Estimation &
Course Review

ELEC 3004: **Systems**: Signals & Controls

Dr. Surya Singh
(with material from Dr. Paul Pounds)

Lecture 25 (The Last!)

elec3004@itee.uq.edu.au
http://robotics.itee.uq.edu.au/~elec3004/

May 31, 2013

Today in Linear Systems…

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response
	12-Apr	z-Transform
7	17-Apr	Noise & Filtering
	19-Apr	Analog Filters
8	24-Apr	Discrete-Time Signals
	26-Apr	Discrete-Time Systems
9	1-May	Digital Filters & IIR/FIR Systems
	3-May	Fourier Transform & DTFT
10	8-May	Introduction to Digital Control
	10-May	Stability of Digital Systems
11	15-May	PID & Computer Control
	17-May	Applications in Industry
12	22-May	State-Space
	24-May	Controllability & Observability
13	29-May	State-Space: Made Clear
	31-May	Summary and Course Review

</assessable>

WARNING: NOT ASSESSABLE

- Nothing beyond this point is on the exam.
- Do not pay attention.
- Do not attempt to learn.

What's Assessable?

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response
	12-Apr	z-Transform
7	17-Apr	Noise & Filtering
	19-Apr	Analog Filters
8	24-Apr	Discrete-Time Signals
	26-Apr	Discrete-Time Systems
9	1-May	Digital Filters & IIR/FIR Systems
	3-May	Fourier Transform & DTFT
10	8-May	Introduction to Digital Control
	10-May	Stability of Digital Systems
11	15-May	PID & Computer Control
	17-May	Applications in Industry
12	22-May	State-Space
	24-May	Controllability & Observability
13	29-May	State-Space: Made Clear
	31-May	Summary and Course Review

Announcements:

- Practice Final Posted:
  - You're welcome ☺
  - While it's final #3 (real, supp, & prac):
    - It shares **no** questions with either the real or the supp.
  - Exam Review Session: June 7,

  ➔It is harder than the real thing!
  (∵ its primarily to practice the material **not the timing**)

- Problem Set 2 Grading:
  - May 31 (& you get your grades during the semester)
  - June 7 (More time, better feedback??)

Goals for the Week

Today:

- State-Space: Controllability & Observability
- State-Space: Solution of Differential Equations
- State-Space: Compensator Design

Friday:

- Everything!

ELEC 3004:
A Review

AKA ELEC 3004:
What do I need to know about *.* ???

To Review:
Back to the **Beginning…Lecture 1 Slide 9**

- Systems
- Signal Abstractions
- Signals as Vectors / Systems as Maps

- Linear Systems and Their Properties
- LTI Systems
- Autonomous Linear Dynamical Systems

- Convolution
- FIR & IIR Systems
- Frequency domain
- Fourier Transform (CT)
- Fourier Transform (DT)

- Even and Odd Signals
- Likelihood
- Causality

- Impulse Response
- Root Locus
- Bode Functions

- Left-hand Plane

- Frequency Response

- Discrete Time
- Continuous Time

- Laplace Transformation
- Feedback and Control
- Additional Applications

- Linear Functions
- Linear Algebra Review
- Least Squares
- Least Squares Problems
- Least Squares Applications
- Matrix Decomposition and Linear Algebra
- Regularized Least Squares

- Least-squares
- Least-squares applications
- Orthonormal sets of vectors
- Eigenvectors and diagonalization
- Linear dynamical systems with inputs and outputs

- Symmetric matrices, quadratic forms, matrix norm, and SVD
- Controllability and state transfer
- Observability and state estimation

- And that, of course, **Linear Systems are Cool!** ☺

Review

- What do you think when you see?

$$\ddot{y} + 2\dot{y} + 3y = u$$

- System?
- ODE?
- Linear Algebra?

- Joy?
- Excitement?
- Shock and Awe??

Linear algebra provides the tools/foundation for working with (linear) differential equations.

Signals & Systems

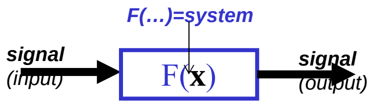
Linear algebra provides the tools/foundation for working with (linear) differential equations.

- Signals are vectors. Systems are matrices.

$$\mathbf{y} = F\mathbf{x} + G\mathbf{u}$$

Linear Systems

Linear Systems in 1-Slide



- Signals Are Vectors
- Systems Are Matricies

Is it Useful?

