Systems Overview

ELEC 3004: **Systems**: Signals & Controls
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Lecture 2

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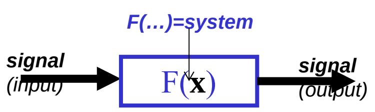
March 1, 2013

Recap from Last Time…
Signals and Systems Together

- A **signal** can be seen as that which goes in and out of a **system**

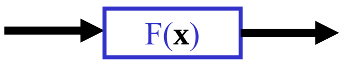


Signals and Systems Together

- A **signal** can be seen as that which goes in and out of a **system**

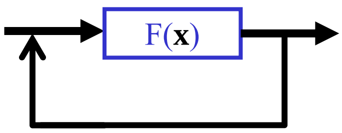
- **Signal Processing / "Filters" :**
  can be seen as a open-loop system

Filter:



- **Feedback Control:**
  can be viewed as the case where the output signal **shapes** the input signal

Feedback:



The Point of the Course

- Introduction to terminology/semantics
- An appreciation of how to frame problems in a linear systems engineering context
- Modeling and learning assumptions/when to trust the model
- Ability to identify critical details from the problem

➔ It's a **shortcut** …
Once you see that a system is **"linear"**
you can then <u>apply the raft of</u>
<u>"linear systems" tools</u>
(time & frequency analysis) to them
without having to do all the analysis from scratch

Announcements:

- A revised Lecture 1 is posted
  ➔ I'll try to incorporate class discussion / commentary to the slides so please do take a look at the posted material(s)

- If you can not make the class ∵ of a clash, please advise us

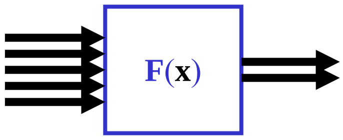
- Tutorials **will** be meeting next week (Week 2)
  To:
  - Allow for clash resolution
    (**please** see if you can swap with someone to your desired time)
  - Discuss peer-review and help guide with the good reviewing
  - Get familiar with Platypus
  - Meet your fellow tutors

Today:

Week	Date	Lecture Title
1	27-Feb	Introduction
	1-Mar	Systems Overview
2	6-Mar	Signals & Signal Models
	8-Mar	System Models
3	13-Mar	Linear Dynamical Systems
	15-Mar	Sampling & Data Acquisition
4	20-Mar	Time Domain Analysis of Continuous Time Systems
	22-Mar	System Behaviour & Stability
5	27-Mar	Signal Representation
	29-Mar	Holiday
6	10-Apr	Frequency Response
	12-Apr	Analog Filters
7	17-Apr	IIR Systems
	19-Apr	FIR Systems
8	24-Apr	Discrete-Time Signals
	26-Apr	Discrete-Time Systems
9	1-May	z-Transform
	3-May	Discrete Time Analysis Using the z-Transform
10	8-May	Introduction to Digital Control
	10-May	Stability of Digital Systems
11	15-May	Digital Filters
	17-May	Information Theory & Communications
12	22-May	State-Space
	24-May	Controllability & Observability
13	29-May	Applications in Industry
	31-May	Summary and Course Review

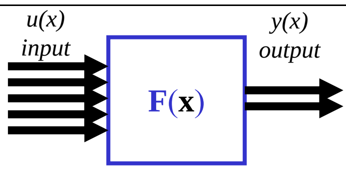
An Overview of Systems

- Today we are going to look at **F**(x)!



- F(x): System Model
  - The rules of operation that describe it's behaviour of a "system"
  - Predictive power of the responses
  - Analytic forms > Empirical ones
    - Analytic formula offer various levels of detail
    - Not everything can be experimented on *ad infinitum*
    - Also offer Design Intuition (let us devise new "systems")
    - Let's us do **analysis**! (determine the outputs for an input)
  - Various Analytic Forms
    - Constant, Polynomial, **Linear**, Nonlinear, Integral, **ODE**, PDE, Bayesian…

Linear Systems



- Model describes the relationship between the input **u**(x) and the output **y**(x)
- If it is a Linear System (wk 3):

$$y(t) = \int_0^t F(t-\tau)\, u(\tau)\, d\tau$$

- If it is also a (Linear and) **lumped**, it can be expressed **algebraically** as:

