Signals as Vectors
Systems as Maps

ELEC 3004: **Systems**: Signals & Controls

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Lecture 9

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March 27, 2013

Announcements:

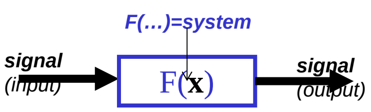
- Assignment 1 due Thursday!
  - As noted in the class email the grade will be the higher of:
  - The median (as prescribed in HW 1)
  - The mean with outliers removed
- Teaching Award Nominations
  http://www.uq.edu.au/teaching-learning/index.html?page=7417
- Extra lab session -- Thursday from 11:00--12:50 pm.
  - Thanks to Nathan and Ian for volunteering to help you!
- There is a pop-quiz today. Surprise! ☺

Today:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response & Fourier Transform
	12-Apr	Analog Filters
7	17-Apr	IIR Systems
	19-Apr	FIR Systems
8	24-Apr	z-Transform
	26-Apr	Discrete-Time Signals
9	1-May	Discrete-Time Systems
	3-May	Digital Filters
10	8-May	State-Space
	10-May	Controllability & Observability
11	15-May	Introduction to Digital Control
	17-May	Stability of Digital Systems
12	22-May	PID & Computer Control
	24-May	Information Theory & Communications
13	29-May	Applications in Industry
	31-May	Summary and Course Review

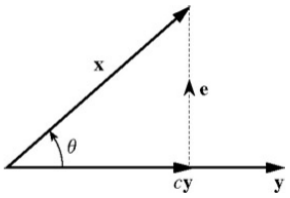
Signals as Vectors

- Back to the beginning!



Vector Refresher

$$\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}| \cos \theta \qquad (6.46)$$



- Length: $|\mathbf{x}|^2 = \mathbf{x} \cdot \mathbf{x}$
- Decomposition: $\mathbf{x} = c_1\mathbf{y} + \mathbf{e}_1 = c_2\mathbf{y} + \mathbf{e}_2$
- Dot Product of ⊥ is 0: $\mathbf{x} \cdot \mathbf{y} = 0$

Signals **Are** Vectors

- A Vector / Signal can represent a sum of its components

  Remember (Lecture 5, Slide 10):
  Total response = Zero-input response + Zero-state response

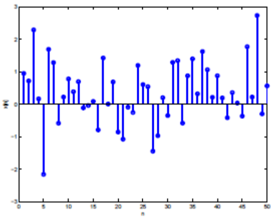
  Initial conditions **External Input**

- Vectors are Linear
  - They have **additivity** and **homogeneity**

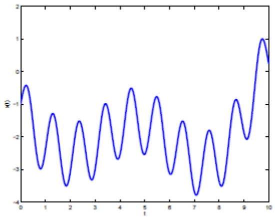
Vectors / Signals Can Be Multidimensional

- A signal is a quantity that varies as a function of an index set
- They can be multidimensional:
  - 1-dim, discrete index (time): x[n]
  - 1-dim, continuous index (time): x(t)
  - 2-dim, discrete (e.g., a B/W or RGB image): x[j; k]
  - 3-dim, video signal (e.g, video): x[j; k; n]

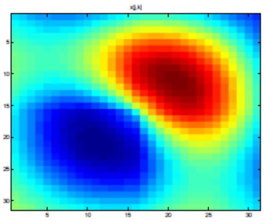
Discrete 1D



Continuous 1D



Discrete 2D

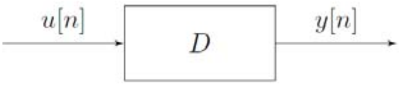


Signals as Vectors

- Represent them as Column Vectors

$$x = \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ \vdots \\ x[N] \end{bmatrix}.$$

Then a System is a **MATRIX**

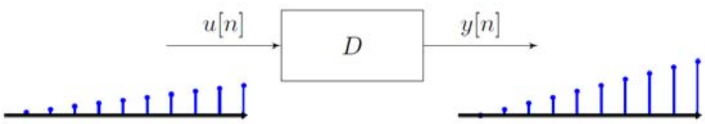


$$y = Du.$$

$$\begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[M] \end{bmatrix} = \begin{bmatrix} D_{11} & D12 & \cdots & D_{1N} \\ D_{21} & D_{22} & \cdots & D_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ D_{M1} & D_{M2} & \cdots & D_{MN} \end{bmatrix} \begin{bmatrix} u[1] \\ u[2] \\ \vdots \\ u[N] \end{bmatrix}.$$

$$y[i] = \sum_j D_{ij} u[j].$$

It’s Just a Linear Map



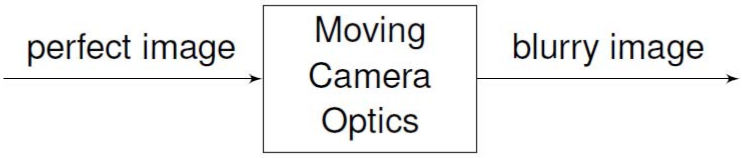
$u_1 = [1, 2, 3, 4, ..., 10]^T$ $y_1 = [0, 2, 4, 6, ..., 18]^T$

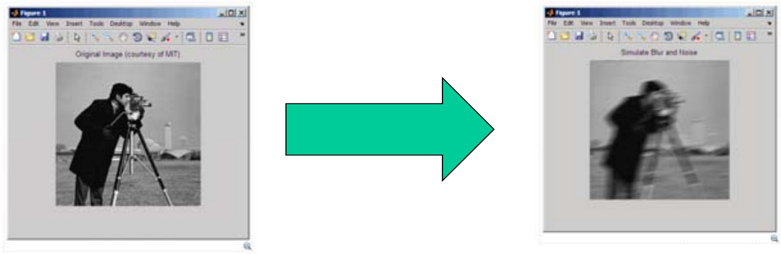
- y[n]=2u[n-1] is a linear map
- BUT y[n]=2(u[n]-1) is **NOT Why?**

- **Because of homogeneity!**

  T(au)=aT(u)

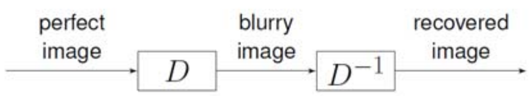
Ex: Deblurring

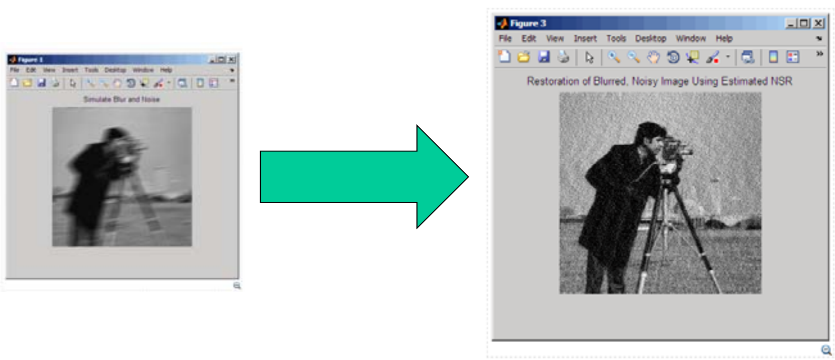




- Matlab: **deconvwnr**

Debluring (II)





Pop Quiz!

Next Time…

- Digital Signals & Filtering

- Review:
  - Chapter 3 of Lathi

- Send (and you shall receive) a positive signal ☺