Yes.

(For example … Next Year – ELEC/METR 3800)

It Can Rock Your Boat Gently Down The Stream: IMU Deaduced Reckoning (Navigation)

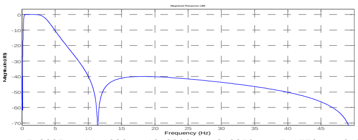
Idea: Integrate your motion (twice for $\ddot{x} \rightarrow x$ and once for $\dot{\theta} \rightarrow \theta$)

Problem:

- (DC) bias in accelerometer → drift

Solution:

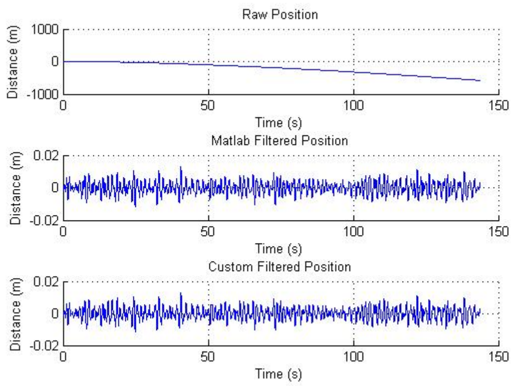
- IIR Bandpass filter (0.1-10 Hz)



```
A = [1.0000  -5.8235  14.1332  -18.2966  13.3259  -5.1772  0.8382];
B = [0.9155  -5.4933  13.7331  -18.3109  13.7331  -5.4933  0.9155];
```


It Can Rock Your Boat Gently Down The Stream: IMU Deaduced Reckoning (Navigation) [2]

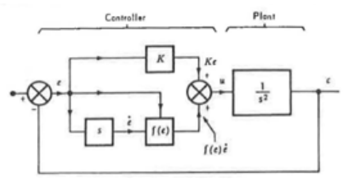
Solution:



Extra Material:
(For Fun!)

Poles are Eigenvalues

Stability of a 2nd order regulator



$$u = Ke + f(e)\dot{e}$$

state equations let $e = x_1$ and $\dot{e} = x_2$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -Kx_1 - f(x_1)\ x_2$$

assume for simplicity that K = 1.

$$0 = x_2^0$$

$$0 = -x_1^0 - f(x_1^0)x_2^0$$

The Jacobian matrix is

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -f(0) \end{bmatrix}$$

- The linear behavior of the system in the close neighborhood of the origin is described by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - f(0)x_2$$

- AND, the characteristic equation is:

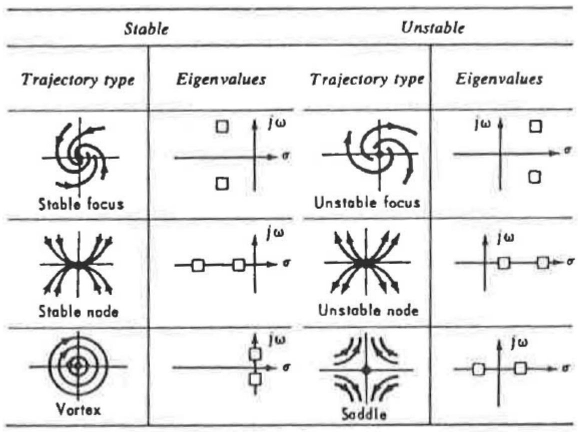
$$s[s + f(0)] + 1 = 0$$

with the eigenvalues

$$\lambda_1 = -\frac{1}{2} f(0) + \sqrt{\frac{1}{4} f^2(0) - 1}$$

$$\lambda_2 = -\frac{1}{2} f(0) - \sqrt{\frac{1}{4} f^2(0) - 1}$$

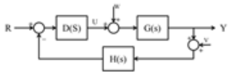
Various Types of Singularities (2nd order systems)



Root Locus
Control Design Method
(See also Q10. Part F)

Root Locus Design: "Evan's Method"

- Imagine "the basic feedback system":



$$TF(s) = \frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)} = \frac{DG}{1 + DGH}$$

→ Characteristic Equation: $1 + DGH = 0$

- Put the characteristic equation in Root Locus form:

$$1 + K \cdot L(s) = 0$$

  - **If** we define $L(s) = \frac{b(s)}{a(s)}$
  - **then** $a(s) + K \cdot b(s) = 0$ and $L(s) = -\frac{1}{K}$
- Thus a Root Locus is a graph of all possible roots of $1 + K \cdot L(s) = 0$ and K as the variable parameter

- ∵ This is the solution to the roots of the closed-loop system characteristic equation and thus the closed-loop poles of the system. The root-locus graph may be viewed as a method for interring the **dynamic properties of a system as K changes**.

