Sampling & Data Acquisition

ELEC 3004: **Systems**: Signals & Controls
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Lecture 6

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Announcements:

- Assignment 1 is Posted:
  - While some questions are "shocking"
  - Most are designed to be fair – not trying to be tricky

➔For next Lecture (#7):
  - Review Fourier Transform materials
  - Please review **Convolution**

- ELEC 7213 Students:
  - You have 2 additional questions
  - Submission instructions for Q9-Q10 via email.
- Space Audit on **March 22** (F) and **March 28** (W)
  - Sounds like an ideal time for a **pop-quiz**
  - Bring a friend or two (just saying ☺)

!

Cool Signal Share

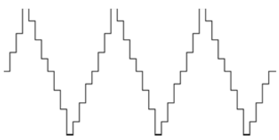
Today:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response & Fourier Transform
	12-Apr	Analog Filters
7	17-Apr	IIR Systems
	19-Apr	FIR Systems
8	24-Apr	z-Transform
	26-Apr	Discrete-Time Signals
9	1-May	Discrete-Time Systems
	3-May	Digital Filters
10	8-May	State-Space
	10-May	Controllability & Observability
11	15-May	Introduction to Digital Control
	17-May	Stability of Digital Systems
12	22-May	PID & Computer Control
	24-May	Information Theory & Communications
13	29-May	Applications in Industry
	31-May	Summary and Course Review

Analog vs Digital

- Analog Signal: An analog or analogue signal is any variable signal continuous in both time and amplitude

- Digital Signal: A digital signal is a signal that is both discrete and quantized



E.g. Music stored in a CD: 44,100 Samples per second and 16 bits to represent amplitude

Digital Signal

- Representation of a signal against a discrete set
- The set is fixed in by computing hardware

$$s \in \mathbb{Z}$$

- Can be scaled or normalized … but is limited

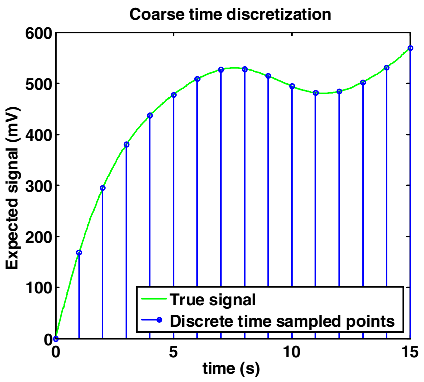
$$s \in \mathbb{Z}(0, \ldots, 2^{16})$$

- Time is also discretized

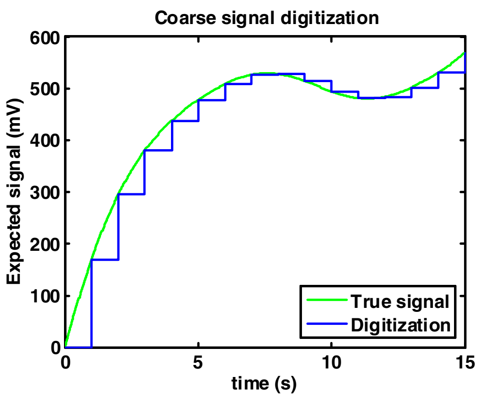
$$s' \in \frac{\mathbb{Z}(0,\ldots,2^{16})}{2^{16}}$$

Representation of Signal

- Time Discretization



- Digitization



Signal: A carrier of (desired) information [1]

- Need **NOT** be electrical:
  - Thermometer
  - Clock hands
  - Automobile speedometer
- Need **NOT** always being given
  - "Abnormal" sounds/operations
  - Ex: "pitch" or "engine hum" during machining as an indicator for feeds and speeds

Signal: A carrier of (desired) information [2]

- Electrical signals
  - Voltage
  - Current
- **Digital signals**
  - **Convert analog electrical signals to an appropriate digital electrical message**
  - **Processing by a microcontroller or microprocessor**

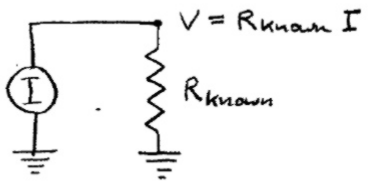
Transduction (sensor to an electrical signal)

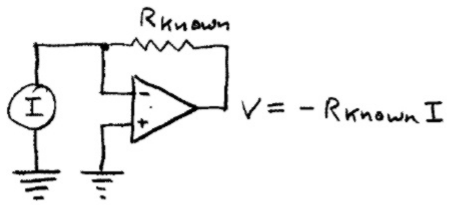
- Sensor reacts to environment (physics)
- Turn this into an electrical signal:
  - V: voltage source
  - I: current source
- Measure this signal
  - Resistance
  - Capacitance
  - Inductance

Ex: Current-to-voltage conversion

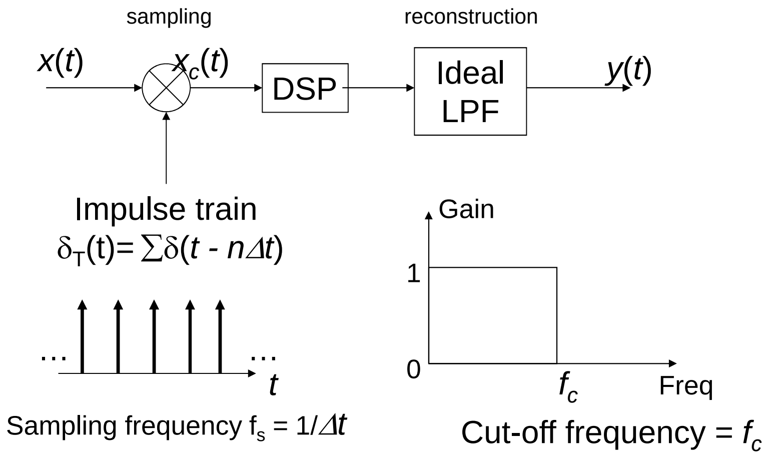
- simple:
  Precision Resistor
- better:
  Use an “op amp”

