Discrete (Digital) Feedback Control:
PID, Regulators, Compensators, Computer Control, etc.

ELEC 3004: **Systems**: Signals & Controls

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Lecture 20

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May 15, 2013

Next Time in Linear Systems ….

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
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Goals for the Week

- Stability of Digital Systems
- Lead/Lag Compensators
- ZOH and Discretisation

Two classes of control design

The system…

- Isn't fast enough
- Isn't damped enough
- Overshoots too much
- Requires too much control action

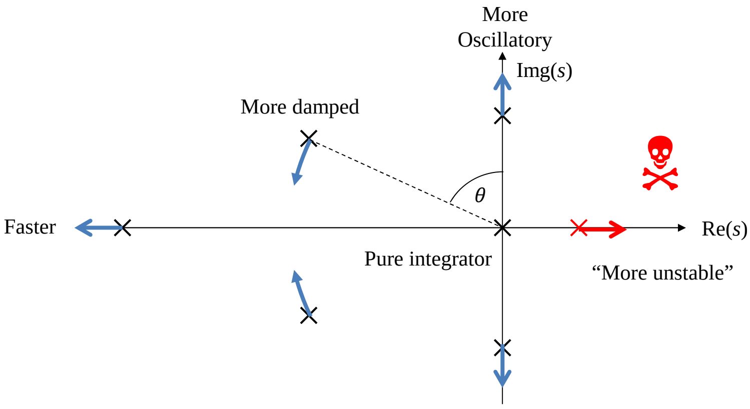
("Performance")

- Attempts to spontaneously disassemble itself

("Stability")

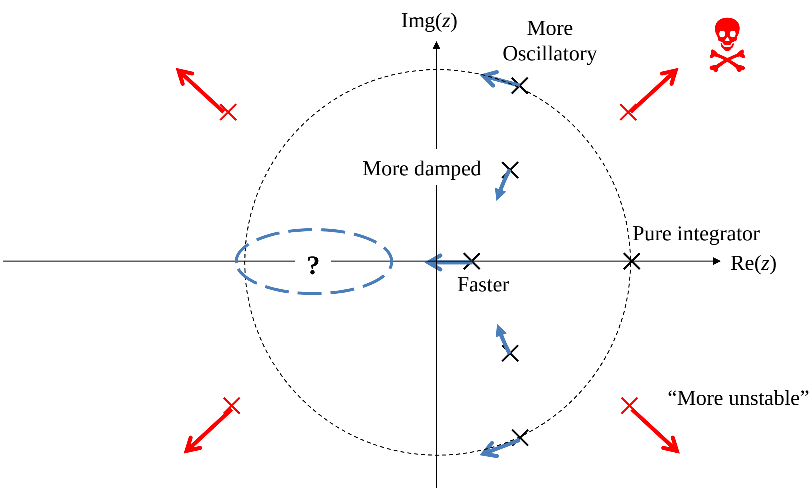
Recall dynamic responses

- Moving pole positions change system response characteristics



Recall dynamic responses

- Ditto the z-plane:



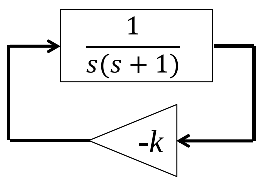
The fundamental control problem

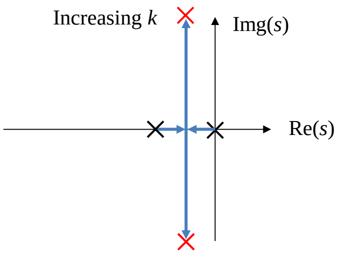
The poles are in the wrong place

How do we get them where we want them to be?