The Direct Method of Digital Controls –

NOT to be confused with
Controller Emulation
(e.g., Tustin's Method)

Direct Design Method Of Ragazzini (See also: FPW 5.7 pp.216-222)

Start with 3 Discrete Transfer Functions:

- **G(z)**: TF[1] of a **plant + a hold** (e.g., from a ZOH)
- **D(z)**: A **controller** TF to do the job (what we want here)
- **H(z)**: The final desired TF between **R** (reference) and **Y** (output)
- Thus[2]:

$$H(z) = \frac{DG}{1+DG}$$
$$\rightarrow D(z) = \frac{1}{G}\frac{H}{1-H}$$

- This calls for a **D(z)** that will **cancel the plant effects** and that will **add whatever is necessary to give the desired result**. The problem is to discover and implement constraints on **H(z)** so that we do not ask for the impossible.
  - This implies that we **need some constraints** on both **H(z)** and **D(z)**

1: Transfer Function
2: Mental Quiz: What does **1+DG** say about the sign of the feedback (positive or negative)? That is, what is the characteristic equation for a system with positive feedback?

Direct Design Method Of Ragazzini [2]: Design Constraints: I. Causality

- Remember/Recall an Interesting Point:
  - From z-transform theory we know that **if D(z)** is causal, **then** as $z \rightarrow \infty$ its transfer function is well behaved & it **does not** have a pole at infinity.

- $D(z) = \frac{1}{G}\frac{H}{1-H}$ implies that **if** G**(z)** = 0 (at ∞), **then D(z)** would have a pole (at ∞) **unless H(z)** cancels it.

∴

H(z) must have a zero (at ∞) of the same order as G**(z)**'s 0s (at ∞)

→ Which means: **If** there is a lag in the plant (**G(z) starts with** z^{-1}) **then** causality requires that the delay of **H(z)** is that the closed-loop system must be at least as long a delay of the plant.

(Whoa! It might sound deep, but it's rather intuitive ☺)

Direct Design Method Of Ragazzini [3]: Design Constraints: II. Stability

- The characteristic equation and the closed loop roots:

$$1 + D(z)G(z) = 0$$

- Define[3] $D = \frac{c}{d}$ and G= $\frac{b}{a}$ ➔ $ad + bc = 0$
- Define $z - \alpha$ as a pole of **G(z)** and a common factor in DG that represents **D(z)** cancelling a pole/zero of **G(z)**.
- **Then** this common factor *remains a factor of the characteristic polynomial.*
- **If** this factor is outside the unit circle, **then** the system is unstable!

∴

1-H(z) must contain as zeros
all the poles of **G(z)** that are outside the unit circle &
H(z) must contain as zeros
all the zeros of G(z) that are outside the unit circle

3: Note the switching of the "alphabetical-ness" of these two fractions

Direct Design Method Of Ragazzini [4]: Design Constraints: III. Steady State Accuracy

- The error from **H(z)** is given by:

$$\mathrm{E(z)} = \mathrm{R(z)}\left(1 - \mathrm{H(z)}\right)$$

- **If** the system is "Type 1" (with a constant velocity/first derivative (K_v)
  - **Then**[4] $E_{ss}^{Step} = 0$ and $E_{ss}^{Ramp} = {}^{1}/_{K_v}$

∴

$$\mathrm{H(z)} = 1$$

&

$$-\mathrm{T_s} \left.\frac{dH(z)}{d(z)}\right|_{z=1} = \frac{1}{K_v}\mathrm{H(z)} = 1$$

4: E_{ss}: steady-state error

Direct Design Method Of Ragazzini [5]: An Example

- Consider the plant: $s^2 + s + 1 = 0$

With T_s=1 → z-Transform: $z^2 + 0.786z + 0.368$=0

- Let's design this **system** such that
  - $K_v = 1$
  - Poles at the roots of the plant equation & additional poles as needed

→$\mathrm{H(z)} = \frac{b_0+b_1z^{-1}+b_2z^{-2}+b_3z^{-3}+\cdots}{1-0.786z^{-1}+0.368\,z^{-2}}$