$$i = \frac{V_{\text{measured}}}{R_{\text{known}}}$$





Mathematics of Sampling and Reconstruction

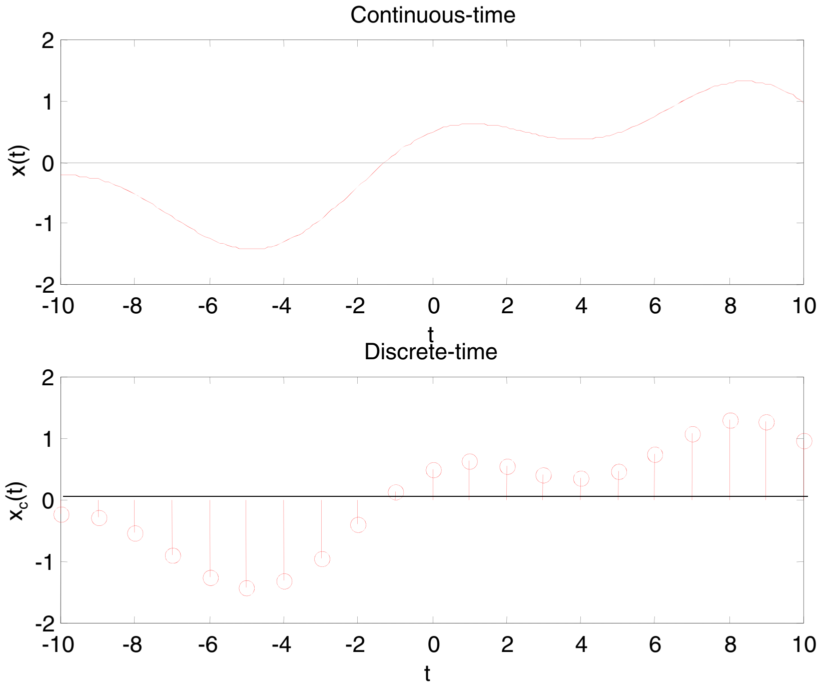


Mathematical Model of Sampling

- x(t) multiplied by impulse train δT(t)

$$
\begin{aligned}
x_c(t) &= x(t)\delta_T(t) \\
&= x(t)\left[\delta(t) + \delta(t-\Delta t) + \delta(t-2\Delta t) + \cdots\right] \\
&= \sum_n x(n\Delta t)\delta(t-n\Delta t)
\end{aligned}
$$

- $x_c(t)$ is a train of impulses of height $x(t)|_{t=n\Delta t}$



Frequency Domain Analysis of Sampling

- Consider the case where the DSP performs no filtering operations
  - i.e., only passes $x_c(t)$ to the reconstruction filter
- To understand we need to look at the frequency domain
- Sampling: we know
  - multiplication in time ≡ convolution in frequency
  - $F\{x(t)\} = X(w)$
  - $F\{\delta_T(t)\} = \sum \delta(w - 2\pi n/\Delta t)$,
  - i.e., an impulse train in the frequency domain

Frequency Domain Analysis of Sampling

- In the frequency domain we have

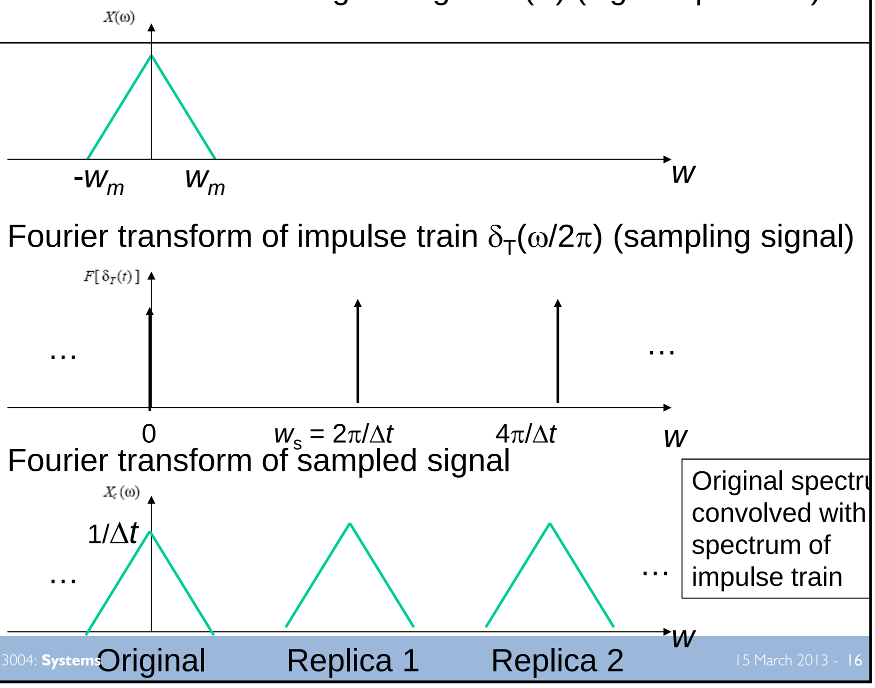
$$X_c(w) = \frac{1}{2\pi}\left(X(w) * \frac{2\pi}{\Delta t} \sum_n \delta\left(w - \frac{2\pi n}{\Delta t} \right) \right)$$

$$= \frac{1}{\Delta t} \sum_n X\left(w - \frac{2\pi n}{\Delta t} \right)$$

Remember convolution with an impulse? Same idea for an impulse train

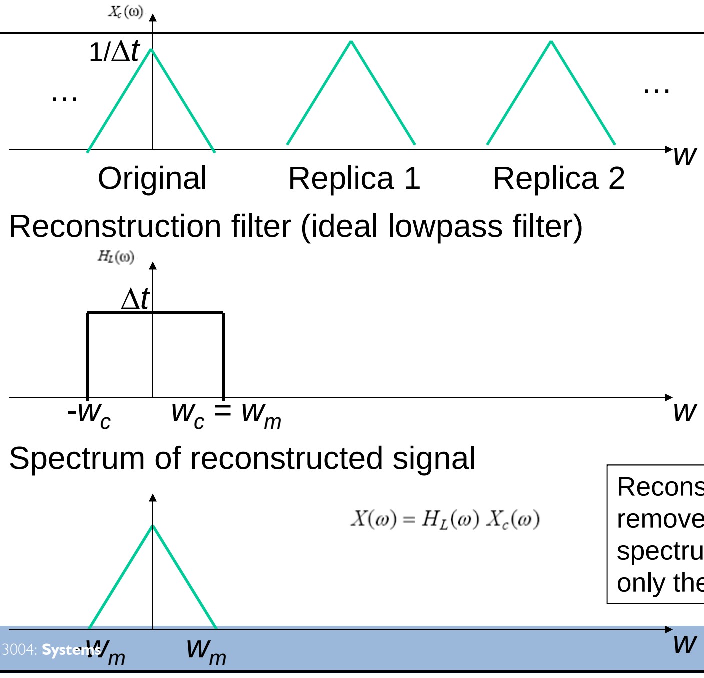
- Let's look at an example
  - where X(w) is triangular function
  - with maximum frequency w_m rad/s
  - being sampled by an impulse train, of frequency w_s rad/s

Fourier transform of original signal X(ω) (signal spectrum)