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t)$$
$$y(t) = C(t)\, x(t) + D(t)\, u(t)$$

- If it is also (Linear and) **time invariant** the matrices can be reduced to:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$\mathscr{L}$aplacian: $$y(s) = F(s)\, u(s)$$

Discrete Variations & Stability

$$y(s) = F(s)\,u(s)$$

- Is in <u>continuous time</u> …
- To move to <u>discrete time</u> it is more than just "sampling" at: 2 × (biggest Frequency)
- Discrete-Time Exponential

$$F(t) \rightarrow F[kT]$$

$$e^{\frac{k}{T}} = \gamma^k$$

$$\frac{1}{T} = \ln\gamma$$

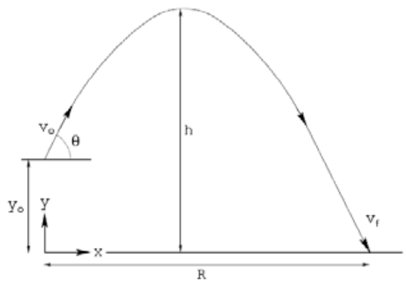
- SISO to MIMO
  - Single Input, Single Output
  - Multiple Input, Multiple Output
- BIBO:
  - Bounded Input, Bounded Output
- Lyapunov:
  - Conditions for Stability
  - ➔ Are the results of the system asymptotic or exponential

Further Classifications of Systems

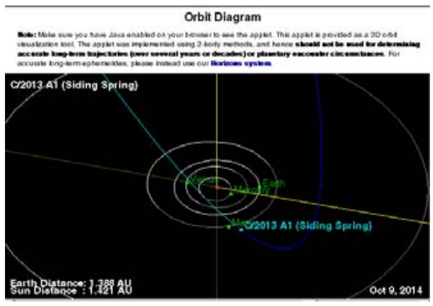
1. Linear and nonlinear systems
2. Constant-parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Modelling order

- Modelling order depends on what you are trying to achieve



The parabolic trajectory of a projectile



http://ssd.jpl.nasa.gov/

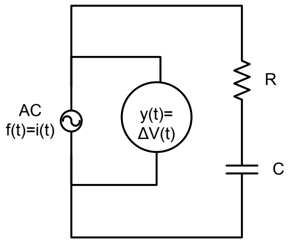
JPL Small-Body Database Browser.url

Modelling Ties Back with ELEC 2004

- Linear Circuit Theorems, Operational Amplifiers
- Operational Amplifiers
- Capacitors and Inductors, RL and RC Circuits
- AC Steady State Analysis
- AC Power, Frequency Response
- Laplace Transform
- Reduction of Multiple Sub-Systems
- Fourier Series and Transform
- Filter Circuits

➔ Modelling Tools!

Example: RC Circuits



$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{t} f(\tau)\, d\tau$$

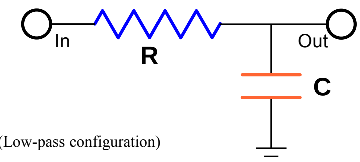
$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{0} f(\tau)\, d\tau + \frac{1}{C}\int_{0}^{t} f(\tau)\, d\tau$$

$$y(t) = v_C(0) + Rf(t) + \frac{1}{C}\int_{0}^{t} f(\tau)\, d\tau$$

$$y(t) = v_C(t_0) + Rf(t) + \frac{1}{C}\int_{t_0}^{t} f(\tau)\, d\tau$$

First Order RC Filter

- Passive, First-Order **R**esistor-**C**apacitor Design:



(Low-pass configuration)

