# ELEC 3004 / 7312 – Systems: Signals & Controls
# Problem Set 2: Digital Signals & Controls

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**Total marks:** 150        **Due Date:** May 24, 2013 (at 11:59pm, AEST -- no extensions)

**Note:** This assignment is worth **30%** of the final course mark. Please submit answers via Platypus. It is requested that solutions, including equations, should be typed please. The grade is the median of the marks from the peer reviews and the staff (with provisions for review). Finally, the tutors will **not** assist you further unless there is real evidence you have attempted the questions.

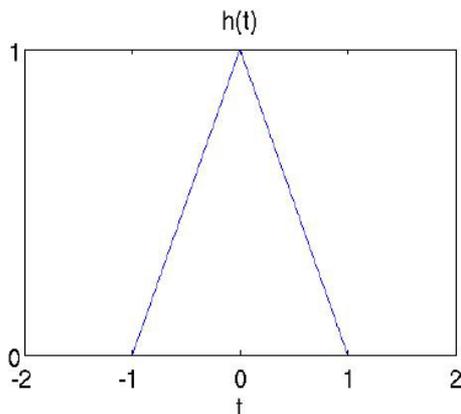Revisions are listed at the end. Changes are indicated in the paper by shading.

## Short Questions Section:
(Please keep it simple)

### Q1. Graphical Convolution [5 points]
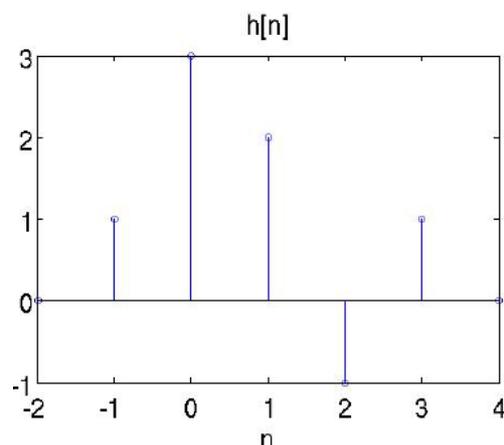Consider an LTI system with impulse response h(t) given below:



Suppose the input to the system is: $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$

Please plot the output of the system, **y(t)**. [**Hint**: please remember to label your plots]

### Q2. More Convolutions [5 points]
Suppose an LTI system has the following impulse response h[n] given below:



Suppose the input to the system is: $x[n] = u[n+1] + u[n-1] - 2u[n-2]$

Please plot the output of the system, **y[n]**. [**Hint**: please remember to label your plots]

## Q3. A Bilateral Understanding [5 points]

Janus wants to apply the $z$-Transform to a noncausal signal. Janus recalls that this can not be handled unilaterally using the unilateral $z$-Transform, but can be done using the bilateral (or two-sided) $z$-Transform. Using this, help Janus determine the z-Transform of:

$x[n] = (0.9)^n u[n] + (1.2)^n u[-(n+1)]$

## Q4. Ordering a Lowpass Filter Around [10 points]

[**Revised May 20:** Note and comment on MATLAB added.
If you have solved this directly, this is fine and acceptable.]

Determine the order, $n$, and corresponding cutoff frequency, $\omega_c$, of a lowpass Butterworth and Inverse Chebyshev filter that satisfies the following specifications:

- $G_p \geq -0.5$ dB and $G_s \leq -20$ dB
- $f_p = 30$ Hz ($\omega_p$=188.5 rad/s) and $f_s = 40$ Hz ($\omega_s$=251.3 rad/s)

[**Note:** Please choose and state the ripple for the Inverse Chebyshev filter. **YES**, you are allowed to use `cheb2ord` in MATLAB (though please explain your code/solution). In this case, you will need to specify the passband ripple (Rp). Rs can be determined from the specification above (i.e., $G_s$)]

[**Hint**: When finding values of $\omega_c$ please find one that meets or oversatisfies the passband specification]

## Q5. Set FIRe to the Reign (of analog filters) [10 points]

[**Revised May 10:** 100 Hz specified]

Design a linear phase FIR filter (using the window design method) to meet the specifications of the lowpass filter problem in Q4. Determine the required filter order, $N$, corresponding cutoff frequency, $\omega_c$, and corresponding delay. The sample rate is **100 Hz**. Then specify the final FIR filter.