Recall the root locus

- We know that under feedback gain, the poles of the closed-loop system move
  - The root locus tells us where they go!
  - We can solve for this analytically*

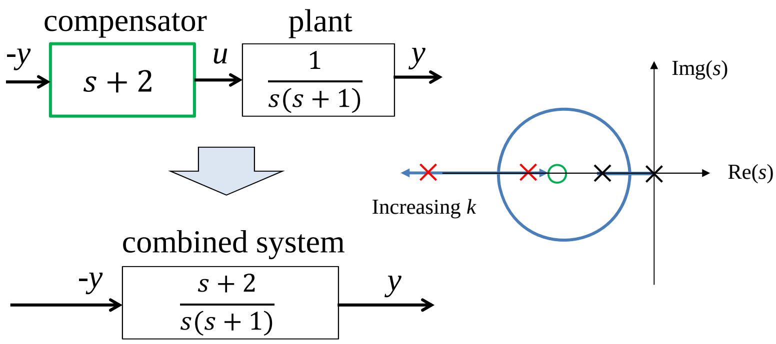




Root loci can be plotted for all sorts of parameters, not just gain!

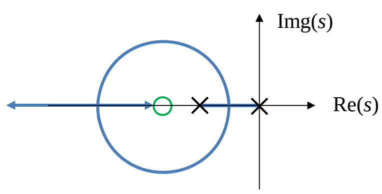
Dynamic compensation

- We can do more than just apply gain!
  - We can add dynamics into the controller that alter the open-loop response



But what dynamics to add?

- Recognise the following:
  - A root locus starts at poles, terminates at zeros
  - “Holes eat poles”
  - Closely matched pole and zero dynamics cancel
  - The locus is on the real axis to the left of an odd number of poles (treat zeros as ‘negative’ poles)



Some standard approaches

- Control engineers have developed time-tested strategies for building compensators
- Three classical control structures:
  - Lead
  - Lag
  - Proportional-Integral-Derivative (PID)
    (and its variations: P, I, PI, PD)

How do they work?

Lead/lag compensation

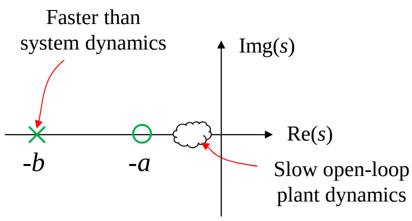
- Serve different purposes, but have a similar dynamic structure:

$$D(s) = \frac{s+a}{s+b}$$

Note:

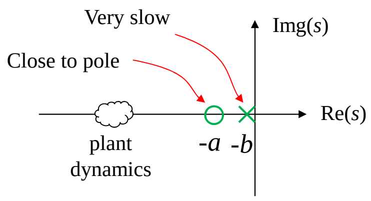
Lead-lag compensators come from the days when control engineers cared about constructing controllers from networks of op amps using frequency-phase methods. These days pretty much everybody uses PID, but you should at least know what the heck they are in case someone asks.

Lead compensation: $a < b$



- Acts to decrease rise-time and overshoot
  - Zero draws poles to the left; adds phase-lead
  - Pole decreases noise
- Set a near desired ω_n; set b at ~3 to 20x a

Lag compensation: $a > b$



- Improves steady-state tracking
  - Near pole-zero cancellation; adds phase-lag
  - Doesn't break dynamic response (too much)
- Set b near origin; set a at ~3 to 10x b

PID – the Good Stuff

- Proportional-Integral-Derivative control is the control engineer's hammer*
  - For P,PI,PD, etc. just remove one or more terms

$$\mathrm{C(s)} = k\left(1 + \frac{1}{\tau is} + \tau ds\right)$$

Proportional
Integral
Derivative

*Everything is a nail. That's why it's called "Bang-Bang" Control ☺

PID – the Good Stuff

- PID control performance is driven by three parameters:
  - k: system gain
  - τ_i: integral time-constant
  - τ_d: derivative time-constant

You're already familiar with the effect of gain.
What about the other two?

Integral

- Integral applies control action based on accumulated output error
  - Almost always found with P control
- Increase dynamic order of signal tracking
  - Step disturbance steady-state error goes to zero
  - Ramp disturbance steady-state error goes to a constant offset

Let's try it!