Original spectrum convolved with spectrum of impulse train

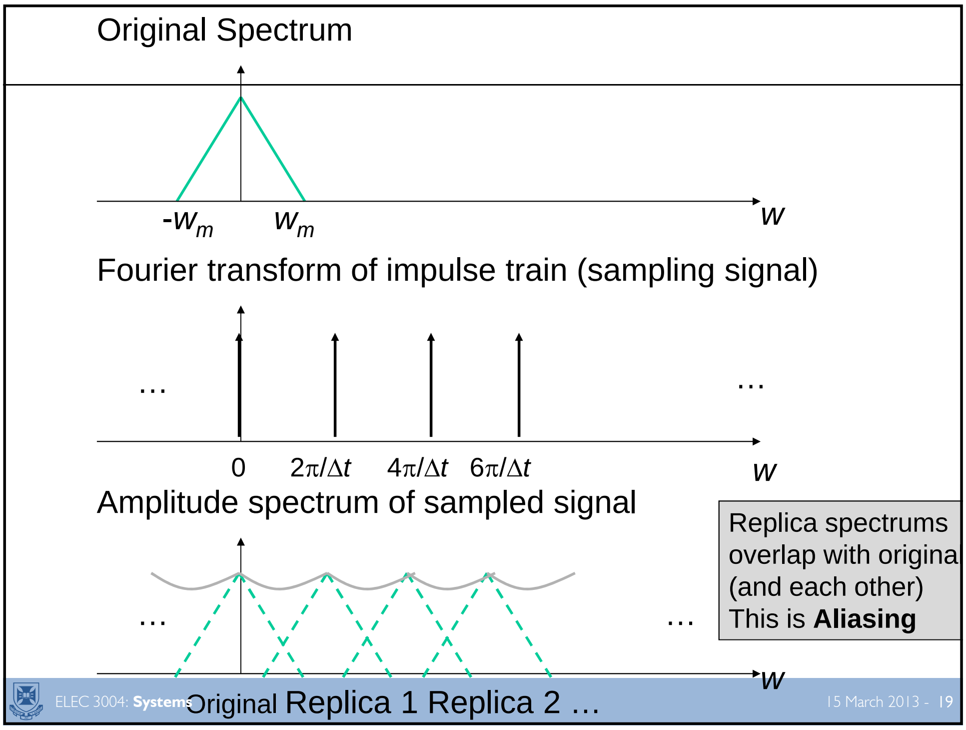
Spectrum of sampled signal



Reconstruction filter removes the replica spectrums & leaves only the original

Sampling Frequency

- In this example it was possible to recover the original signal from the discrete-time samples
- But is this always the case?
- Consider an example where the sampling frequency w_s is reduced
  - i.e., Δt is increased


Original Spectrum
-Wm
Wm
W
Fourier transform of impulse train (sampling signal)
…
…
0
2π/Δt
4π/Δt
6π/Δt
W
Amplitude spectrum of sampled signal
Replica spectrums overlap with original (and each other)
This is Aliasing
…
…
W
Original Replica 1 Replica 2 …
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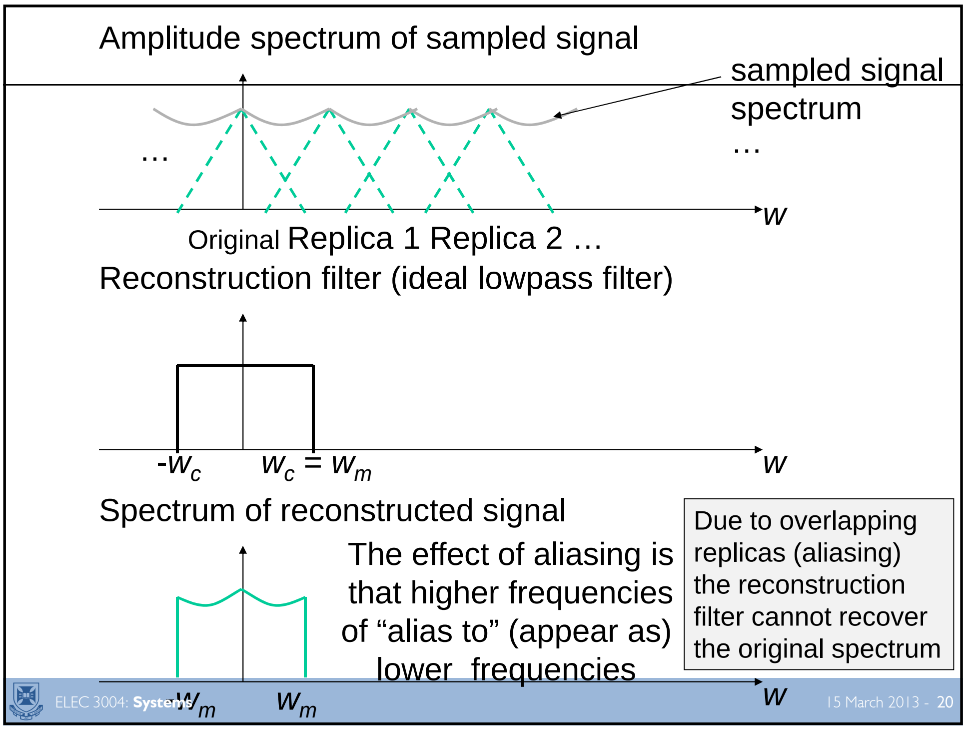
Amplitude spectrum of sampled signal
sampled signal spectrum
…
…
W
Original Replica 1 Replica 2 …
Reconstruction filter (ideal lowpass filter)
-Wc
Wc = Wm
W
Spectrum of reconstructed signal
The effect of aliasing is that higher frequencies of "alias to" (appear as) lower frequencies
Due to overlapping replicas (aliasing) the reconstruction filter cannot recover the original spectrum
-Wm
Wm
W
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Sampling Theorem

- The Nyquist criterion states:

To prevent aliasing, a ***bandlimited*** *signal of bandwidth w_B rad/s must be sampled at a rate greater than $2w_B$ rad/s*