- $G_p \geq -0.5$ dB and $G_s \leq -20$ dB
- $f_p = 30$ Hz ($\omega_p$=188.5 rad/s) and $f_s = 40$ Hz ($\omega_s$=251.3 rad/s)

[**Hint**: The stopband and passband ripple requirements may be inferred from $G_p$ and $G_s$. ]

## Q6. I Hear You [10 points]

Uncle Robert, having escaped electrocution while fixing the lights, has set his interests on sound. To get rid of noise he has built a simple second-order Butterworth lowpass filter for his stereo. Although the system performs well, the ever entrepreneurial Robert, wants to improve it. He reckons "twice the filtering gives twice the performance," and suggests filtering the audio signal not once but twice with a cascade of two identical filters. His idealistic cousin is skeptical and suggests, "if you are using identical filters, it makes no difference whether you filter once or twice."

Who is correct (both, either, or neither)? and why?

## Q7. Driving a Hard Drive [15 points]

Consider a digital controller for a disk drive system in which the disk spins with a constant speed and the has a time between readings of T.  This may be modeled as:
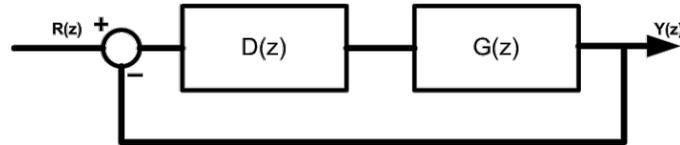


Where $G(s) = [ZOH]\left[\frac{5}{s(s+20)}\right] = \left[\frac{1-e^{-sT}}{s}\right]\left[\frac{5}{s(s+20)}\right]$

a. Determine $G(z)$ [please state your assumptions (e.g., ZOH)]

b. For $T = 1ms$, a proportional controller ($D(z) = K$), what is the open loop transfer function?
   [**Hint:** What happens to G(s) (or how can G(s) be simplified) if the fast pole ($s = -20$) is neglected?]

c. What is the closed-loop transfer function?
   [**Hint:** Apply the same assumptions as was done in Part b]

d. **[Revised May 17 for the case where K=4000 is unstable]** For $K = 4000$, what is response?  if $K = 4000$ is stable, what is the settling time?  If $K = 4000$ is unstable, what is the maximum K for which the system is stable?

# Long Questions Section:

## Q8. Analog vs Digital Filters [20 points]

[**Revised May 9:** (1) Cheby-II to Cheby, (2) 12th to 8th order]
[**Revised May 20**: (ii) Setting values to **100kΩ**. IF you have chosen another value, this is okay, just be certain to mention it **clearly.**]

a) Design a $8^{th}$ order Chebyshev lowpass filter with a cutoff frequency of 5000 Hz
(i.e. $\omega_c = 2\pi(5000)$ ) by completing the following:
i) Locate and plot the filter's poles and zeros in the complex plane. Plot the corresponding magnitude response $|H_{LP}(j\omega)|$ to verify proper design.
ii) Setting all resistor values to **100kΩ, determine the capacitor values** to implement the filter using a cascade of four second-order Sallen-Key filters (as below). On a single plot, plot the magnitude response of each section as well as the overall magnitude response. Identify the poles that correspond to each section's magnitude response curve.
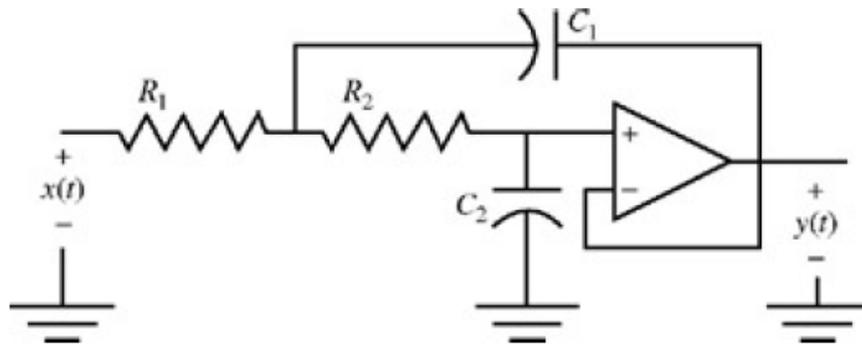


b) Design an equivalent $8^{th}$ order digital lowpass filter with $\Omega_c = \pi/3$
[**Hint:** Try Matlab's `cheby1` function] Plot the filter's poles and zeros in the complex $z$ plane and the decibel magnitude response $20log_{10}|H(e^{j\Omega})|$.