Integral

- Consider a first order system with a constant load disturbance, w; (recall as $t \to \infty$, $s \to 0$)

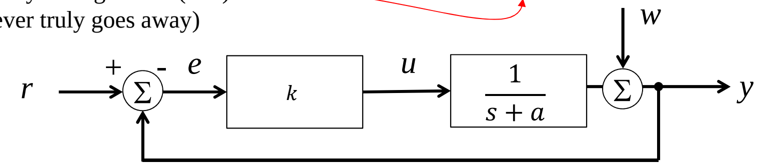
$$y = k\frac{1}{s+a}(r-y) + w$$

$$(s+a)y = k\,(r-y) + (s+a)w$$

$$(s+k+a)y = kr + (s+a)w$$

$$y = \frac{k}{s+k+a}r + \frac{(s+a)}{s+k+a}w$$

Steady state gain = a/(k+a)
(never truly goes away)



Now with added integral action

$$y = k\left(1 + \frac{1}{\tau_i s}\right)\frac{1}{s+a}(r - y) + w$$

$$y = k\frac{s + \tau_i^{-1}}{s}\frac{1}{s+a}(r - y) + w$$

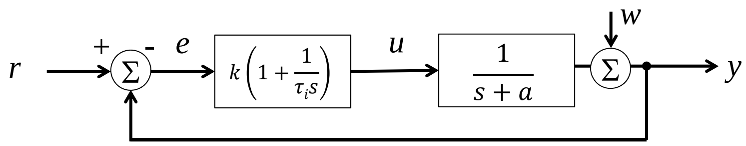
Same dynamics

$$s(s+a)y = k(s + \tau_i^{-1})(r - y) + s(s+a)w$$

$$(s^2 + (k+a)s + \tau_i^{-1})y = k(s + \tau_i^{-1})r + s(s+a)w$$

$$y = \frac{k(s + \tau_i^{-1})}{(s^2 + (k+a)s + \tau_i^{-1})}r + \frac{s(s+a)}{k(s + \tau_i^{-1})}w$$

Must go to zero for constant *w*!



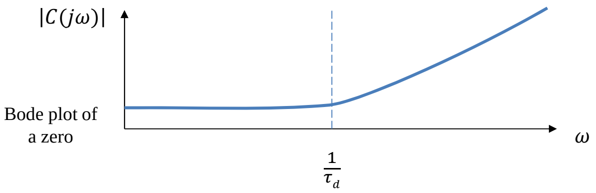
Derivative

- Derivative uses the rate of change of the error signal to anticipate control action
  - Increases system damping (when done right)
  - Can be thought of as 'leading' the output error, applying correction predictively
  - Almost always found with P control*

**What kind of system do you have if you use D, but don't care about position?* Is it the same as P control in velocity space?

Derivative

- It is easy to see that PD control simply adds a zero at $s = -\frac{1}{\tau_d}$ with expected results
  - Decreases dynamic order of the system by 1
  - Absorbs a pole as $k \to \infty$
- Not all roses, though: derivative operators are sensitive to high-frequency noise



PID

- Collectively, PID provides two zeros plus a pole at the origin
  - Zeros provide phase lead
  - Pole provides steady-state tracking
  - Easy to implement in microprocessors
- Many tools exist for optimally tuning PID
  - Zeigler-Nichols
  - Cohen-Coon
  - Automatic software processes

Be alert

- If gains and time-constants are chosen poorly, all of these compensators can induce oscillation or instability.
- However, when used properly, PID can stabilise even very complex unstable third-order systems

Now in discrete

- Naturally, there are discrete analogs for each of these controller types:

Lead/lag: $\frac{1-\alpha z^{-1}}{1-\beta z^{-1}}$

PID: $k\left(1+\frac{1}{\tau_i(1-z^{-1})}+\tau_d(1-z^{-1})\right)$

But, where do we get the control design parameters from?
The s-domain?