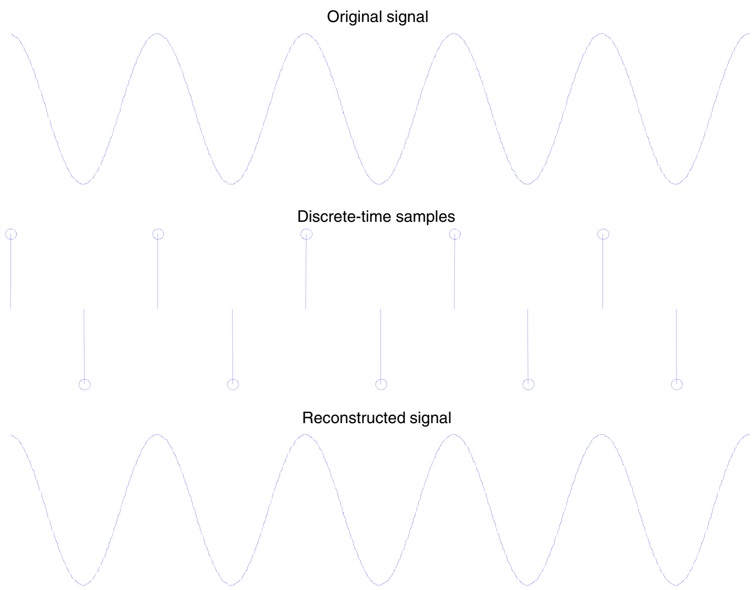
$$-w_s > 2w_B$$

Note: this is a > sign not a ≥

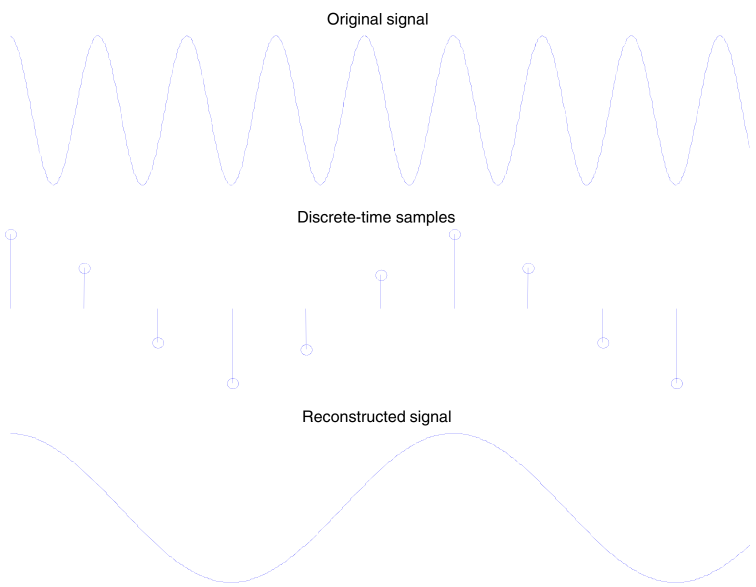
Also note: Most real world signals require band-limiting with a lowpass (anti-aliasing) filter

Time Domain Analysis of Sampling

- Frequency domain analysis of sampling is very useful to understand
  - sampling ($X(w)*\sum \delta(w - 2\pi n/\Delta t)$)
  - reconstruction (lowpass filter removes replicas)
  - aliasing (if $w_s \leq 2w_B$)
- Time domain analysis can also illustrate the concepts
  - sampling a sinewave of increasing frequency
  - sampling images of a rotating wheel

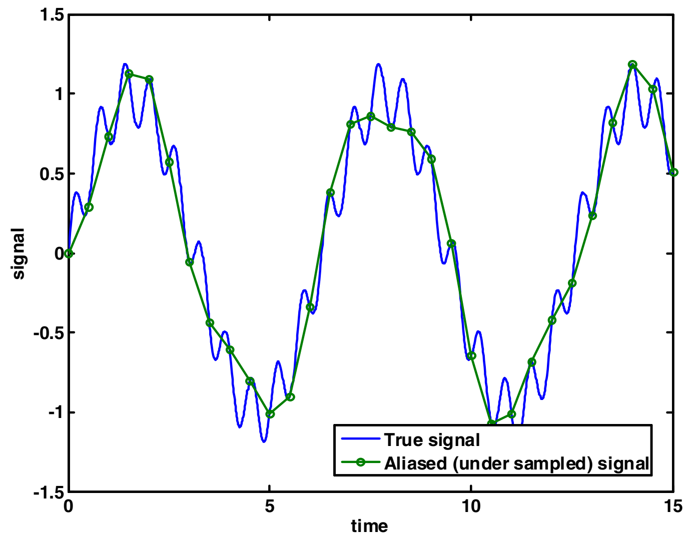


A signal of the original frequency is reconstructed

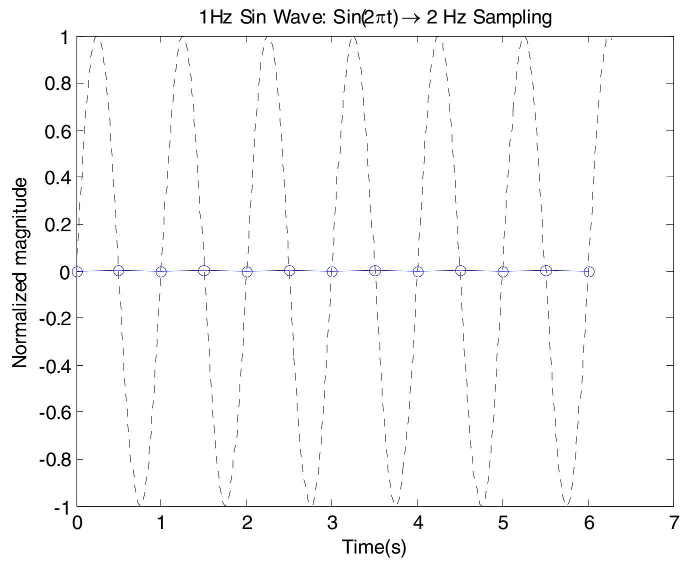


A signal with a reduced frequency is recovered, i.e., the signal is aliased to a lower frequency (we recover a replica)