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

- 3dB (½ Signal Power):

$$\omega = 2\pi f$$

$$f_c = \frac{1}{2\pi \mathbf{RC}}$$

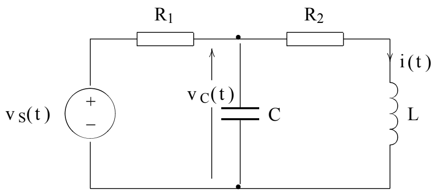
- Magnitude:

$$|V_{\text{out}}| = \sqrt{\frac{1}{(\omega RC)^2}}\,|V_{\text{in}}|$$

- Phase:

$$\phi = \tan^{-1}(-\omega RC)$$

Example of 2nd Order: RLC Circuits

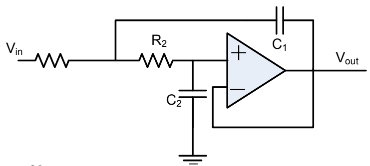


- KCL: $\frac{V_s(t) - V_c(t)}{R_1} = C\frac{d}{dt}V_c(t) + i(t)$
- KVL: $V_c(t) = L\frac{d}{dt}i(t) + R_2 i(t)$
- Combining:

$$V_s(t) = R_1 LC\frac{d^2}{dt^2}i(t) + (L + R_1 R_2 C)\frac{d}{dt}i(t) + (R_1 + R_2)\, i(t)$$

2nd Order Active RC Filter (Sallen–Key)

- 2nd Order System Sallen–Key Low-Pass Topology:

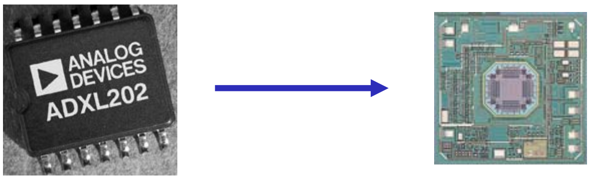


Build this for Real in **ELEC 4403**

- KCL: $\frac{v_{\text{in}} - v_x}{R_1} = C_1 s\,(v_x - v_{\text{out}}) + \frac{v_x - v_{\text{out}}}{R_2}$
- Combined with Op-Amp Law:
$\frac{v_{\text{in}} - v_{\text{out}}(C_2 s R_2 + 1)}{R_1} = C_1 s v_{\text{out}}\,(C_2 s R_2 + 1) - v_{\text{out}} + \frac{v_{\text{out}}(C_2 s R_2 + 1) - v_{\text{out}}}{R_2}$
- Solving for Gives a 2nd order System:

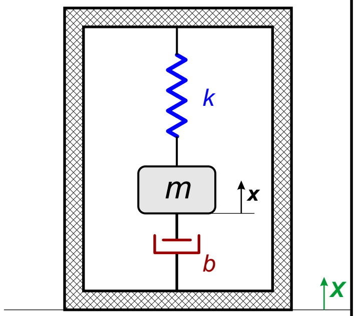
$$\frac{v_{out}}{v_{in}} = \frac{1}{C_1 C_2 R_1 R_2 s^2 + C_2(R_1 + R_2)s + 1}$$

Another 2nd Order System:
Accelerometer or Mass Spring Damper (MSD)



- General accelerometer:
  - Linear spring (*k*) (0^{th} order w/r/t o)
  - Viscous damper (*b*) (1^{st} order)
  - Proof mass (*m*) (2^{nd} order)

➔ Electrical system analogy:
  - resistor (R) : damper (b)
  - inductance (L) : spring (k)
  - capacitance (C) : mass (m)



Measuring Acceleration:
Sense *a* by measuring spring motion *Z*

- Start with Newton's 2^{nd} Law:

$$ma = F$$

- Substitute:

$$m\frac{d^2x}{dt^2} = k(X - x) + b\frac{d(X - x)}{dt}$$

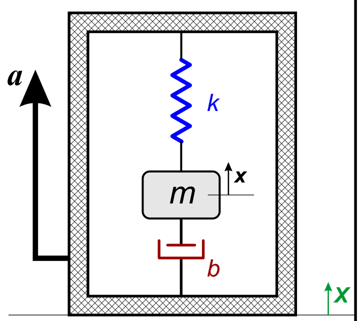
$$\mathbf{Z} \equiv (X - x) \rightarrow x = X - \mathbf{Z}$$

$$\Rightarrow m\frac{d^2X}{dt^2} = m\frac{d^2\mathbf{Z}}{dt^2} + k\mathbf{Z} + b\frac{d\mathbf{Z}}{dt}$$

