## — SP&S - 3/6/2013 — Lecture 3

## — Measure of a Signal

### — Signal Size

$$s = \int_0^T u(t)\, dt$$

← $u(t)$ is just some arbitrary "signal"

- might integrate to zero
- ∴ ① Absolute value
  - ② Magnitude ⇒ $\|u(t)\|^2$

### — Signal Energy

$$E = \int_{-\infty}^{\infty} u^2(t)\, dt$$

- But what if $u(t)$ is not bounded — for example a sine wave —
- Then $\infty$ Energy

### — Signal Power

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t)\, dt$$

P: is the time average mean of $(\text{amplitude})^2$

∴ $\sqrt{P}$ = RMS (root mean square) value of the amplitude

— "Power" is not "power" in a typical energy sense (not in Watts)

→ infinite signals can have finite power

— Ex: unit step (which is not periodic or statistically regular) does have finite power

# → Signal Classifications ←

## ① Continuous - Time


(...) for continuous

u(t) is in continuous time: t is of a continuous set

$t \in \mathbb{R}$ (reals)

or any other continuous set such as $t \in$ Imaginary, etc.

## ② Discrete - Time


$u[t_n]$ or $u[n]$ ← [...] for discrete

relationship to continuous time: $u[n] = u(nT_s)$

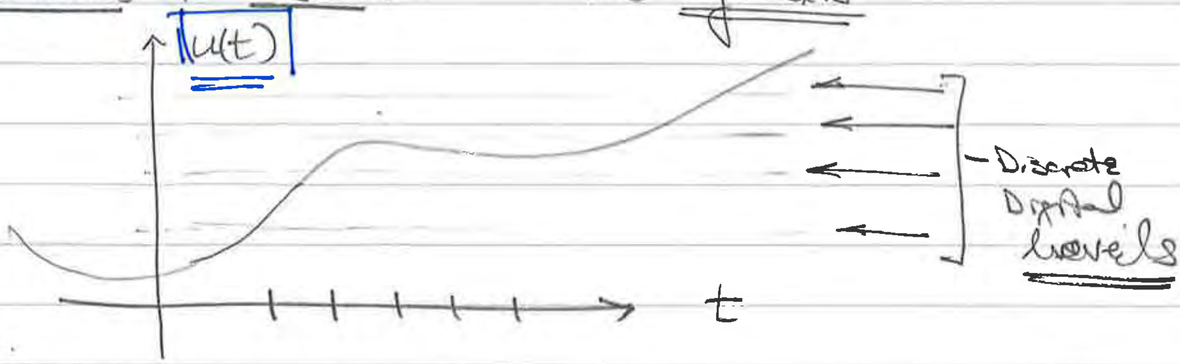
$t \in \mathbb{I}$ (integers)

$t_n = \{t_0, t_1, t_2, \ldots, t_n\}$

## (3) Analog & Digital Signals

Continuous & Discrete on the y-axis

analog




$u(t) \doteq$ Continuous Values $(\in \mathbb{R})$ : Analog

$u(t) =$ Discrete Values : Digital

— Digital is much better for communications "

∴ It is noise robust

→ small changes in value due to noise don't change "the signal"

## (4) Real & Complex Signals

$x(t) = x_1(t) + j x_2(t)$

or $x(t) = x_1(t) + i x_2(t)$

set $x_1(t), x_2(t)$ to constants

$\Rightarrow$ consider $s = x_1 + x_2 j$

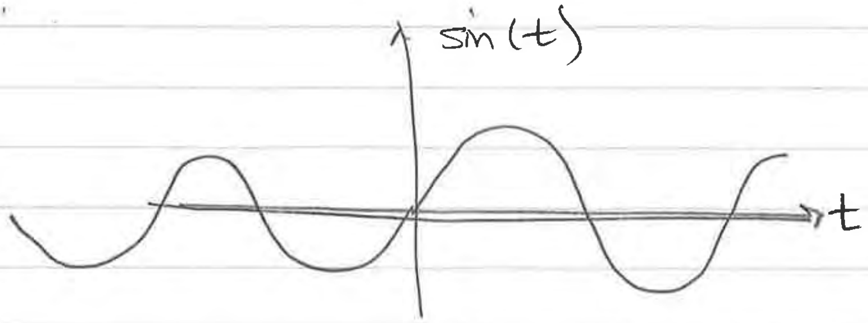
(if $x_1 = \sigma$ & $x_2 = \omega$, then this is the familiar $s = \sigma + j\omega$)