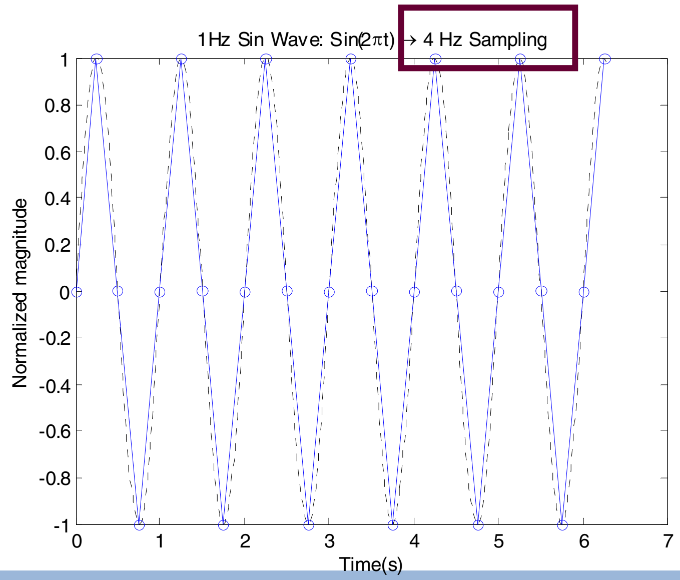
Sampling < Nyquist → Aliasing



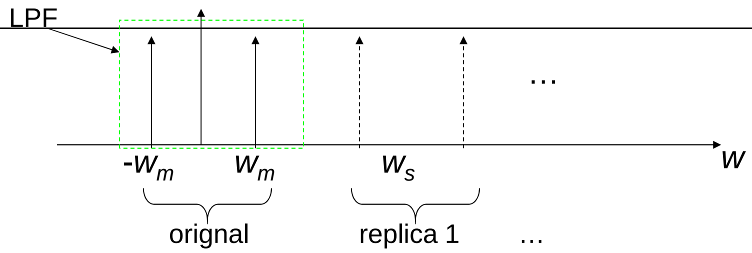
Nyquist is not enough …



A little more than Nyquist is not enough …

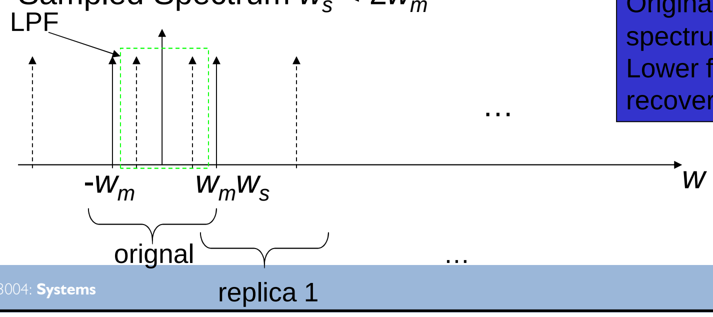


Sampled Spectrum $w_s > 2w_m$



original freq recovered

Sampled Spectrum $w_s < 2w_m$

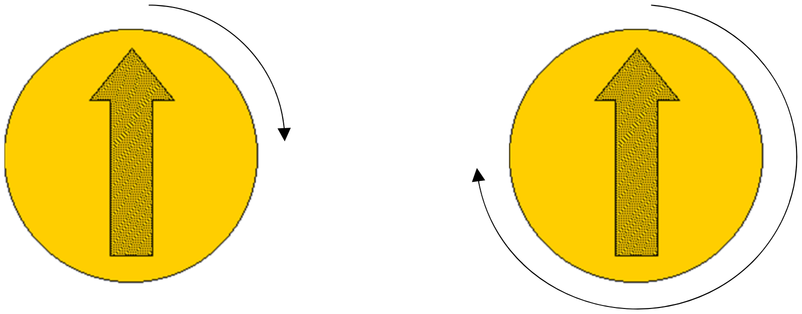


Original and replica spectrums overlap
Lower frequency recovered ($w_s - w_m$)

Temporal Aliasing

90^o clockwise rotation/frame
clockwise rotation perceived

270^o clockwise rotation/frame
(90^o) anticlockwise rotation perceived i.e., aliasing

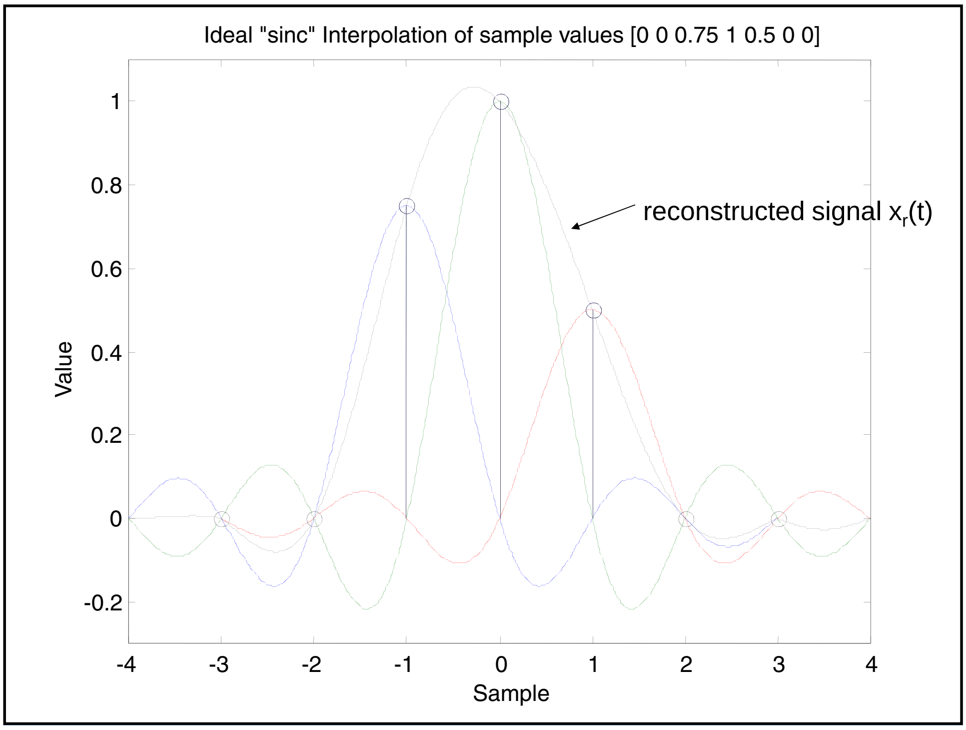


Require LPF to 'blur' motion

Time Domain Analysis of Reconstruction

- Frequency domain: multiply by ideal LPF
  - ideal LPF: 'rect' function (gain Δt, cut off w_c)
  - removes replica spectrums, leaves original
- Time domain: this is equivalent to
  - convolution with 'sinc' function
  - as $F^{-1}\{\Delta t \ \mathrm{rect}(w/w_c)\} = \Delta t \, w_c \, \mathrm{sinc}(w_c t/\pi)$
  - i.e., weighted sinc on every sample
- Normally, $w_c = w_s/2$