The "displacement" measured by the unit (the motion of *m* relative the accelerometer frame)

- Solve ODE:

$$X(t) = X_0 e^{i\omega t} \qquad \mathbf{Z}(t) = Z_0 e^{i\omega t}$$



Measuring Acceleration [2]

- Substitute candidate solutions:

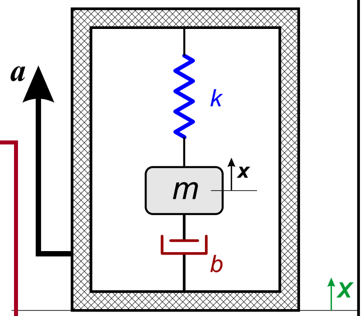
$$m\frac{d^2\left(X_0e^{i\omega t}\right)}{dt^2} = m\frac{d^2\left(Z_0e^{i\omega t}\right)}{dt^2} + k\left(Z_0e^{i\omega t}\right) + b\frac{d\left(Z_0e^{i\omega t}\right)}{dt}$$

$$-m\omega^2X_0e^{i\omega t} = -m\omega^2Z_0e^{i\omega t} + kZ_0e^{i\omega t} + (i\omega)\,bZ_0e^{i\omega t}$$

- Define Natural Frequency (ω_0)
  & Simplify for Z_0
  (the spring displacement "magnitude"):

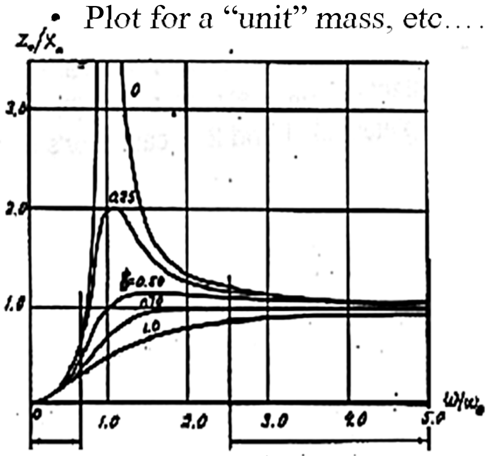
$$\omega_0 \equiv \sqrt{\frac{k}{m}}$$

$$Z_0 = \frac{m\omega^2X_0}{m\omega^2 - k - i\omega b} = \frac{X_0}{\sqrt{1 - \frac{\omega_0{}^2}{\omega^2} - \frac{b^2}{m^2\omega^2}}}$$



Acceleration: 2nd Order System

- Plot for a "unit" mass, etc….



Accelerometer Seismometer

- For $\omega << \omega_0$:

