System Behaviour
& Stability

ELEC 3004: **Systems**: Signals & Controls
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Lecture 8

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Announcements:

- Assignment 1 is up on Platypus!
  - Due at the end of next week!
  - Next week's tutorial sessions are Q&A meetings

Lab 1:

- Extra Session Needed ????

- Space Audit this ~~**Friday**~~ **Today** & next **Wednesday**
  - Sounds like an ideal time for a **pop-quiz**

Today:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response & Fourier Transform
	12-Apr	Analog Filters
7	17-Apr	IIR Systems
	19-Apr	FIR Systems
8	24-Apr	z-Transform
	26-Apr	Discrete-Time Signals
9	1-May	Discrete-Time Systems
	3-May	Digital Filters
10	8-May	State-Space
	10-May	Controllability & Observability
11	15-May	Introduction to Digital Control
	17-May	Stability of Digital Systems
12	22-May	PID & Computer Control
	24-May	Information Theory & Communications
13	29-May	Applications in Industry
	31-May	Summary and Course Review

Recall from Last Time:
Graphical Understanding of Convolution

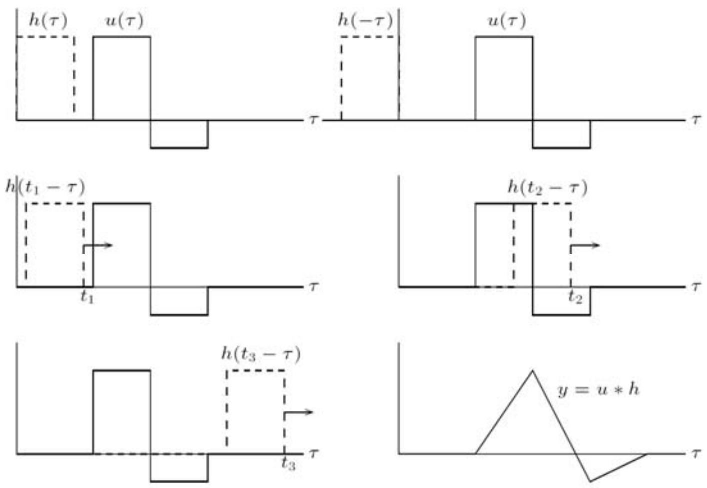
➔ For c(τ)= $(f*g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau$:

1. Keep the function $f(\tau)$ fixed
2. **Flip** (invert) the function $g(\tau)$ about the vertical axis (τ=0)
   = this is $g(-\tau)$
3. **Shift** this frame ($g(-\tau)$) along τ (horizontal axis) by t_0.
   = this is $g(t_0 - \tau)$

➔ For c(t_0):

4. $c(t_0)$ = the area under the product of $f(\tau)$ **and** $g(t_0 - \tau)$

5. Repeat this procedure, shifting the frame by different values (positive and negative) to obtain **c(t) for all values of t**.



Some Properties of Convolution Systems:

1. convolution systems are **linear**: for all signals u_1, u_2 and all α, $\beta \in \mathbf{R}$,

$$h * (\alpha u_1 + \beta u_2) = \alpha(h * u_1) + \beta(h * u_2)$$

2. convolution systems are **causal**: the output $y(t)$ at time t depends only on past inputs $u(\tau)$, $0 \le \tau \le t$

3. convolution systems are **time-invariant**: if we shift the input signal u over $T > 0$, *i.e.*, apply the input

$$\tilde{u}(t) = \begin{cases} 0 & t < T \\ u(t-T) & t \ge 0 \end{cases}$$

to the system, the output is

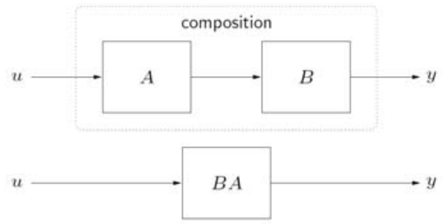
$$\tilde{y}(t) = \begin{cases} 0 & t < T \\ y(t-T) & t \ge 0 \end{cases}$$

in other words: convolution systems commute with delay

Some Properties of Convolution Systems:

4. **composition** of convolution systems corresponds to

- multiplication of transfer functions
- convolution of impulse responses



ramifications:

- can manipulate block diagrams with transfer functions as if they were simple gains
- convolution systems commute with each other

Interpretation of Convolution

$$y(t) = \int_0^t h(\tau)u(t-\tau)\,d\tau$$

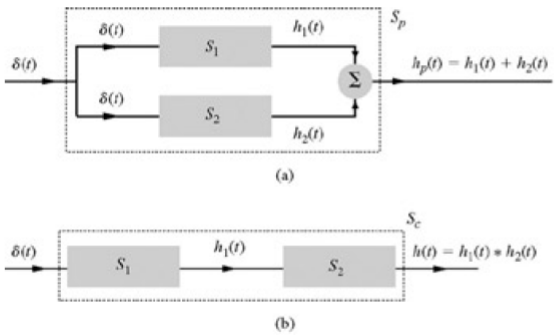
- $y(t)$ is current output; $u(t-\tau)$ is what the input was τ seconds ago
- $h(\tau)$ shows how much current output depends on what input was τ seconds ago

for example,

- $h(21)$ big means current output depends quite a bit on what input was, 21sec ago
- if $h(\tau)$ is small for $\tau > 3$, then $y(t)$ depends mostly on what the input has been over the last 3 seconds
- $h(\tau) \to 0$ as $\tau \to \infty$ means $y(t)$ depends less and less on remote past input

- LTI + causal = LTI Causal System = "Convolution System"
- Any Convolution System is LTI and causal
- The converse is also true: Any LTI causal system can be represented by a convolution system