Emulation vs Discrete Design

- Remember: polynomial algebra is the same, whatever symbol you are manipulating:

  eg. $s^2 + 2s + 1 = (s+1)^2$

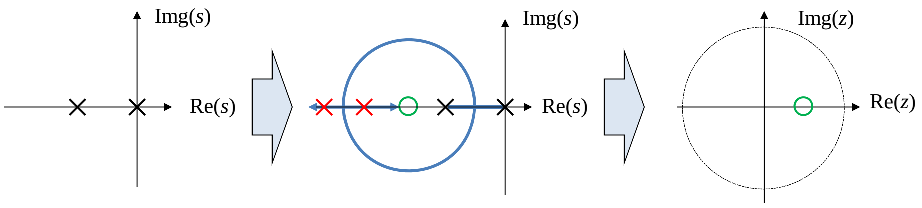
  $z^2 + 2z + 1 = (z+1)^2$

  Root loci behave the same on both planes!

- Therefore, we have two choices:
  - Design in the s-domain and digitise (emulation)
  - Design only in the z-domain (discrete design)

Emulation design process

1. Derive the dynamic system model ODE
2. Convert it to a continuous transfer function
3. Design a continuous controller
4. Convert the controller to the z-domain
5. Implement difference equations in software



Emulation design process

- Handy rules of thumb:
  - Use a sampling period of 20 to 30 times faster than the closed-loop system bandwidth
  - Remember that the sampling ZOH induces an effective *T*/2 delay
  - There are several approximation techniques:
    - Euler's method
    - Tustin's method
    - Matched pole-zero
    - Modified matched pole-zero

Tustin's method

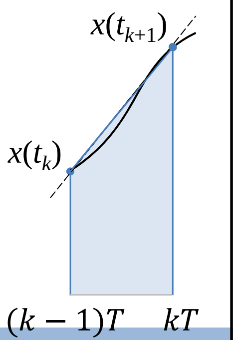
- Tustin uses a trapezoidal integration approximation (compare Euler's rectangles)
- Integral between two samples treated as a straight line:

$$u(kT) = \frac{T}{2}[x(k-1) + x(k)]$$

Taking the derivative, then z-transform yields:

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

which can be substituted into continuous models



Matched pole-zero

- If $z = e^{sT}$, why can't we just make a direct substitution and go home?

$$\frac{Y(s)}{X(s)} = \frac{s+a}{s+b} \Rightarrow \frac{Y(z)}{X(z)} = \frac{z-e^{-aT}}{z-e^{-bT}}$$

- Kind of!
  - Still an approximation
  - Produces quasi-causal system (hard to compute)
  - Fortunately, also very easy to calculate.

Matched pole-zero

The process:

1. Replace continuous poles and zeros with discrete equivalents:

$$(s+a) \Rightarrow (z-e^{-aT})$$

2. Scale the discrete system DC gain to match the continuous system DC gain

3. If the order of the denominator is higher than the enumerator, multiply the numerator by $(1-z^{-1})$ until they are of equal order*

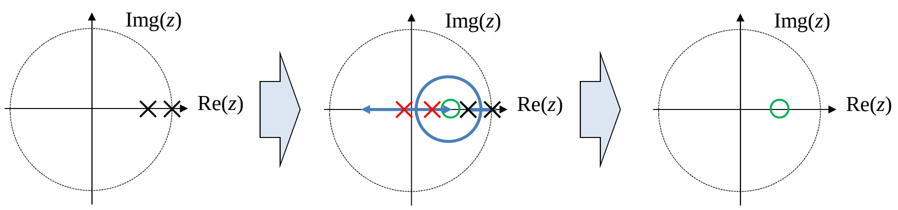
* This introduces an averaging effect like Tustin's method

Modified matched pole-zero

- We're prefer it if we didn't require instant calculations to produce timely outputs
- Modify step 2 to leave the dynamic order of the numerator one less than the denominator
  - Can work with slower sample times, and at higher frequencies

Discrete design process

1. Derive the dynamic system model ODE
2. Convert it to a discrete transfer function
3. Design a digital compensator
4. Implement difference equations in software
5. Platypus Is Divine!