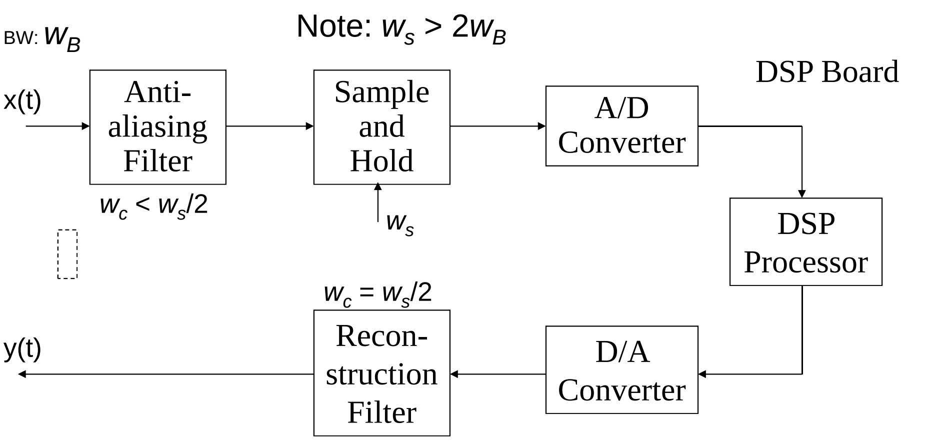
$$x_r(t) = \sum_{n=-\infty}^{\infty} x(n\Delta t)\Delta t w_c \,\mathrm{sinc}\left(\frac{w_c(t - n\Delta t)}{\pi}\right)$$



Sampling and Reconstruction Theory and Practice

- Signal is bandlimited to bandwidth W_B
  - Problem: real signals are not bandlimited
    - Therefore, require (non-ideal) anti-aliasing filter
- Signal multiplied by ideal impulse train
  - problems: sample pulses have finite width
  - and not ⊗ in practice, but sample & hold circuit
- Samples discrete-time, continuous valued
  - Problem: require discrete values for DSP
    - Therefore, require A/D converter (quantisation)
- Ideal lowpass reconstruction ('sinc' interpolation)
  - problems: ideal lowpass filter not available
    - Therefore, use D/A converter and practical lowpass filter

Practical DSP System



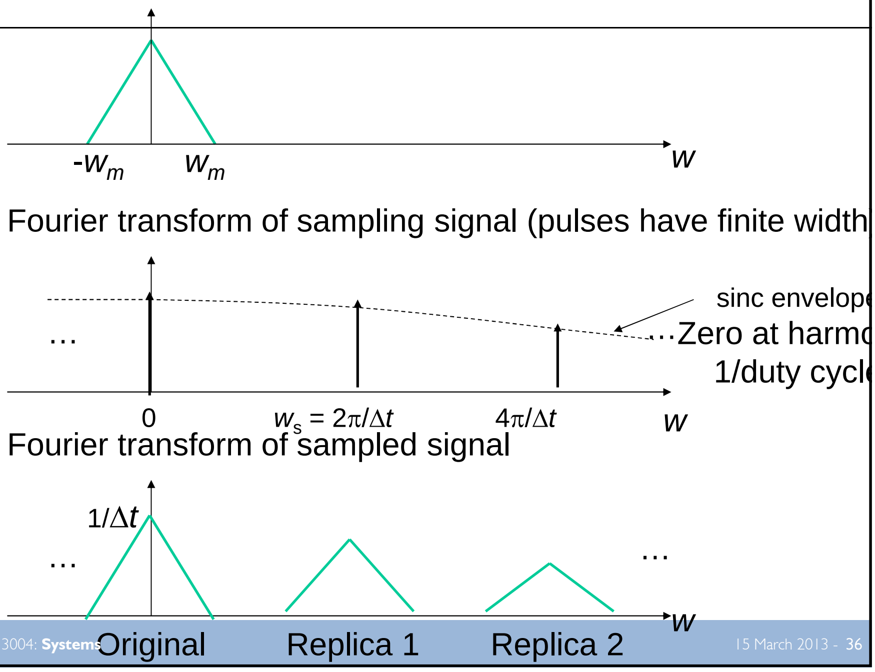
Practical Anti-aliasing Filter

- Non-ideal filter
  - wc = ws /2
- Filter usually 4th – 6th order (e.g., Butterworth)
  - so frequencies > wc may still be present
  - not higher order as phase response gets worse
- Luckily, most real signals
  - are lowpass in nature
    - signal power reduces with increasing frequency
  - e.g., speech naturally bandlimited (say < 8KHz)
  - Natural signals have a (approx) 1/f spectrum
  - so, in practice aliasing is not (usually) a problem

Finite Width Sampling

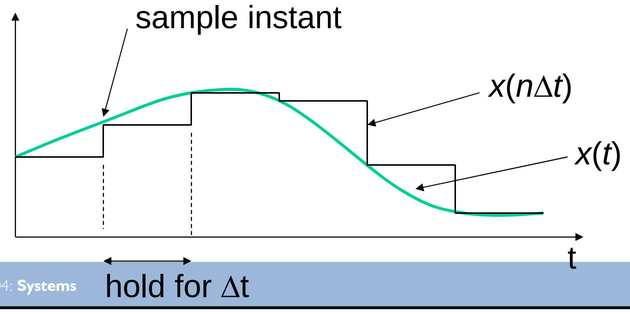
- Impulse train sampling not realisable
  - sample pulses have finite width (say nanosecs)
- This produces two effects,
- Impulse train has sinc envelope in frequency domain
  - impulse train is square wave with small duty cycle
  - Reduces amplitude of replica spectrums
    - smaller replicas to remove with reconstruction filter ☺
- Averaging of signal during sample time
  - effective low pass filter of original signal
    - can reduce aliasing, but can reduce fidelity ☹
    - negligible with most S/H ☺

Amplitude spectrum of original signal



Practical Sampling

- Sample and Hold (S/H)
  1. takes a sample every Δt seconds
  2. holds that value constant until next sample
- Produces 'staircase' waveform, $x(n\Delta t)$



Quantisation

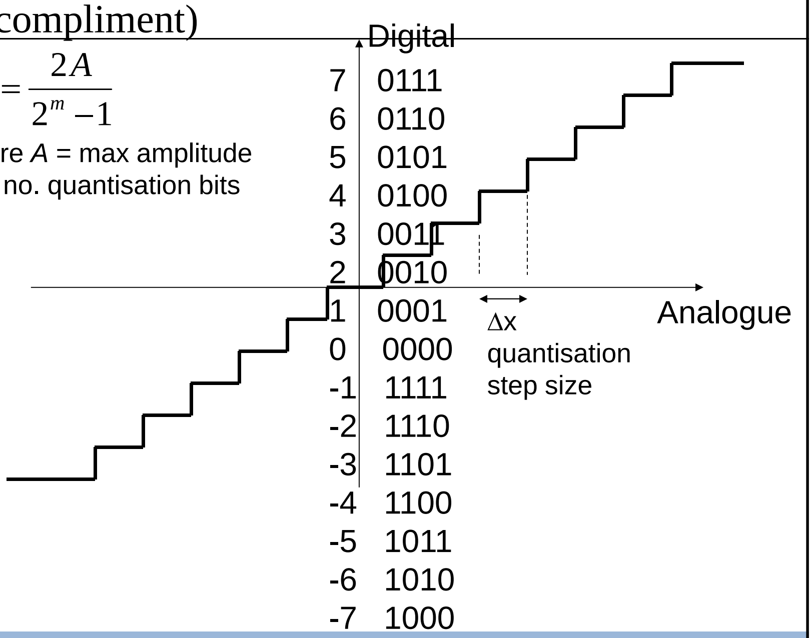
- Analogue to digital converter (A/D)
  - Calculates nearest binary number to $x(n\Delta t)$
    - $x_q[n] = q(x(n\Delta t))$, where q() is non-linear rounding fctn
  - output modeled as $x_q[n] = x(n\Delta t) + e[n]$
- Approximation process
  - therefore, loss of information (unrecoverable)
  - known as 'quantisation noise' ($e[n]$)
  - error reduced as number of bits in A/D increased
    - i.e., Δx, quantisation step size reduces