$$Z_0 \approx \frac{\omega^2X_0}{\omega_0^2} = \frac{a}{\omega_0^2}$$

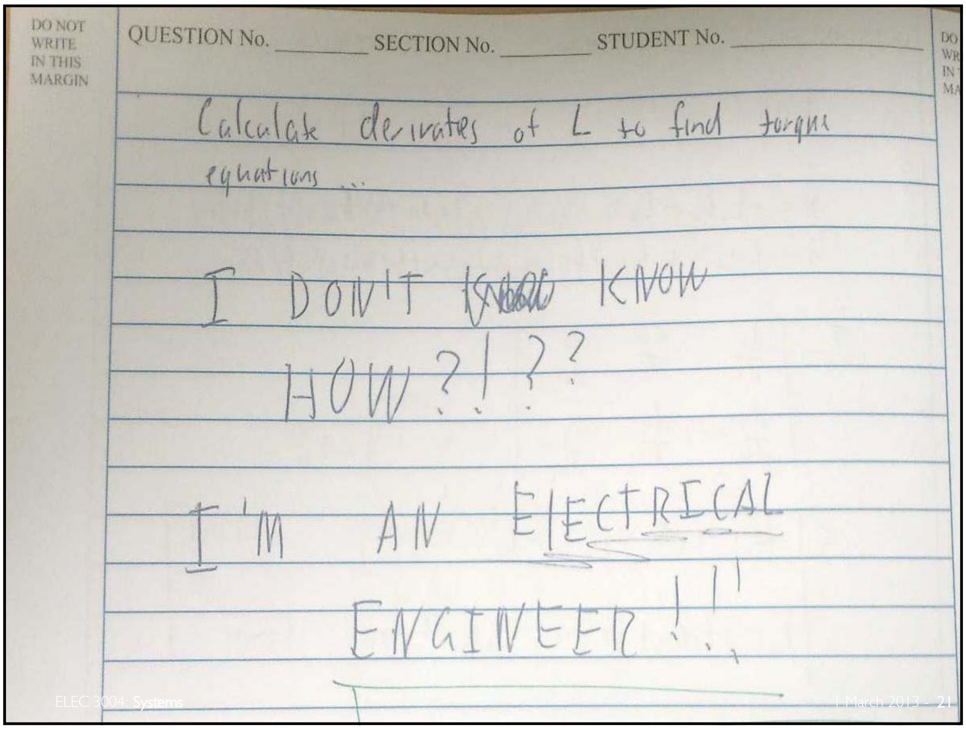
$$\rightarrow a = Z_0\omega_0^2$$

→ **it's an Accelerometer**

- For $\omega \sim \omega_0$
  - As: b→ 0, Z → ∞
  - Sensitivity ↑
- For $\omega >> \omega_0$:

$$Z_0 \approx X_0$$

→ **it's a Seismometer**



Equivalence Across Domains

Table 2.1 Summary of Through- and Across-Variables for Physical Systems

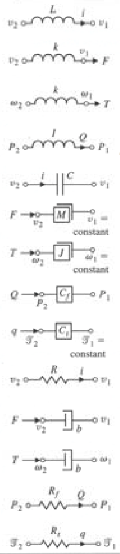
System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 73

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}{}^2$
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2{}^2$
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2{}^2$
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}{}^2$
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}{}^2$
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}{}^2$
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}{}^2$
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}{}^2$
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$

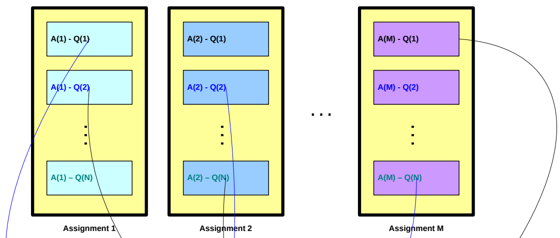
Symbol



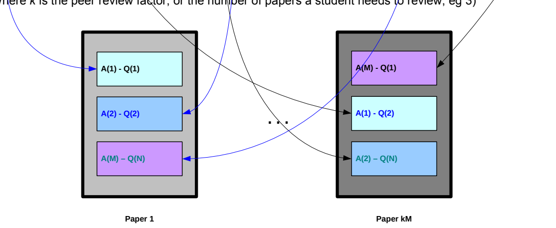
Source: Dorf & Bishop, *Modern Control Systems*, 12th Ed., p. 74

Peer Reviewing – Platypus: How does it work?

I. Collect Assignments (one per student)



II. Randomly Shuffle Questions between assignments to create ***kM*** "Papers"
(Where *k* is the peer review factor, or the number of papers a student needs to review, eg 3)



Some Philosophy

- Let's start with Why …
- To learn something is to teach it
  - The function of a teaching is not so much to cover the topics, but more to discover them
- It is actually **more** work for us!
  - We have to teach you how to reflect & then assess this as well as how to do the assignment
- It helps you understand it by giving you a different perspective
- We're a community
  - You (alone) can't do everything … that's why we work together
  - The notion of "free speech" ➔ Trust emerges → efficiency (η)

Next Time…

- We'll talk about Signals
- Review:
  - Phasers, complex numbers, polar to rectangular, and general functional forms.
  - Chapter 1 of Lathi (particularly the first sections on signals & classification thereof)
- Register on Platypus
- Try the practise assignment (background review problems)