$$\Rightarrow e^{st} = e^{(x_1 + x_2 j)} = e^{x_1 \cdot t} e^{x_2 t j}$$

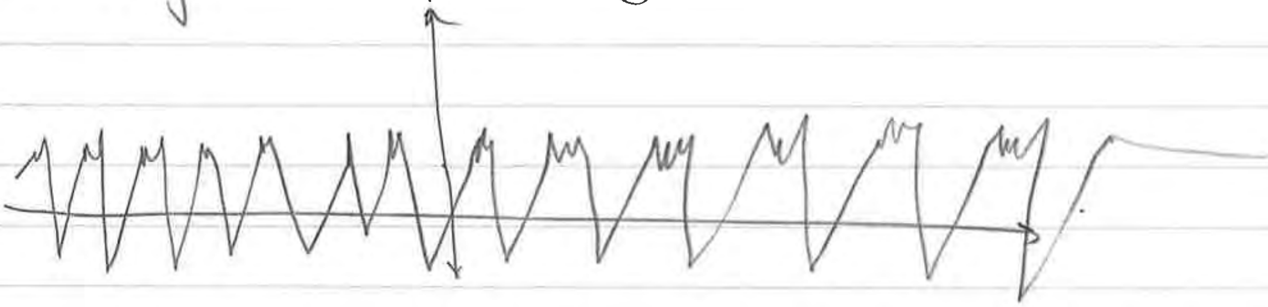
$$= e^{x_1 \cdot t}\left(\cos(x_2 \cdot t) + \sin(x_2 t) j\right)$$

## 5 Deterministic & Random

Deterministic are ones whose description is known completely

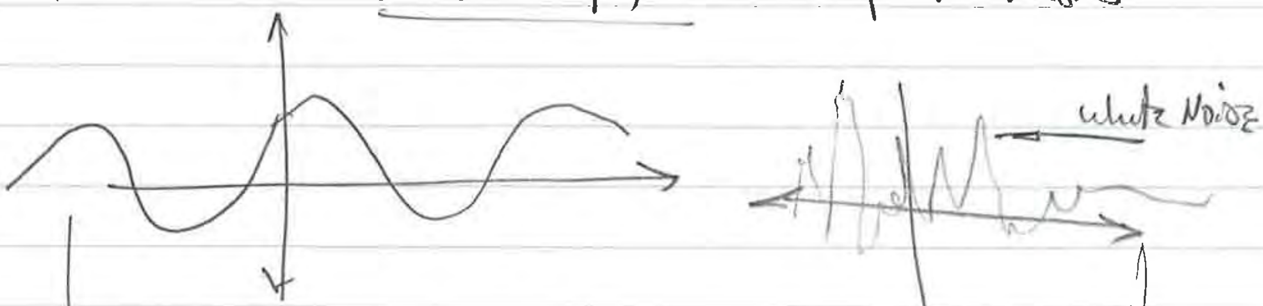



these may be complex (E.g. wavelets)



Random
— Those signals whose description is known incompletely via statistical or probabilistic descriptors

→ Ex: $\sin(\omega t + \phi)$ where $\phi$ is unknown



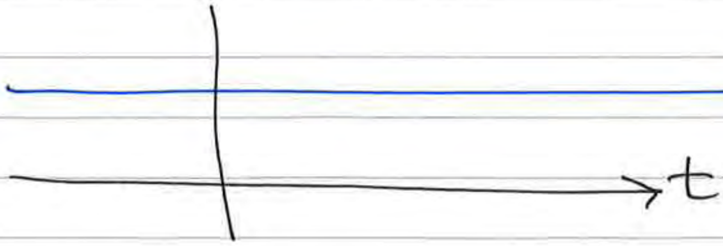

Avg is that of a sine

The mean is zero

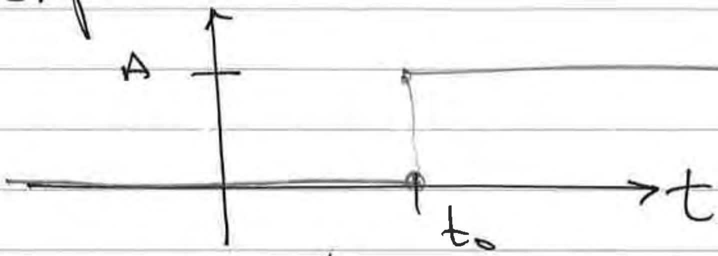
(A zero-meaned, Gaussian of var $\sigma$) → $[N(0, \sigma)]$