## Q9. Digital Control [30 points]

**Part I:** [10 points]
Sketch the root locus of the system: $x'' = -5x + 2u$ in feedback with a 5Hz controller implementing:
a) a lead compensator
b) a lag compensator
Which is more robust to arbitrarily large system gains?

**Part II:** [20 points]
For what range of proportional feedback gains are the following systems stable?
i) $H(z) = \frac{0.1052}{z - 1.105}$
ii) $H(s) = \frac{2}{s^2 + 6s + 9}$, 20 Hz (approximate with Tustin's method where $s = \frac{2}{T}\frac{z-1}{z+1}$)
iii) $H(s) = \frac{2}{s^2 + 6s + 9}$, 20 Hz (using exact $z$-Transform)

# The Applied Systems Section:

This section considers the application of these concepts to an applied system (e.g. such as a levitating bearing system as seen in many electric motors)

[For this section you may propose a new application problem of similar scope. Proposed problems **must** be reviewed and accepted by a tutor and lecturer at least one week in advance and must be accompanied by a petition with at least eight students (~5% class) who want to do the custom problem if accepted. If accepted the petitioners must to the custom problem to which they have signed a petition.]

## Q10. It's Not Easy Being Stable                                          [40 points]

[**Revised May 21:** Root Locus now optional (+5 Extra Credit). The values in (g) have been updated to be more realistic to actual parts -- Matlab tools are allowed (though remember to justify the solution)].

Professor Honeydew has a new machine, Leviathan. Much like a magnetic bearing in turbomachinery, this suspends ferromagnetic material (or a weight to which a magnet has been attached) by means of an electromagnet whose current is controlled by the position of the mass[1].

**Leviathan -- A Magnetic Levitator[2] [3]:**



The equation of motion is: $m\frac{d^2x}{dt^2} = -mg + f(x,I)$ where the force on the ball due to the electromagnet is given by $f(x,I)$. At equilibrium the magnet force balances the gravity force.

Let $I_0$ represent the current at equilibrium, then we can linearize this system about $x = 0$ and $I = I_0$ to get:

$$m\frac{d^2x}{dt^2} = k_1 x + k_2 i$$

The Professor's assistant, Beaker, has estimated the values of the constants as:

**m=1 kg, $k_1$=50N/m,** and **$k_2$=1 N/A.**
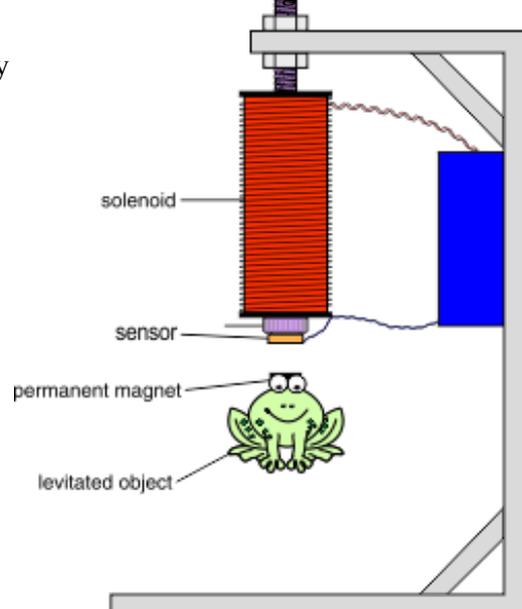
a. **Continuous transfer function**. Compute the transfer function from $I$ to $x$ (i.e., $G(s) = \frac{X(s)}{I(s)}$ ).