Discrete design process

- Handy rules of thumb:
  - Sample rates can be as low as twice the system bandwidth
    - but 5 to 10× for "stability"
    - 20 to 30 × for better performance
  - A zero at $z = -1$ makes the discrete root locus pole behaviour more closely match the s-plane
  - Beware "dirty derivatives"
    - dy/dt terms derived from sequential digital values are called 'dirty derivatives' – these are especially sensitive to noise!
    - Employ actual velocity measurements when possible

Announcements:

- Final Exam:
  - 15 Questions (60% Short Answer, 40% Regular Problems)
  - 3 Hours
  - Closed-book
  - Took tutor ~90min to complete
  - Equation sheet will be provided (in addition to your own)
    [See Prac Final – Coming out next week]
  - Yes, it has an unexpected twist at the end, but you'll like it. ☺

➔Saturday, June 15 at 9:30 AM (sorry!)

ELEC3004	Signals, Systems & Control	15/06/2013	9:30 AM		Agarwal, Atu - Vasuian, Fab	UQ Union Complex (21) - Innes Room
					Vermeulen, Nat - Zulkarnain, Moh	UQ Union Complex (21) - Heath Room

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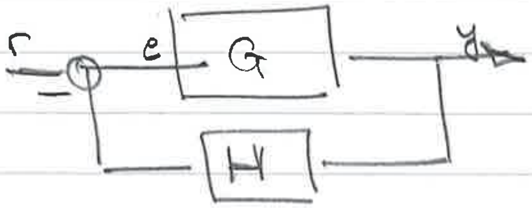
→ **feedback**



$y = G(e)$

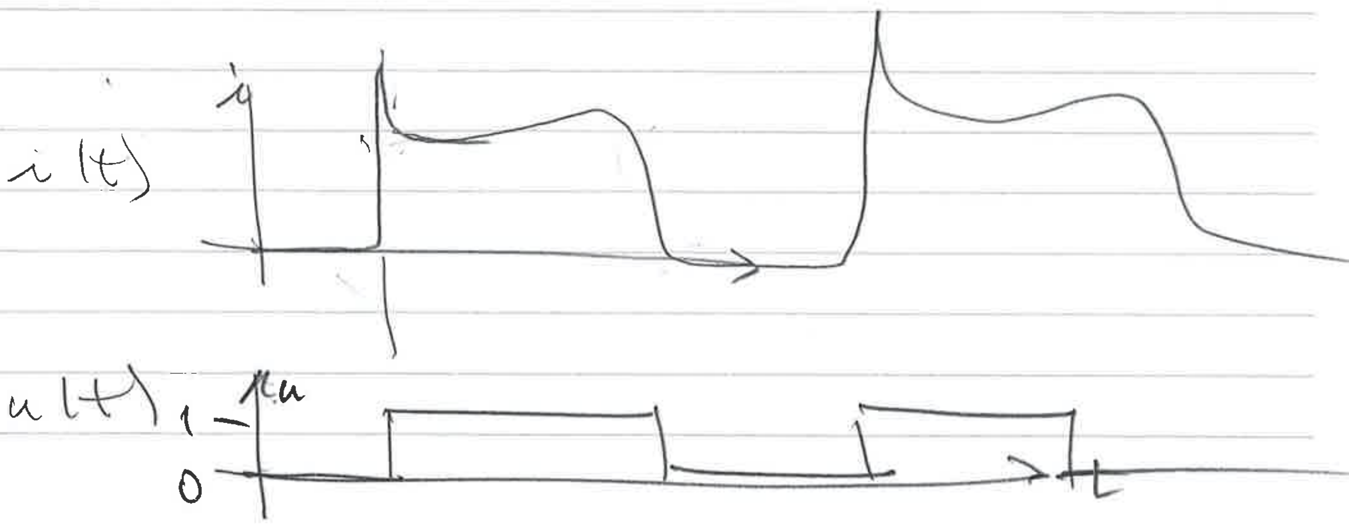
$$\frac{y}{r} = \frac{G}{1+GH}$$

What's bang-bang control

$$u(t) = \begin{cases} 1 & \text{for certain conditions} \\ 0 & \text{else} \end{cases}$$

