Signals & Signal Models

ELEC 3004: **Systems**: Signals & Controls
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Lecture 3

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March 6, 2013

Recall From Last Time …

- Equivalence Across Domains

Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

Announcements:

- Assignment 0 is online
  - Please complete by **tomorrow**!

- Adam Keys is looking for volunteers as part of a study of motion and video simulations.

- Thank you for your participation in the Tutorials !
  ➔ Based on feedback we'll run **Tutorials on Week 4** as well!!

- Practise Shuffle / Peer Feedback on Friday.

- Assignment 1 comes out next Monday – March 11

…

Announcements [2]:

- Space Audit on **March 22** (F) and **March 28** (W)
  - They will be going around in **black hats**
  - P&F will be checking on us
  - This effects class funding (read tutor hours!!)
  - Sounds like an ideal time for a **pop-quiz** (just saying ☺)

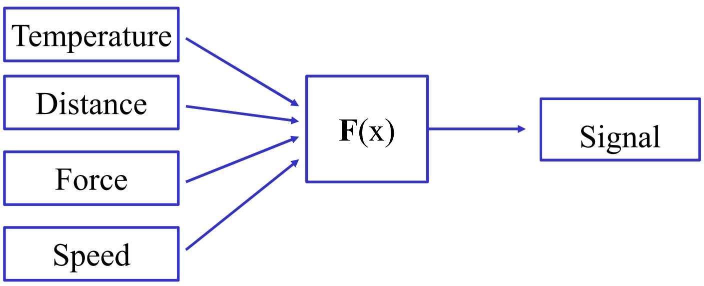
- **Q & A Forum Software**
  - Sadly, there is no "StackOverflow" for classes
  1. Blackboard Forums – Allows Ranking (**) and is "local"
  2. Piazza – Yet another IT system
  3. news:uq.itee.elec3004 -- A classic!

➔**Matlab Transfer Function Example**
➔By R. Simpson on Tutorial Page. Thank you!

Today:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response & Fourier Transform
	12-Apr	Analog Filters
7	17-Apr	IIR Systems
	19-Apr	FIR Systems
8	24-Apr	z-Transform
	26-Apr	Discrete-Time Signals
9	1-May	Discrete-Time Systems
	3-May	Digital Filters
10	8-May	State-Space
	10-May	Controllability & Observability
11	15-May	Introduction to Digital Control
	17-May	Stability of Digital Systems
12	22-May	PID & Computer Control
	24-May	Information Theory & Communications
13	29-May	Applications in Industry
	31-May	Summary and Course Review

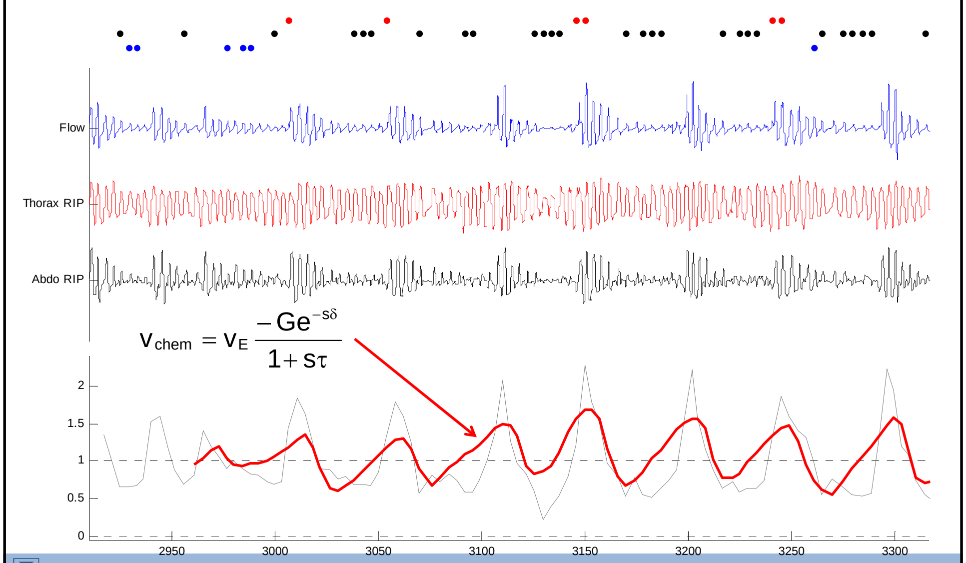
What is a Signal?



- Communicates information
- Varies in Time and/or Space

➔ Everywhere:
- The type
- The screen
- Your 5 senses
- Telephony
- Interest rates
- Leaf colour

Signals!



