→ SPRING – 3/13/2013 — LINEAR DYNAMICAL SYSTEMS — LECTURE 5 (Via online!)

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(CHE Style)

→ Impulse response Example

$$\underbrace{(D^2 + 3D + 2)}_{\text{2nd order}}\ y(t) = \underline{D} \cdot x(t)$$

$D^2 + 3D + 2$ — characteristic polynomial →

$\lambda = -1, -2$

$D^2 + 3D + 2 \longrightarrow (\lambda+1)(\lambda+2)$

$$\mathcal{L}^{-1} \Rightarrow \underline{y(t) = c_1 e^{-t} + c_2 e^{-2t}}$$

$Dy(0) = 1$

$y(0) = 0$

$\Rightarrow c_1 = 1,\ c_2 = -1$

$$\hookrightarrow \boxed{y(t) = e^{-t} - e^{-2t}} = \underline{h(t)}$$

→ 2nd order Systems

$$a y'' + b y' + c y = 0$$

$\mathcal{L}$:
$$a\left[s^2 Y(s) - s\,y(0) - y'(0)\right]$$

$$+ b\left[s Y(s) - y(0)\right]$$

$$+ c\left[Y(s)\right] = 0$$

$\Rightarrow$ SOLVE FOR $Y(s)$

$$Y(s) = \frac{a y(0) \cdot s + a y'(0) + b y(0)}{a s^2 + b s + c}$$

$$= \frac{\alpha s + \beta}{a s^2 + b s + c}$$

$$= \boxed{\frac{(\alpha/a) s + (\beta/a)}{s^2 + (b/a) s + (c/a)}}$$

$$Y(s) = \frac{\bar{\alpha} s + \bar{\beta}}{s^2 + \frac{B}{A} \cdot s + \frac{C}{A}}$$

$$\omega_n^2 = \frac{C}{A} \qquad 2\zeta\omega_n = \frac{B}{A} \Rightarrow \zeta = \frac{B}{2\sqrt{AC}}$$

$$Y(s) = \frac{\bar{\alpha} s + \bar{\beta}}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \bar{\alpha} = 0, \ \bar{\beta} = \omega_n^2$$

$\boxed{1}$ UNDERDAMPED $\quad 0 < \zeta < 1$

$$\frac{Y(s)}{\text{underdamped}} = \frac{\omega_n^2}{\underbrace{(s + \zeta\omega_n + j\underbrace{(\omega_n\sqrt{1-\zeta^2})}_{\omega_d})}_{\mathrm{I}}\underbrace{(s + \zeta\omega_n - j\omega_d)}_{\mathrm{II}}}$$

NOTE THE SIGN – ITS A SIGNAL!

UNIT IMPULSE = $\underline{\underline{1}}$

STEP INPUT = $\underline{\underline{\frac{1}{s}}}$

$$\mathcal{L}^{-1}[\mathrm{I}] = e^{-\zeta\omega_n t} \cdot \cos(\omega_d t)$$

$$\mathcal{L}^{-1}[\mathrm{II}] = e^{-\zeta\omega_n t} \cdot \sin(\omega_d t)$$

II CRITICALLY DAMPED $\zeta = 1$

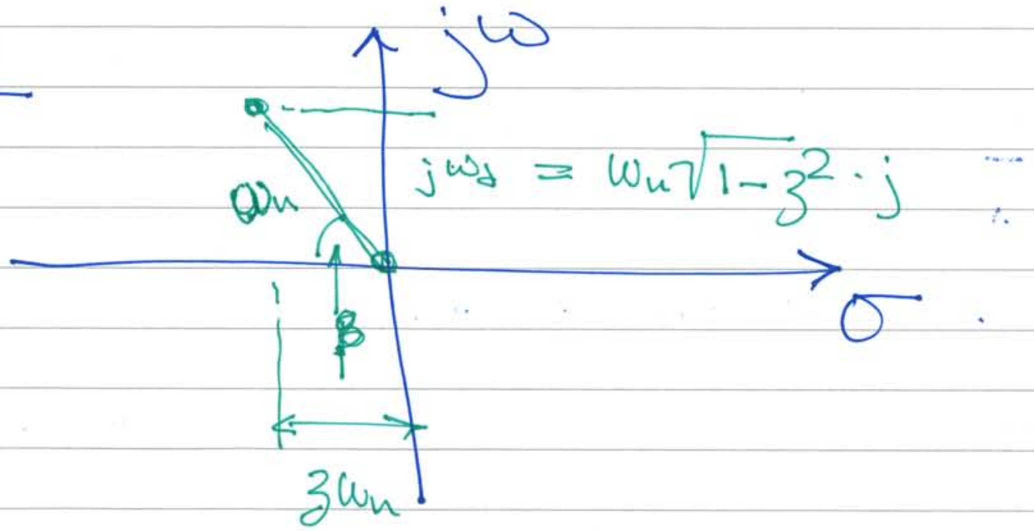
$$\rightarrow C(s) = \frac{1}{s} \quad \rightarrow T(s) = \frac{\omega_n^2}{(s+\omega_n)^2 \cdot s}$$

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$$

III OVERDAMPED CASE $(\zeta > 1)$

$$c(t) = 1 - \exp\left[-\left(\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t\right]$$

S-PLANE



$$s = \sigma + j\omega$$