Fridge:

$$u_f(t) = \begin{cases} 1 & \text{if } T > T^* \\ 0 & T \le T^* \end{cases}$$

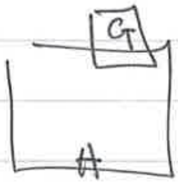


→ D controller + Noise

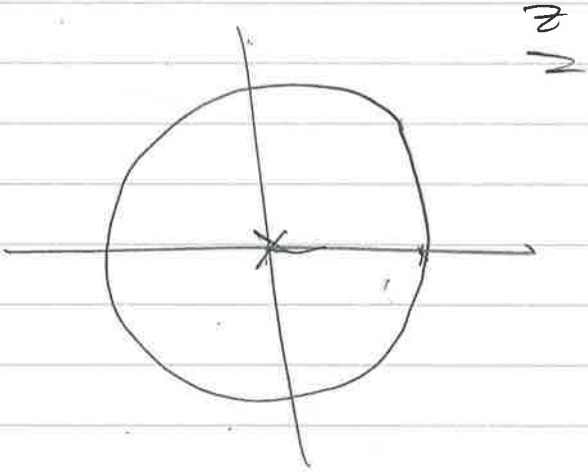
$$y(t) = \sin(\omega t) + 0.01\cdot\sin(100\omega t)$$

$$y'(t) = \omega\left[\cos(\omega t) + 100\cdot(0.01)\cdot\cos(100\omega t)\right]$$

$$y''(t) = -\omega^2\left[\sin(\omega t) + 100^2(0.01)\cdot\sin(\omega t)\right]$$



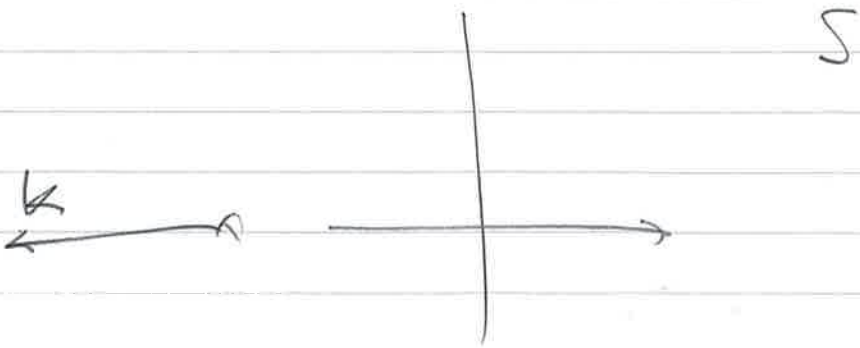
$$\frac{G}{1+GH}$$



H_z

$$\frac{H}{1+kH}$$

$1+kH=0$

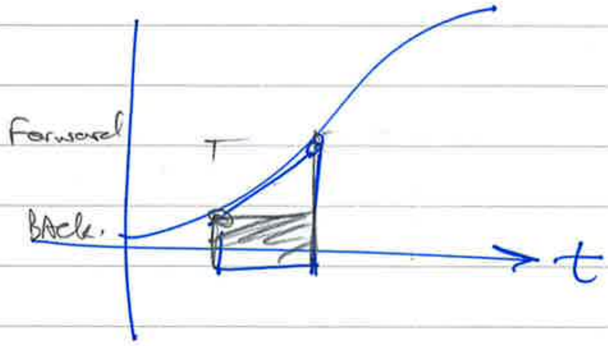


$$k\cdot\frac{0.1052}{z-1.105}$$

→ Numerical integration

$$\frac{Y(s)}{X(s)} = H(s) = \frac{a}{s+a}$$

$$\dot{y} + a y = a x$$



Trapazoidal

$$H_T = \left[\frac{a}{\left(\frac{2}{T}\right)\left[\frac{z-1}{z+1}\right] + a} \right]$$

Forward :

$$\frac{z-1}{T}$$

Backward: $\frac{z-1}{Tz}$

TRAP $\left[\frac{2}{T} \cdot \left(\frac{z-1}{z+1}\right)\right]$