System **Models**

ELEC 3004: **Systems**: Signals & Controls
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Lecture 4

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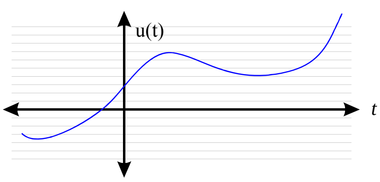
Recall From Last Time ...

Signal Measures

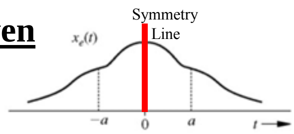


$$x(t) = x_1(t) + x_2(t)\, j$$
$$s = x_1 + x_2 j$$

$$\rightarrow e^{st} = e^{(x_1 + x_2 j)} = e^{x_1 t} e^{x_2 t j}$$
$$= e^{x_1 t}\left(\cos(x_2 t) + \sin(x_2 t)\, j\right)$$

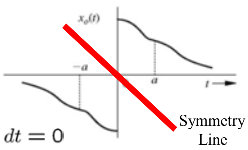
Signal Classifications

- **Even**



$$\int_{-a}^{a} u_e(t)\, dt = 2\int_{0}^{a} u_e(t)\, dt$$

- *Odd*



$$\int_{-a}^{a} u_e(t)\, dt = 0$$

- Every signal can be expressed as a sum of **even** and ***odd*** components

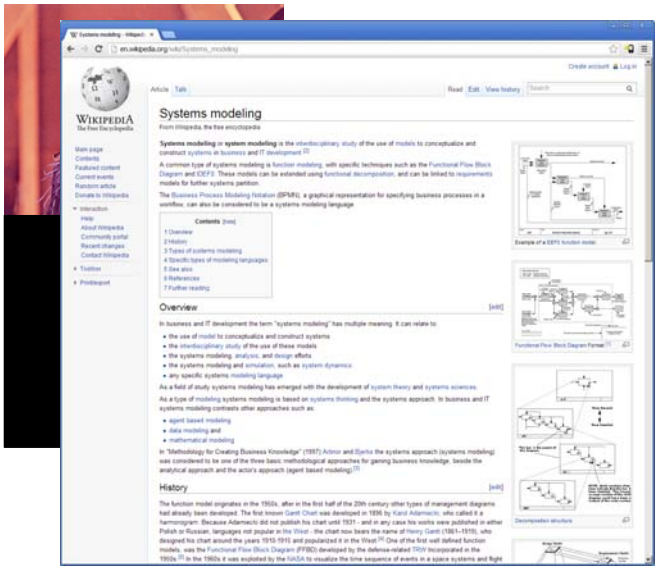
Announcements:

- Assignment 0 is up for peer review
  - Answers will be posted this afternoon!
  - Please complete by **Monday**!
- Practise Shuffle / Peer Feedback on Friday.
- Assignment 1 comes out next Monday – March 11
- Space Audit on **March 22** (F) and **March 28** (W)
  - They will be going around in **black hats**
  - P&F will be checking on us
  - This effects class funding (read tutor hours!!)
  - Sounds like an ideal time for a **pop-quiz** (just saying ☺)

Today:

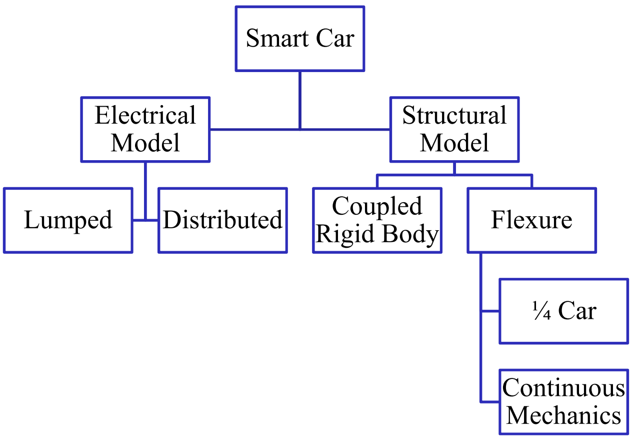
Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response & Fourier Transform
	12-Apr	Analog Filters
7	17-Apr	IIR Systems
	19-Apr	FIR Systems
8	24-Apr	z-Transform
	26-Apr	Discrete-Time Signals
9	1-May	Discrete-Time Systems
	3-May	Digital Filters
10	8-May	State-Space
	10-May	Controllability & Observability
11	15-May	Introduction to Digital Control
	17-May	Stability of Digital Systems
12	22-May	PID & Computer Control
	24-May	Information Theory & Communications
13	29-May	Applications in Industry
	31-May	Summary and Course Review

What be System Models ? …



Modelling

- The same physical system may have many different models
- The best choice depends on the problem at hand:



Which Model?