# → SIGNAL MODELS ←

(0) Constant



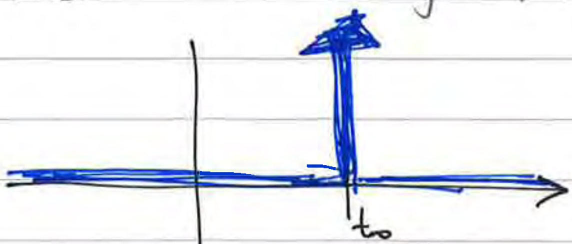

(1) Unit Step




$$u(t-t_0) = \begin{cases} A & t > t_0 \\ 0 & t < t_0 \end{cases}$$

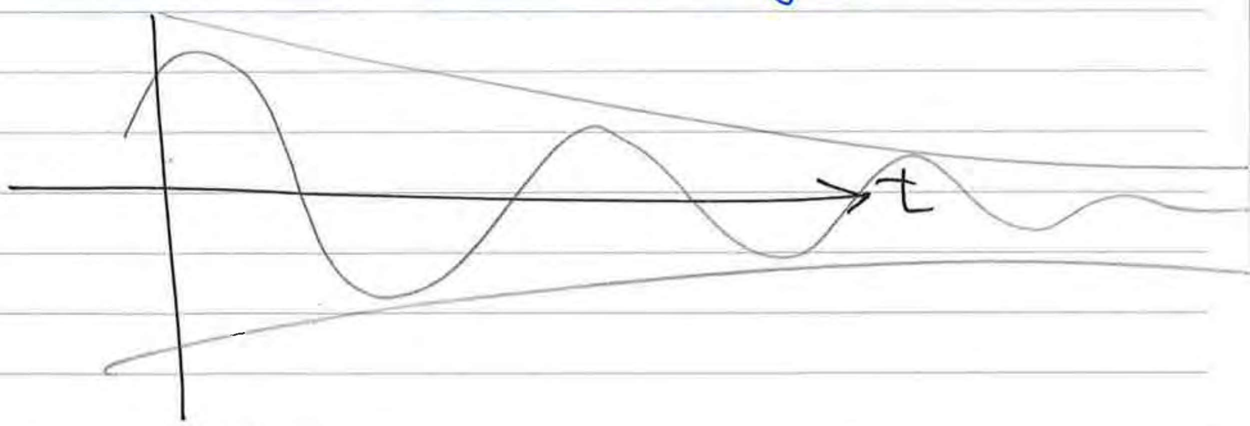
if $A=1 \Rightarrow$ Heaviside function

(2) Unit Impulse
↳ Dirac delta function




$$\delta(t) = \begin{cases} 0 & t \neq t_0 \\ \infty & t = t_0 \end{cases}$$

## (3) Sinusoidal & Exponential decay

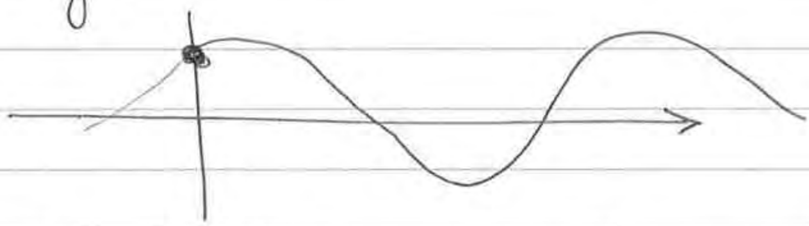



Sinusoidal signal

$$x(t) = A\cos(\omega_0 t + \theta)$$

A = amplitude | $\omega$ = frequency ($\omega = 2\pi f$) | $\theta$ = Phase angle

→ A, $\omega$ : Self-explanatory (mostly)
$\theta$ = Phase angle !



we define Phase relative to y-axis crossing
∴ for t = 0

$$x(0) = A\cos(\theta)$$

$$\therefore \theta = \left[\cos^{-1}\left(A/x(0)\right)\right]$$

Period : $T_0 = \frac{2\pi}{\omega_0}$ (which intuits, Period $\propto \frac{1}{freq}$)

→ Note that the sinusoid is related to the exponential via Euler's formula

if we recall this from the discussion on real and complex signals we get

$$s = \sigma + j\omega$$

$$e^{st} = \exp(\sigma + j\omega)\, t = \exp(\sigma t)\exp(j\omega t)$$
$$= \exp(\sigma t) \cdot [\cos(\omega t) + j \sin(\omega t)]$$

There are four cases for this

(0) $s = 0 \Rightarrow$ A constant (trivial)

(1) $s = \sigma,\ \omega = 0 \Rightarrow$ A monotonic exponential

(2) $s = \pm j\omega,\ \sigma = 0 \Rightarrow$ A sinusoid

(3) $s = \sigma \pm j\omega \Rightarrow$ An exponentially varying sinusoid

# EVEN & ODD FUNCTIONS

## DEFINITION

EVEN FUNCTIONS

$$f_e(t) = f_e(-t)$$

ODD FUNCTIONS

$$f_{ODD}(t) = -f_{ODD}(-t)$$

SO WHAT? (SOME PROPERTIES)

**EVEN**

⇒ Has the same value at the instant $t$ and $-t$

∴ Symmetric about the vertical axis

AREA $\int_{-a}^{a} f_e(t)\,dt = 2\int_{0}^{a} f_e(t)\,dt$

**ODD**

⇒ Its value at instant $t$ is the negative of its value at the instant $-t$

∴ $f_{odd}(t)$ is anti-symmetrical about the vertical axis (symmetrical about the line $y=-x$)

$$\int_{-a}^{a} f_{odd}(t) = 0$$

→ Every signal can be expressed as a sum of even and odd components

$$f(t) = \underbrace{\frac{1}{2}\left[f(t) + f(-t)\right]}_{\text{EVEN}} + \underbrace{\frac{1}{2}\left[f(t) - f(-t)\right]}_{\text{ODD}}$$