$$|e[n]| \leq \frac{\Delta x}{2}$$

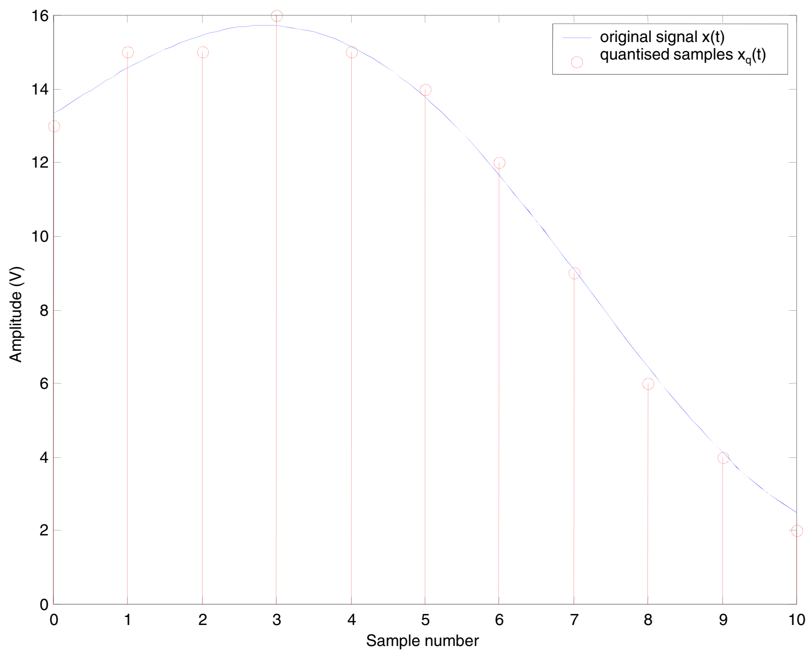
Input-output for 4-bit quantiser (two's compliment)

$$\Delta x = \frac{2A}{2^m - 1}$$

where A = max amplitude
m = no. quantisation bits



Example: error due to signal quantisation



Signal to Quantisation Noise

- To estimate SQNR we assume
  - *e*[*n*] is uncorrelated to signal and is a
  - uniform random process
- assumptions not always correct!
  - not the only assumptions we could make…
- Also known a 'Dynamic range' (R_D)
  - expressed in decibels (dB)
  - ratio of power of largest signal to smallest (noise)

$$R_D = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

Dynamic Range

1 extra bit halves Δ:

~~i.e., 20log10(1/2) =~~

Need to estimate:

1. Noise power
   - uniform random process: $P_{noise} = \Delta x^2/12$
2. Signal power
   - (at least) two possible assumptions
   1. sinusoidal: $P_{signal} = A^2/2$
   2. zero mean Gaussian process: $P_{signal} = \sigma^2$
      - Note: as $\sigma \approx A/3$: $P_{signal} \approx A^2/9$
      - where σ^2 = variance, A = signal amplitude

Regardless of assumptions: R_D increases by 6dB for every bit that is added to the quantiser

Practical Reconstruction

Two stage process:

1. Digital to analogue converter (D/A)
   - zero order hold filter
   - produces 'staircase' analogue output
2. Reconstruction filter
   - non-ideal filter: $w_c = w_s/2$
   - further reduces replica spectrums
   - usually $4^{th} - 6^{th}$ order e.g., Butterworth
     - for acceptable phase response

D/A Converter

- Analogue output y(t) is
  - convolution of output samples $y(n\Delta t)$ with $h_{ZOH}(t)$

$$y(t) = \sum_n y(n\Delta t) h_{ZOH}(t - n\Delta t)$$

$$h_{ZOH}(t) = \begin{cases} 1, & 0 \le t < \Delta t \\ 0, & \text{otherwise} \end{cases}$$

$$H_{ZOH}(w) = \Delta t \exp\left(\frac{-jw\Delta t}{2}\right) \frac{\sin(w\Delta t/2)}{w\Delta t/2}$$

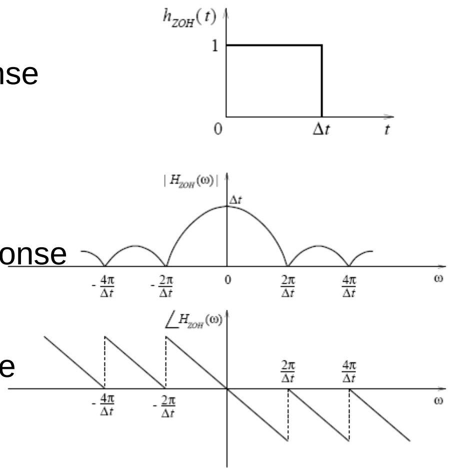
D/A is lowpass filter with sinc type frequency response
It does not completely remove the replica spectrums
Therefore, additional reconstruction filter required

Zero Order Hold (ZOH)