I. Causality: $\mathrm{H(z)}|_{z=\infty} = 0 \to b_0 = 0$

II. Stability: All poles/zeros of G(z) are in the unit circle
  - except for b_0, which is taken care of by $b_0 = [Const] = 0$

III. Tracking:
  - $\mathrm{H}(1) = b_1 + b_2 + b_3 + \cdots = 1 \cdot (1 - 0.786 + 0.368)$ &
  - $-\{1\} \frac{dH(z)}{d(z^{-1})}\Big|_{z=1} = \frac{1}{\{1\}}$ → $\frac{b_1+2b_2+3b_3+\cdots -[-.05014]}{(1-0.786+0.368)}$ (note the z^{-1})
  - Truncate the number of unknowns to 2 "zeros" … thus solve for b_1 and b_2 (& set $b_3,b_4,\ldots$=0)

$\therefore \mathrm{H(z)} = \frac{b_1z+b_2}{z^2-0.786z+0.368}$ →

$$D(z) = \frac{(z-1)(z-0.9048)(0.6321)}{(0.04837)(z+0.9672)}\frac{(z-0.07932)}{(z-1)(z-0.4180)}$$

$$= 13.07\frac{(z-0.9048)}{(z+0.9672)}\frac{(z-0.07932)}{(z-0.4180)}.$$

Digital Controls (Magic PID Made Easy Equations)

Implementation of Digital PID Controllers

We will consider the PID controller with an s-domain transfer function

$$\frac{U(s)}{X(s)} = G_c(s) = K_P + \frac{K_I}{s} + K_D s. \quad (13.54)$$

We can determine a digital implementation of this controller by using a discrete approximation for the derivative and integration. For the time derivative, we use the **backward difference rule**

$$u(kT) = \left.\frac{dx}{dt}\right|_{t=kT} = \frac{1}{T}(x(kT) - x[(k-1)T]). \quad (13.55)$$

The z-transform of Equation (13.55) is then

$$U(z) = \frac{1 - z^{-1}}{T}X(z) = \frac{z-1}{Tz}X(z).$$

The integration of $x(t)$ can be represented by the **forward-rectangular integration** at $t = kT$ as

$$u(kT) = u[(k-1)T] + Tx(kT), \quad (13.56)$$

Source: Dorf & Bishop, Modern Control Systems, §13.9, pp. 1030-1

Implementation of Digital PID Controllers (2)

where $u(kT)$ is the output of the integrator at $t = kT$. The z-transform of Equation (13.56) is

$$U(z) = z^{-1}U(z) + TX(z),$$

and the transfer function is then

$$\frac{U(z)}{X(z)} = \frac{Tz}{z-1}.$$

Hence, the z-domain transfer function of the **PID controller** is

$$\boxed{G_c(z) = K_P + \frac{K_I T z}{z-1} + K_D \frac{z-1}{Tz}.} \quad (13.57)$$

The complete difference equation algorithm that provides the PID controller is obtained by adding the three terms to obtain [we use $x(kT) = x(k)$]

$$\begin{aligned} u(k) &= K_P x(k) + K_I[u(k-1) + Tx(k)] + (K_D/T)[x(k) - x(k-1)] \\ &= [K_P + K_I T + (K_D/T)]x(k) - K_D T x(k-1) + K_I u(k-1). \quad (13.58) \end{aligned}$$

Equation (13.58) can be implemented using a digital computer or microprocessor. Of course, we can obtain a PI or PD controller by setting an appropriate gain equal to zero.

Source: Dorf & Bishop, Modern Control Systems, §13.9, pp. 1030-1

Implementation of Digital PID Controllers (2)

- In FPW Terminology (FPW 5.8.4 p. 224

5.8.4 PID Control

Combining all the above yields the PID controller

$$\boxed{D(z) = K_p\left(1 + \frac{Tz}{T_I(z-1)} + \frac{T_D(z-1)}{Tz}\right).} \qquad (5.61)$$

This form of control law is able satisfactorily to meet the specifications for a large portion of control problems and is therefore packaged commercially and sold for general use. The user simply has to determine the best values of K_p, T_D, and T_I.

Now, What's Next?

What's Next?

Research:

- Thesis Projects:
  - Signal Processing (Eulerian Video Magnification)
  - Digital Control (OpenROVs)
  - Robotics
  - More!