b. **Continuous to discrete.** Assume the input is passed through a zero order hold (ZOH) with a sampling frequency of 100 Hz. Compute the equivalent discrete-time plant transfer function.

c. **Digital controller.** Design a digital control for the magnetic levitation device so that the closed-loop system has a rise time of less than 160 milliseconds ($t_r \le 0.159s$), a settling time of less than 460 milliseconds ($t_s \le 0.46s$), and overshoot ($M_p \le 20\%$).

d. **Step Response.** Plot the step response of your design to an initial (unit) disturbance. Show this for both the output response (x(t)) and the control effort (i(t)).

e. **Noise**. Beaker replaces the sensor with a new one that uses mains. This new sensor is very noisy with 50Hz noise of the same amplitude at the signal (SNR=1).
Design a filter to process this sensor's output (include the specific type and order of FIR or IIR or analog filter). What will happen if the order is too great?

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[1] Woodson and Melcher, *Electromechanical Dynamics Part 1: Discrete Systems*, 1968
[2] Based on Figure 1 of K. Lilienkamp and K. Lundberg, "Low-cost magnetic levitation project kits for teaching feedback system design," *Proceedings of the 2004 American Control Conference*, vol.2, pp.1308-1313, 2004.
[3] Tamara, Two Tone Frog Clip Art, http://goo.gl/fAfUf

f.   [Optional: +5 Extra Credit] **Root Locus**. Plot a root locus with respect to $k_1$. Please discuss the possibility of using your closed-loop system to balance frogs of various masses and with more damping (e.g. greater air resistance or if it has wings) .
     You may (for upto +3 points) draw the root locus with respect to the overall system gain (sometimes written "$k$" not to be confused with $k_1$ the linearization about the position, $x$).

g.   [ Optional: +5 Extra Credit] **Saturation**. What would happen if the solenoid has a limit **100A** and a latency of **10 milliseconds** (remember that we are sampling at **100 Hz**)?


# For ELEC 7312 Only:

**Q11. Getting Down With Linearity**                                          **[10 points]**
Consider a downsamping system that generates an output $f_c[k]$ by downsampling an input signal $u[k]$ to give the output via the following function:

$$f_c[k] = \begin{cases} u[k], & \text{if } k \text{ is even} \\ 0, & \text{if } k \text{ is odd} \end{cases}$$

Is this system, $f_c[k]$, linear and time invariant?    Briefly Why?


**Q12. MATLAB is Getting Circulant**                                          **[15 points]**
Please submit a MATLAB function `C = circulant(row)` that constructs a circulant matrix for the first row. Have fun with it. For extra credit develop a second function that will colour cells of the same value and use this to "show" that the circulant matrix has the nature expected. A suggested header is:

```
function C = circulant(row)
% CIRCULANT: Given row (row) make a circulant matrix
```

**Change Log**

- 09/05/2013 - **Q8**.  Updated filter to a Chebyshev Filter
  (it was an Inverse Chebyshev Filter (or Cheby-II) filter, but this as zeros)
- 10/05/2013 - **Q5**.  Updated FIR question with sampling rate of the digital filter of 100Hz.
- 17/05/2013 - **Q7**.  Updated part d to address the case where K=4000 might be unstable.
  (Different assumptions result in different outcomes).  In this case, please determine the maximum
  gain ($K_{max}$) that would be stable.
- 20/05/2013 - **Q4.** Matlab's `cheb2ord` explicitly allowed (with explanation of the results).  **Q8**
  was missing the resistor value.  This has been added.  In both cases, if you had a previous
  solution (i.e., solved the problem algebraically (**Q4**) or , **it will be accepted**.  Though we
  request that you mark this with some text at the beginning of your solution (e.g., "**[This problem
  was solved before the clarification and used the following assumptions]**").
- 21/05/2013 - Google Docs has recently changed its pagination algorithm making a mess of the
  "original" document (hooray!).  The pages layout has changed because of this.
- 21/5/2013 - **Q10**. Root locus is now optional.  The current limit and delay values in part (g)
  have been updated so as to be more representative of practical parts.  Matlab tools are
  explicitly allowed in the solution (they were always allowed, but now their use is explicitly
  noted).