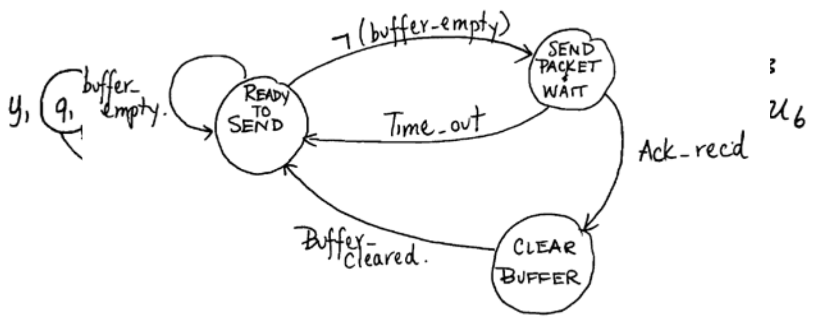
- Utility: What is trying to achieved?
- Predictive Power: Ability to forecast what the system will do?
- Better models:
  A basic tenet of engineering is that better models are those with:
  – compact, abstract principles
  – (as compared to exhaustive listings of rules)
- Occam's razor:
  – Among competing models, take the simplest one

Models & Time

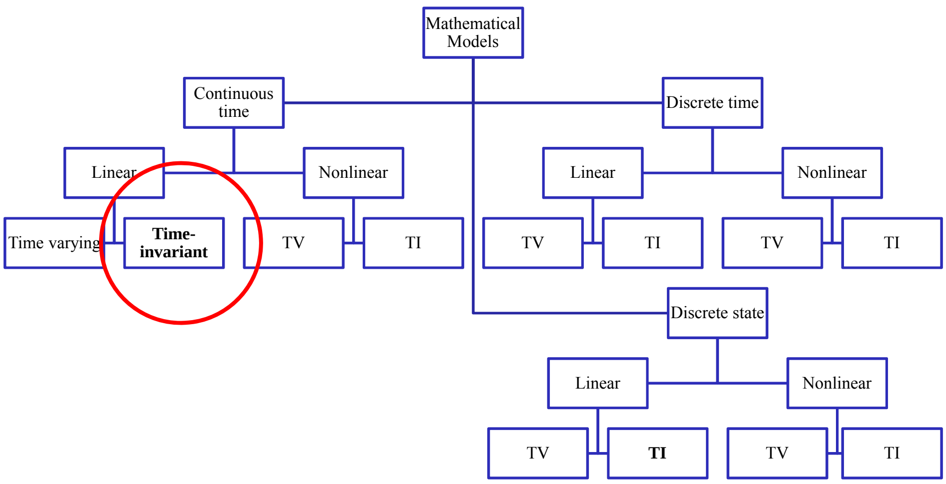
- Temps, tiempo, tempo, tijd, … zeit, 時間, время,
- ***t*** is almost universally reserved for time
- Continuous time
  – $t \in \mathbb{R}$
- Discrete time
  – Synchronous: $t \in \{nT\}, t \in \mathbb{Z}$
  – Asynchronous: $t \in \{t_1, t_2, t_3, \ldots, t_n\}$

Discrete States: Finite State Automata

- Suppose the system can be in a finite number of states:
  - when this happens the system is usually modelled by a finite states
  - Discrete "state machine"



Dynamical Systems: Mathematical Models



Classifications of Systems

1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Linear Systems: **The Concept Of Linearity**

- Additvitiy

$$x_1 \longrightarrow y_1 \quad \text{and} \quad x_2 \longrightarrow y_2$$

$$x_1 + x_2 \longrightarrow y_1 + y_2$$

- Homogeneity (scaling)

$$kx \longrightarrow ky$$

- Superposition

$$k_1x_1 + k_2x_2 \longrightarrow k_1y_1 + k_2y_2$$

Linear Systems and ODE's

- Linear system described by differential equation

$$a_0 y + a_1 \frac{dy}{dt} + \cdots + a_n \frac{d^n y}{dt^n} = b_0 x + b_1 \frac{dx}{dt} + \cdots + b_m \frac{d^m x}{dt^m}$$