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Understanding & Classifying Signals

Metrics:

- Size
- Signal Energy
- Signal Power

- Frequency
- Phase

- Entropy
  - Deterministic signals
  - Random signals

Classifications:

- Continuous-Time
- Discrete-Time

- Analog
- Digital

- Even
- Odd

Signal Size

- The size of any entity is a number that indicates the largeness or strength of that entity. Generally speaking, the signal **amplitude** varies with **time**.

$$S = \int_0^T u(t)dt$$

- However, this will be a defective measure because even for a large signal $x(t)$, its positive and negative areas could cancel each other, indicating a signal of small size.

Signal Energy

- Consider the area under a signal $x(t)$ as a possible measure of its size, because it takes account not only of the amplitude but also of the duration.

- Instead we look at it's energy or *signal energy*

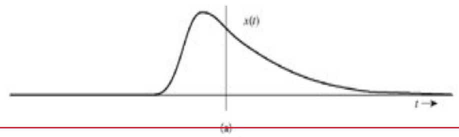
$$E = \int_{-\infty}^{\infty} u^2\left(t\right)dt$$

- But this can be **infinite!**

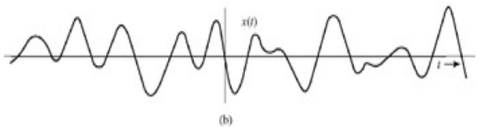
Signal Power:

- Generalize this to a finite measure via a RMS (root means square measurand)

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |u(t)|^2 dt$$



➔ Finite Energy
Finite Power



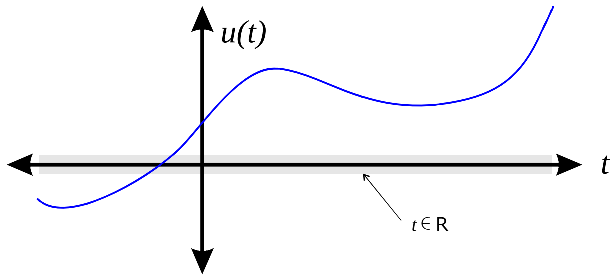
➔ ∞ Energy
Finite Power

Signal Classifications

- Classifications
  - A "pair-wise" way of characterizing the signal by putting it into a "descriptive bin"
  - Ex: How to describe a person – well we could measure them (weight, height, power, etc.) or we could sort by category ("North/South-side," Australian, etc.) ➔ Neither is perfect

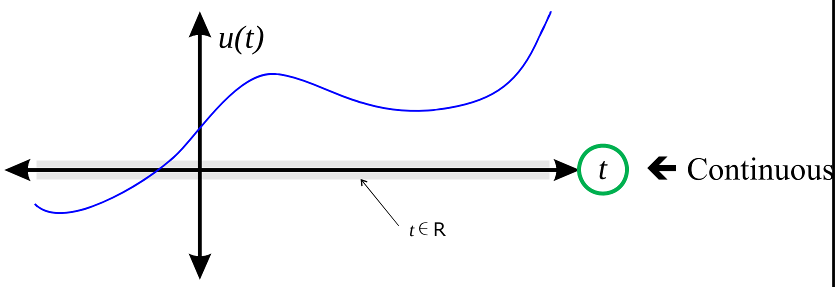
- Some common classifications:
  1. Continuous-time and discrete-time signals
  2. Analog and digital signals
  3. Periodic and aperiodic signals
  4. Real and complex signals
  5. Deterministic and probabilistic signals

Continuous-Time



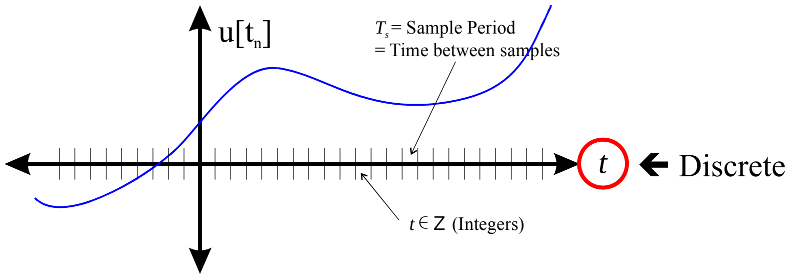
- The independent variable (x-axis) – in this case **t** – is continuous (which may be the ℝeals, but can be others)
- This does not dictate the form of u(t) -- it may be either continuous or discrete.

