefly** justify your answer)

$$y(t) = \frac{dx(t)}{dt} x(t) + x(t) + 1$$

Non-linear

∵ the first term involves a multiplication of $\dot{x}x$, which is a nonlinear term to see this. Consider

① Additivity $x_1 \rightarrow y_1$ + $x_2 \rightarrow y_2$ then $x_1 + x_2 \rightarrow y_1 + y_2$

$$y_1 = \frac{dx_1}{dt} \cdot x_1, \quad y_2 = \frac{dx_2}{dt} \cdot x_2 \quad \Bigg| \quad y_3 = \frac{d(x_1+x_2)}{dt} \cdot (x_1+x_2)$$

$$y_1 + y_2 = \frac{dx_1}{dt} x_1 + \frac{dx_2}{dt} x_2$$

$$= \left[\frac{dx_1}{dt} + \frac{dx_2}{dt} \right] \cdot [x_1 + x_2]$$

$$= \frac{dx_1}{dt} \cdot x_1 + \frac{dx_2}{dt} x_1 + \frac{dx_1}{dt} x_2 + \frac{dx_2}{dt} x_2$$

2. Convolution

Recall that convolution is given by:

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t - \tau) d\tau$$

What are the following convolutions?

(a) $x(t) * \delta(t)$

$$x(t) \quad (\text{by definition})$$

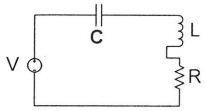
(b) $x(t) * x(t)$

$$t x(t)$$

→ Recall that convolution in the time domain is a multiplication in the Laplace domain. ∴ $\mathcal{L}^{-1}[X(s)^2]$.

3. Second Order Systems

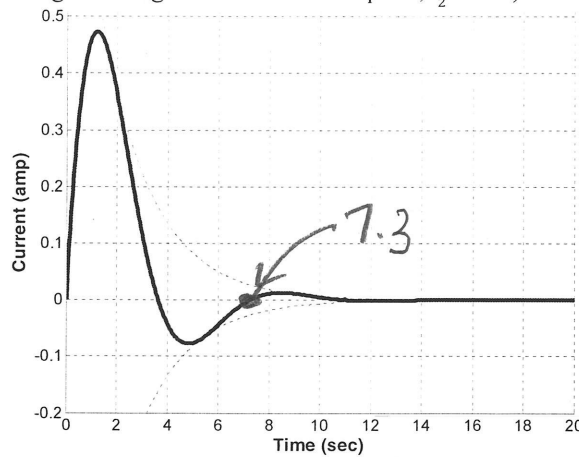
[5 points]



Recall that for a series RLC circuit, the response is second order and that $\omega_0 = \frac{1}{\sqrt{LC}}$, $\alpha = \frac{R}{2L}$ and $\zeta = \frac{\alpha}{\omega_0}$

Determine the C for a circuit with $R=1\Omega$, $L=1H$ and the following response:

(hint: the values might be larger than "common" parts, $\frac{\sqrt{3}}{2}=0.866$)



$$\textcircled{1} T = 7.3 \text{ seconds} \rightarrow f_d = \frac{1}{T} = \frac{1}{7.3} \text{ Hz} \rightarrow \omega_d = 2\pi f_d = \underline{\underline{\frac{2\pi}{7.3}}}$$

$$\begin{aligned} \textcircled{2} \omega_d &= \omega_0 \sqrt{1-\zeta^2} \\ &= \left(\frac{1}{\sqrt{LC}}\right) (1-\zeta^2)^{\frac{1}{2}} \\ &= \frac{1}{\sqrt{LC}} \cdot \left(1 - \frac{\zeta^2}{4}\right)^{\frac{1}{2}} \end{aligned}$$

$$\textcircled{3} \zeta = \frac{R}{2L} \cdot \frac{\sqrt{LC}}{1} = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$= \frac{\sqrt{C}}{2} \quad (\text{using values from above})$$

$$\therefore \zeta^2 = \frac{C}{4}$$

$$\textcircled{4} \left(\frac{2\pi}{7.3}\right) = \left[\frac{1}{C} \left(1 - \frac{C}{4}\right)\right]^{\frac{1}{2}}$$

$$\left(\frac{2\pi}{7.3}\right)^2 = \frac{4-C}{4C}$$

$$\text{Solve for } C \rightarrow \underline{\underline{C \approx 1 \text{ F}}}$$

Note: If we read the period as $\underline{\underline{2\pi}}$

then

$$T = 6.28 \text{ sec} \rightarrow \omega_d = 1$$

$$\Rightarrow \left(\frac{2\pi}{2\pi}\right)^2 = \frac{4-C}{4C}$$

$$\text{solve for } C \rightarrow \underline{\underline{C \approx \frac{4}{5} \text{ F}}}$$