- Which using Laplace Transforms can be written as

$$a_0 Y(s) + a_1 sY(s) + \cdots + a_n s^n Y(s) = b_0 X(s) + b_1 sX(s) + \cdots + b_m s^m X(s)$$

$$A(s)Y(s) = B(s)X(s)$$

where *A(s)* and *B(s)* are polynomials in *s*

Linear System: Transfer Function

- Transfer function can be written as

$$H(s) = \frac{Y(s)}{X(s)} = \frac{B(s)}{A(s)} = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n}$$

- Transfer functions have:
  - **Poles**
    - Infinite value of *H(s)*, i.e., when *A(s)* = 0 (roots of *A(s)*)
  - **Zeros**
    - Zeros value of *H(s)*, i.e., when *B(s)* = 0 (roots of *B(s)*)

The poles & zeros of H(s) define frequency response & stability

Poles & zeros: Example

- Transfer function

$$H(s) = \frac{s^2 + 2s + 2}{s^2 + 4s + 13}$$

- Zeros at $s^2 + 2s + 2 = 0$

$$s = \frac{-2 \pm \sqrt{2^2 - 4(2)}}{2} = -1 \pm j.$$

Poles/zero plot drawn on complex s-plane

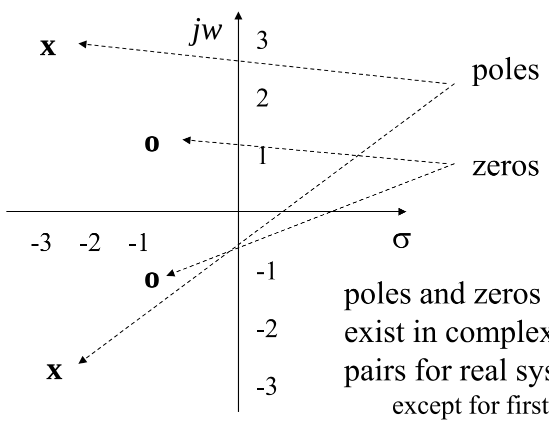
- Poles at

$$s^2 + 4s + 13 = 0$$

$$s = -2 \pm j3$$

Pole zero plot

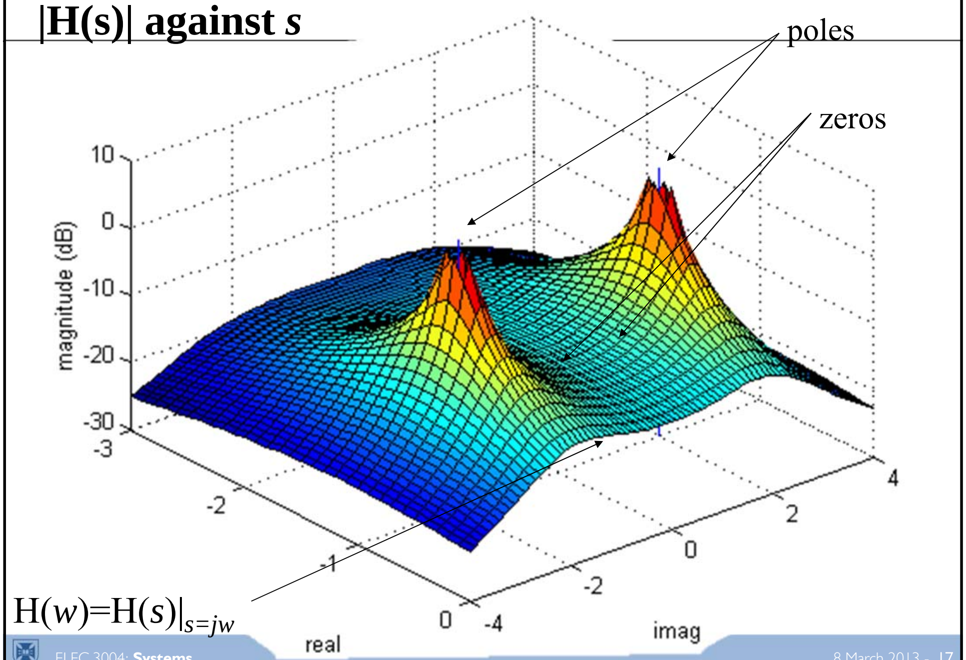
s-plane



poles and zeros always exist in complex conjugate pairs for real systems

except for first order systems when they occur on the σ axis

Linear Systems: Poles & Zeros
|H(s)| against *s*



Next Time…

- Linear Dynamical System Models
- Review:
  – Chapter 2 of Lathi
- PLEASE do give us some peer feedback to Assignment 0