Continuous Time



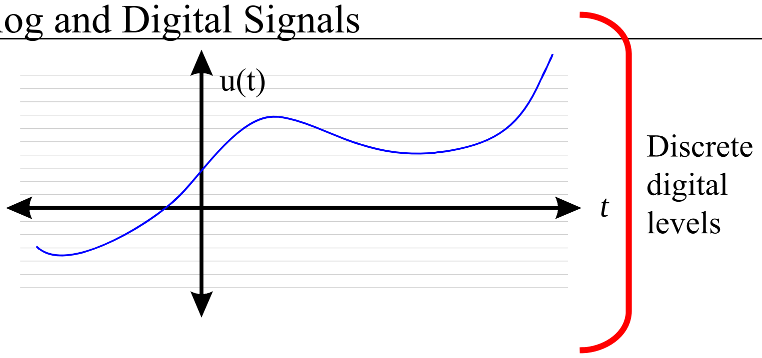
- The independent variable (x-axis) – in this case **t** – is continuous (which may be the ℝeals, but can be others)
- This does not dictate the form of u(t) -- it may be either continuous or discrete.

Discrete Time



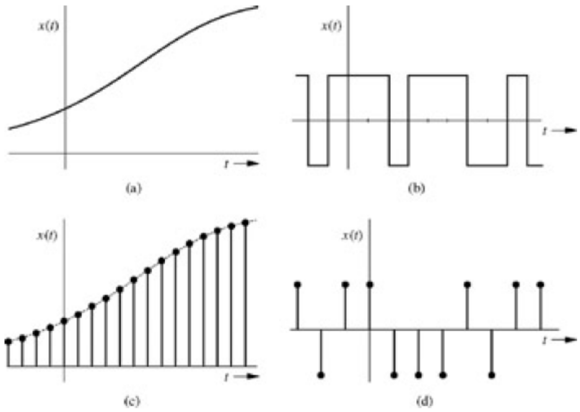
- The independent variable is only set for fixed, discrete values
- Ex: $t \in$ Integers
- Written in [Square Brackets]
- $u[t_n] = u[n]$
- $t_n=\{t_0, t_1, t_2, \ldots, t_n\}$
- Relationship to continuous time: $u[n]=u(nT_s)$

Analog and Digital Signals



- Continuous and Discrete on the **y-axis**
- u(t) = Continuous ➔ Analog
- u(t) = Discrete ➔ Digital
  - Number of levels can be many
  - Simplest case is for two ➔ Binary digit (or bit) signal

Analog and Digital Signals

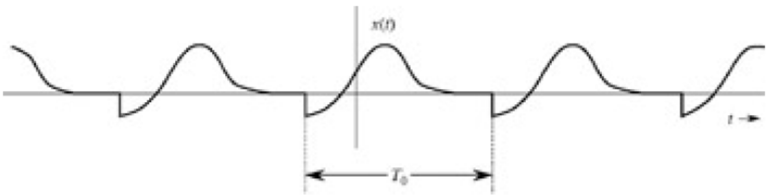


- **(a)** analog, continuous time | **(b)** digital, continuous time
- **(c)** analog, discrete time and **(d)** digital, discrete time

Periodic and Aperiodic Signals

- A signal $x(t)$ is *periodic* if for some positive constant T_0

$$x(t) = x(t + T_0)$$



- For periodic signals $x(t)$ of period T_0 :
  - the area under $x(t)$ over any interval of duration T_0 is the same

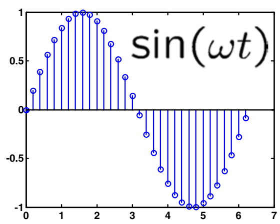
$$A = \int_{a}^{a+T_o} u(t)\, dt = \int_{b}^{b+T_o} u(t)\, dt$$

Real and Complex Signals

$$x(t) = x_1(t) + x_2(t)j$$
$$s = x_1 + x_2 j$$

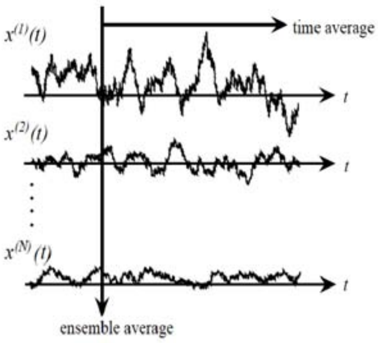
$$\rightarrow e^{st} = e^{(x_1 + x_2 j)} = e^{x_1 t} e^{x_2 t j}$$
$$= e^{x_1 t}\left(\cos(x_2 t) + \sin(x_2 t) j\right)$$

Deterministic and Random



- Deterministic
  - are those whose description is known completely
- Random
  - Those signals whose descriptions is unknown incompletely via statistical or probabilistic descriptions.