Courses

- ELEC4620: Digital Signal Processing
- ELEC4630: Image Processing & Computer Vision
- **METR 4202:** Advanced Control & Robotics
- CSSE4010: Digital System Design

Today's Lecture is Brought To You By the Number 5

SECATs: One more Systems Example

- Is ELEC 3004 Linear?

- ELEC Controllability?

- Is it / the instructor Stable?

SECATs: Let's look back at the topic list from Lecture 1

The course is has a huge mandate:

- It is really 3 · ½ courses in one !
  - Linear Systems
  - Signal Processing
  - Controls & Digital Controls

- ∴ It is **b r o a d !!**

- There is a logic to it
  - They share the same mathematical nature (poles & zeros)
  - The math is common to more than just circuits!

Lots of Stuff To Cover…

- ✔ Systems
- ✔ Signal Abstractions
- ✔ Signals as Vectors / Systems as Maps

- ✔ Linear Systems and Their Properties
- ✔ LTI Systems
- ✔ Autonomous Linear Dynamical Systems

- ✔ Convolution
- ✔ FIR & IIR Systems
- ✔ Frequency domain
- ✔ Fourier Transform (CT)
- ✗ Fourier Transform (DT)

- ✔ Even and Odd Signals
- ✔ Likelihood
- ✔ Causality

- ✔ Impulse Response
- ✔ Root Locus
- ✔ Bode Functions

- ✔ Left-hand Plane

- ✔ Frequency Response

- ✔ Discrete Time
- ✔ Continuous Time

- ✔ Laplace Transformation
- ✔ Feedback and Control
- ✔ Additional Applications

- ✔ Linear Functions
- ✔ Linear Algebra Review
- ✔ Least Squares
- ✗ Least Squares Problems
- ✔ Least Squares Applications
- ✗ Matrix Decomposition and Linear Algebra
- ✗ Regularized Least Squares

- ✔ Least-squares
- ✔ Least-squares applications
- ✗ Orthonormal sets of vectors
- ✔ Eigenvectors and diagonalization
- ✔ Linear dynamical systems with inputs and outputs

- ✔ Symmetric matrices, quadratic forms, matrix norm, and SVD
- ✔ Controllability and state transfer
- ✔ Observability and state estimation

- And that, of course, **Linear Systems are Cool!** ☺ ✔

Yes, this is Hard! Why?

- Breath
  - Books, books, everywhere, yet we're all on Wikipedia!!
  - Authors tend to be "too generalizable"

- Assumptions:
  - Numerous conditions that need to be remembered

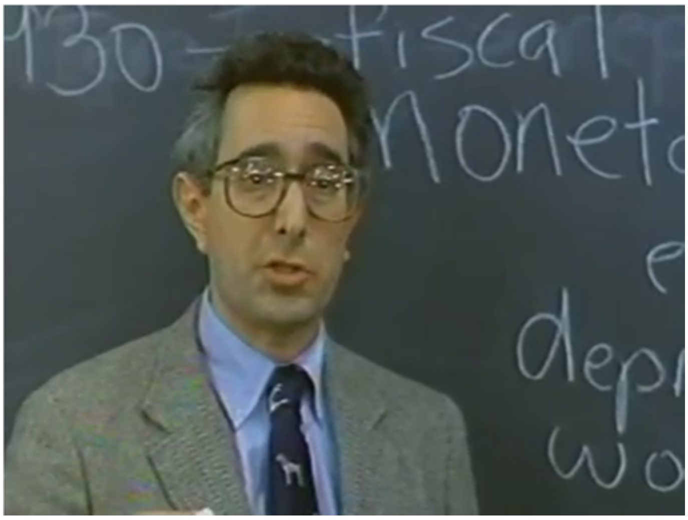
- Tacit Details:

  →The need for examples (but these are few and always seem the same)

- Time consuming

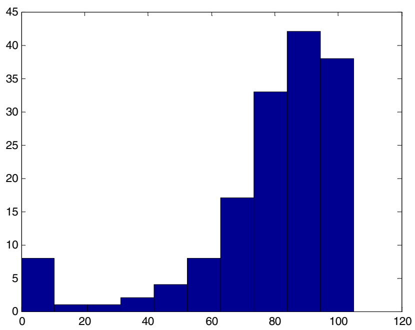
“4” Is Average

- What is a 3?



Grades: Better than you think!