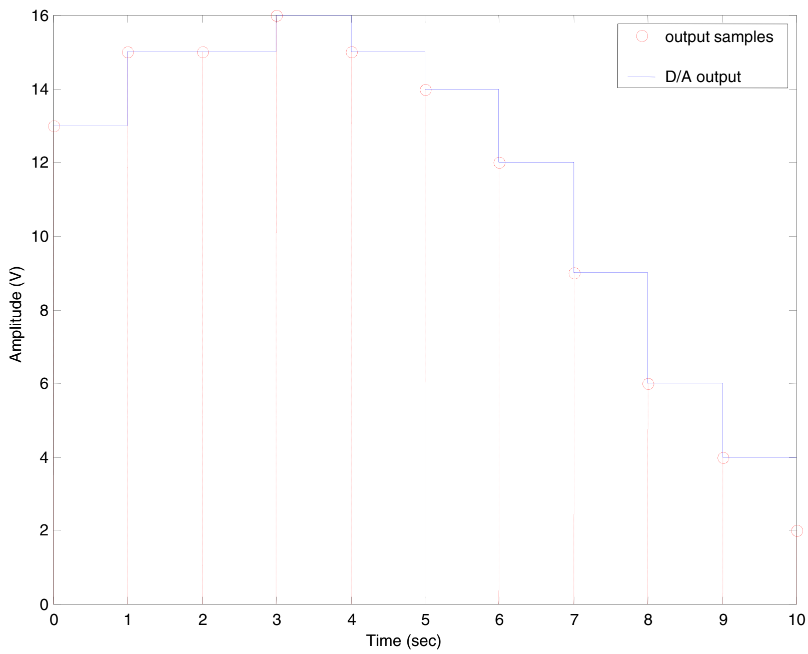
ZOH impulse response

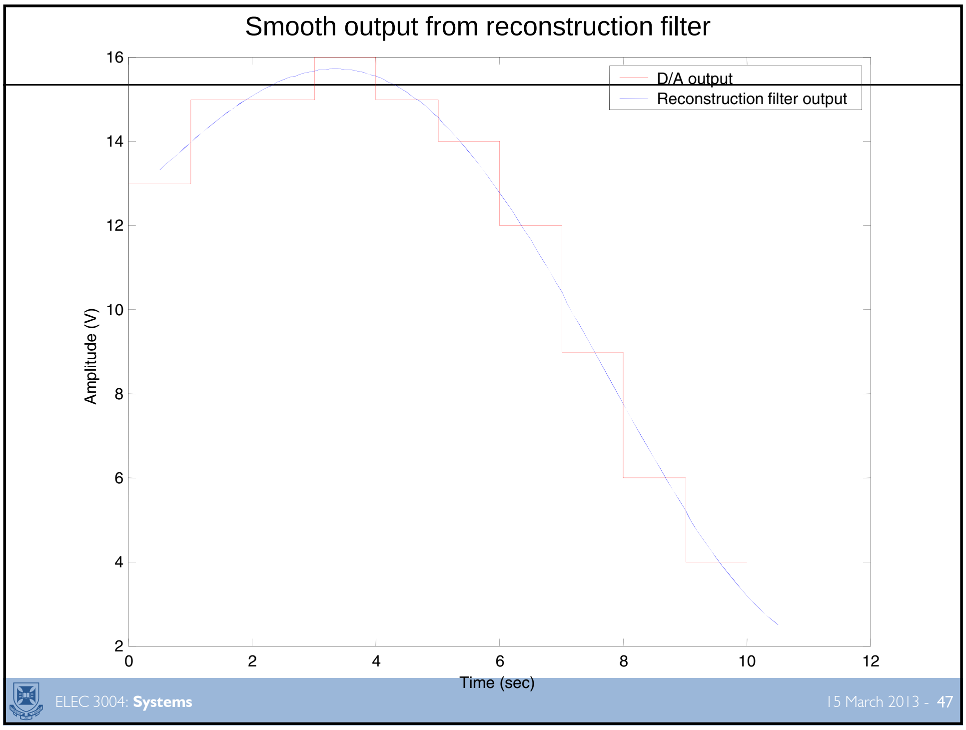
ZOH amplitude response

ZOH phase response



‘staircase’ output from D/A converter (ZOH)




Smooth output from reconstruction filter
D/A output
Reconstruction filter output
Amplitude (V)
Time (sec)
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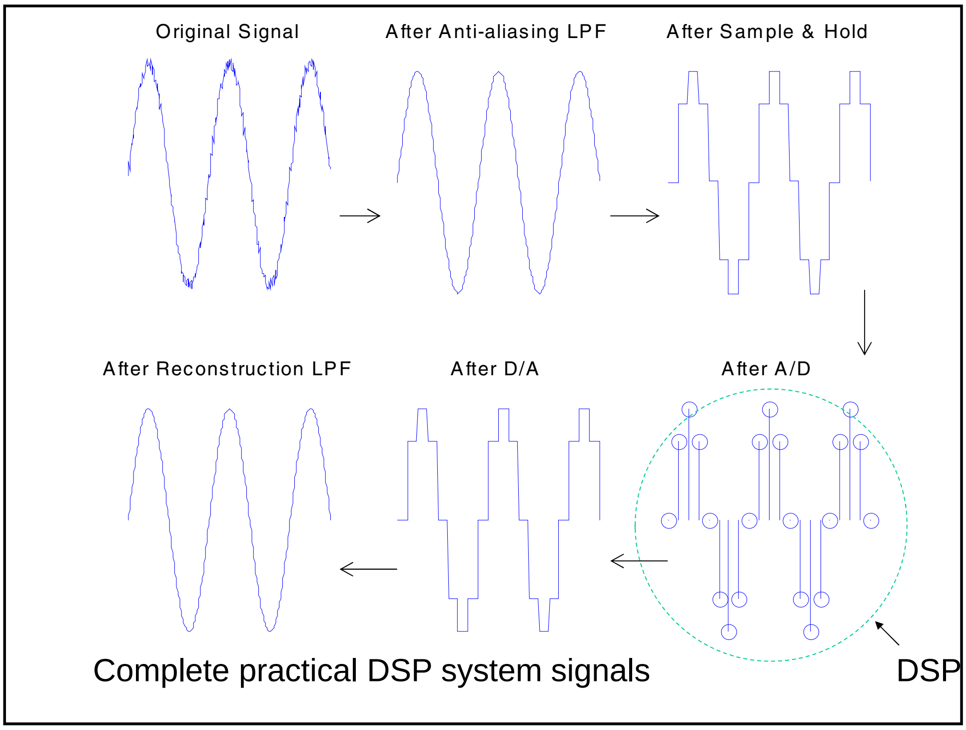
Original Signal
After Anti-aliasing LPF
After Sample & Hold
After Reconstruction LPF
After D/A
After A/D
Complete practical DSP system signals
DSP

Summary

- Theoretical model of Sampling
  - bandlimited signal (wB)
  - multiplication by ideal impulse train (ws > 2wB)
    - convolution of frequency spectrums (creates replicas)
  - Ideal lowpass filter to remove replica spectrums
    - wc = ws /2
    - Sinc interpolation
- Practical systems
  - Anti-aliasing filter (wc < ws /2)
  - A/D (S/H and quantisation)
  - D/A (ZOH)
  - Reconstruction filter (wc = ws /2)

Don't confuse theory and practice!

Next Time…

- Back to Systems Modelling
  - Convolution

- Review:
  - Latter Parts of Chapter 2 of Lathi

- Send (and you shall receive) a positive signal ☺