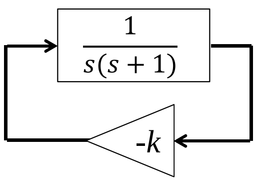
Interconnected Systems

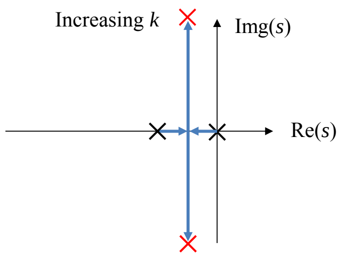


- For more see also Lathi Fig. 2.15

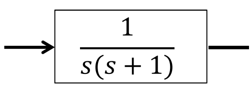
Recall **Root Locus: Pole-Zero Diagram**

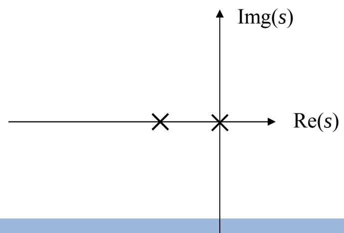
- Feedback Root Locus:





- Open Loop Root Locus: Pole Zero Diagram





Stable & Unstable Systems

- Systems can also be classified as **stable** or **unstable** systems
- Stability can be **internal** or **external:**
  - For external cases: If every bounded input applied at the input terminal results in a bounded output
  - The external stability can be ascertained by measurements at the external terminals (input and output) of the system.

➔ Stability in the BIBO (bounded-input/bounded-output) sense.

BIBO (bounded-input/bounded-output)

- For an LTI System:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

Therefore

$$|y(t)| \le \int_{-\infty}^{\infty} |h(\tau)||x(t-\tau)|\,d\tau$$

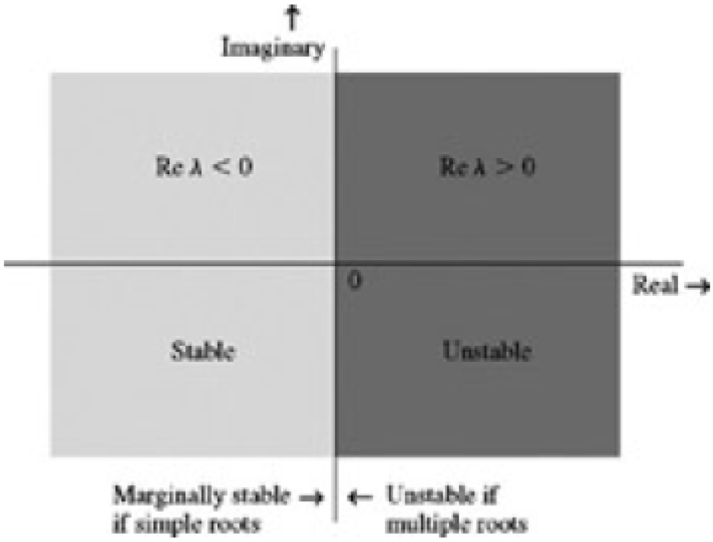
- If x(t) is bounded:

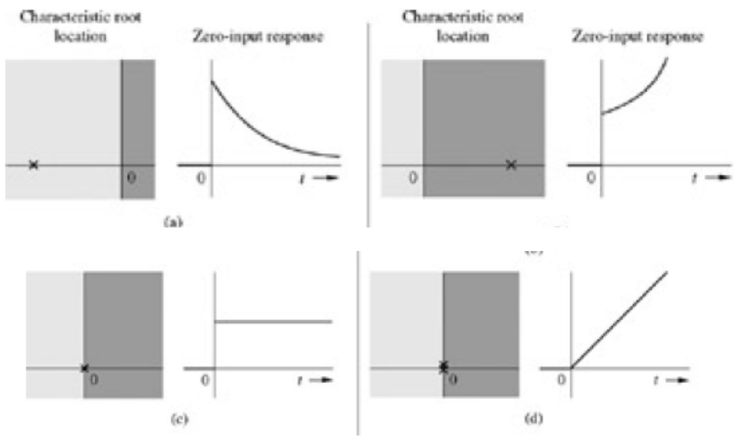
then $|(x-\tau)t < K_1 < \infty$, and $|y(t)| \le K_1 \int_{-\infty}^{\infty} |h(\tau)|\,d\tau$

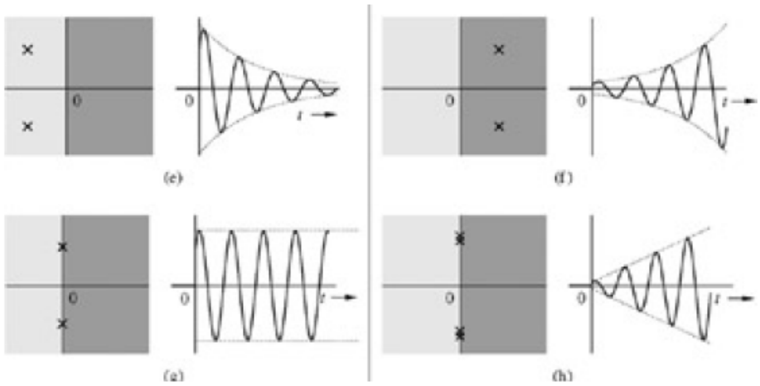
- Hence:

$$\int_{-\infty}^{\infty} |h(\tau)|\,d\tau < \infty$$

BIBO


Imaginary
Re λ < 0
Re λ > 0
0
Real →
Stable
Unstable
Marginally stable → if simple roots
← Unstable if multiple roots


Characteristic root location
Zero-input response
Characteristic root location
Zero-input response
(a)
(c)
(d)


0
0
t
(e)
0
0
t
(f)
0
0
t
(g)
0
0
t
(h)