- Problem Set 1



- Extra Credit

	Lab 1	Lab 2	Lab 3	Lab 4
Mean	3.683486239	3.305084746	3.65	3.229167
Median	4	3	4	3
St. Dev	0.853112355	0.764780622	0.47697	0.62047

- Quiz 1, 2, & 3:

	Quiz 1	Quiz 2	Quiz 3
Mean	7.637868	7.535433	4.706522
Median	7.5	8	5
St. Dev	1.651598	2.056468	1.115971

SECaTs: Some Lessons in the Works for Next Year

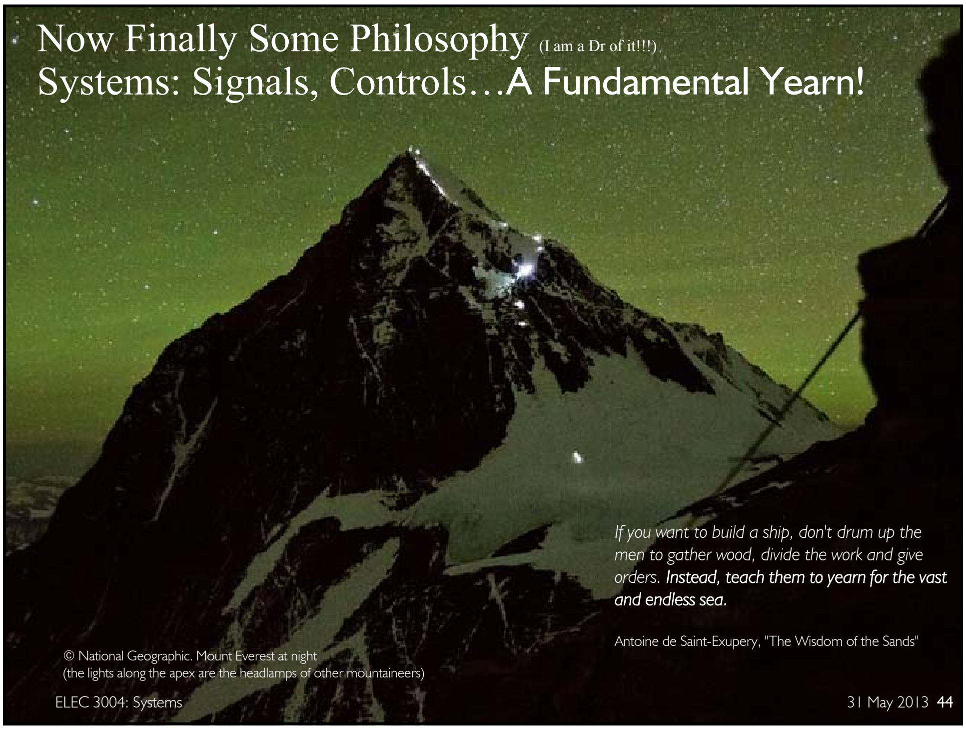
- I shall only use my own slides
- Less is more!
  - Smaller assignments
  - More time for Examples
- Revised Platypus (using TinyMCE)
- Better organization
  - Better (more structured) tutorials
  - More examples!!
  - I get that. But, we've come a long way

➔ To make this happen I need your support!

Next Time in Linear Systems ….

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response
	12-Apr	z-Transform
7	17-Apr	Noise & Filtering
	19-Apr	Analog Filters
8	24-Apr	Discrete-Time Signals
	26-Apr	Discrete-Time Systems
9	1-May	Digital Filters & IIR/FIR Systems
	3-May	Fourier Transform & DTFT
10	8-May	Introduction to Digital Control
	10-May	Stability of Digital Systems
11	15-May	PID & Computer Control
	17-May	Applications in Industry
12	22-May	State-Space
	24-May	Controllability & Observability
13	29-May	State-Space: Made Clear
	31-May	Summary and Course Review

- We're at the End. It's (the) final!
- Thank you folks!


Now Finally Some Philosophy (I am a Dr of it!!!)
Systems: Signals, Controls…A Fundamental Yearn!
If you want to build a ship, don't drum up the men to gather wood, divide the work and give orders. Instead, teach them to yearn for the vast and endless sea.
Antoine de Saint-Exupery, "The Wisdom of the Sands"
© National Geographic. Mount Everest at night
(the lights along the apex are the headlamps of other mountaineers)
ELEC 3004: Systems
31 May 2013 44