Ergodicity



1. **Over time:** multiple readings of a quantity over time
   - "stationary" or "ergodic" system
   - Sometimes called "integrating"
2. **Over space:** single measurement (summed) from multiple sensors each distributed in space
3. **Same Measurand**: multiple measurements take of the **same observable quantity** by multiple, related instruments

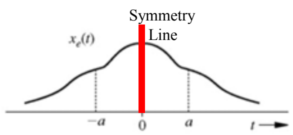
   e.g., measure position & velocity simultaneously

   → Basic "sensor fusion"

Classifying Signals: Even and Odd

- **Even**

  $u_{\text{even}}(t) = u_{\text{even}}(-t)$

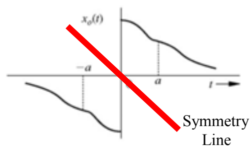


  – Area:

$$\int_{-a}^{a} u_e(t)\,dt = 2\int_{0}^{a} u_e(t)\,dt$$

- *Odd*

  $u_{\text{odd}}(t) = -u_{\text{odd}}(-t)$

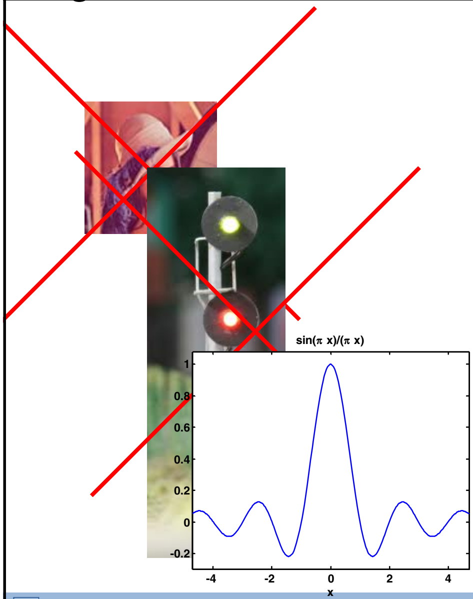


  – Area:

$$\int_{-a}^{a} u_e(t)\,dt = 0$$

➔ Every signal can be expressed as a sum of **even** and ***odd*** components

Signal Models



➔A “model” of signals

- Key functions include:
  - Step
  - Impulse
  - Exponential functions play
- Basis for representing other signals,
- Simplify many aspects of the signals and systems.

Signal Models

1] Unit Step

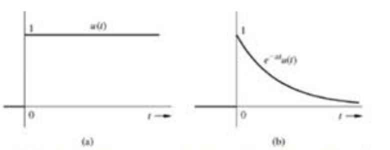
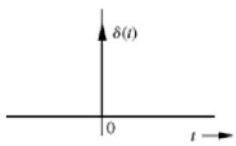


Figure 1.14: **(a)** Unit step function $u(t)$. **(b)** Exponential $e^{-at}u(t)$.

$$u(t) = \begin{cases} A & t > t_0 \\ 0 & t < t_0 \end{cases}$$

- if A=1, t_0=0
  - Heaviside function

2] Unit Impulse



$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

- Can be approximated as:

$$\delta_a(x) = \lim_{a \to 0} \frac{1}{a\sqrt{\pi}} e^{-x^2/a^2}$$

Signal Models

3] Sinusoidal & Exponential Signals

$$u(t) = A\cos(\omega t + \phi)$$

- A, ω : Amplitude and Frequency (ω=2πf)
- ϕ: phase angle

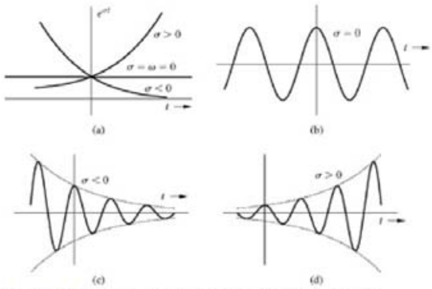


Figure 1.21: Sinusoids of complex frequency $\sigma + j\omega$.

Next Time…

- We'll talk about System Models
- Pleae Review:
  - B1-B5 of Lathi:
    - Complex numbers, sinusoids, partial fraction expansion
  - Chapter 1.7 to 2.1 of Lathi
    - Systems, classifications of systems, and system responses.
- Try the practise assignment – by tomorrow please!
- Assignment 1 comes out Next Monday (March 11).