ELEC 3004 / 7312 – Systems: Signals & Controls

Problem Set 1: Linear Systems

Total marks: 100 **Due Date:** March 28, 2013 (at 11:59pm, AEST)

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. It is requested that solutions, including equations, should be typed please. The final grade is the median of the marks from the peer reviews and the staff (with provisions for review). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

Short Questions

(Please keep it simple. These questions are "all or nothing")

Q1. Even/Odd Signals [5 points]

An aperiodic signal is defined as $x(t) = \sin(\pi t)u(t)$, where $u(t)$ is the continuous-time step function. Is the odd portion of this signal, $x_0(t)$, periodic? Justify your answer (include any definitions and diagrams you deem relevant to your explanations).

Q2. What's the Difference? [5 points]

We may describe a system as being:

- linear[1]
- time-invariant[2]
- causal[3]

Determine which of these three properties hold (**yes**/**no**) for:

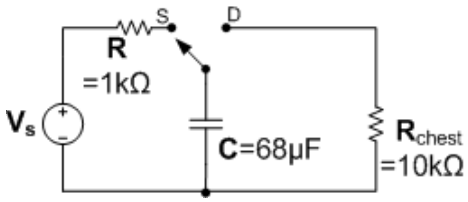
(a) Backward differencer (`diff` in MATLAB): $y[t] = x[t] - x[t-1]$

(b) Forward differencer: $y[t] = x[t+1] - x[t]$

(c) Central differencer: $y(t) = x(t+\frac{1}{2}) - x(t-\frac{1}{2})$

Q3. I ♥ Systems [5 points]

A defibrillator is used to deliver a strong shock across the chest of a person in cardiac arrest (i.e., fibrillation). A simple design for one may be constructed using the following circuit:



With the switch in the standby mode ("S"), the 68μF capacitor is charged using a controller (having a Thevenin equivalent of $\mathbf{V}_s$ and $\mathbf{R}$=1kΩ). To defibrillate, the switch is thrown (to "D") and the capacitor discharges across the patient's chest, which can be approximated as a 10kΩ resistor.

Determine $\mathbf{V}_s$ so that the dose is 150J (Assume the capacitor is fully charged when the switch is thrown)? Using this value, specify how long it takes (in seconds) to deliver 95J?

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[1] Superposition holds, that is for the input/output pairs $\mathbf{x_1[n]} \rightarrow \mathbf{y_1[n]}$ and $\mathbf{x_2[n]} \rightarrow \mathbf{y_2[n]}$, the combination $\mathbf{ax_1[n] + bx_2[n]} \rightarrow \mathbf{ay_1[n] + by_2[n]}$

[2] Characteristics of the system are fixed over time, so for a system with input **x[n]** and output **y[n]**, the input **x[n-m]** will give the output **y[n-m]**

[3] The output only depends on time inputs at the present and past times. The system is nonanticipative.

Q4. Signal Sampling: Touché! **[5 points]**

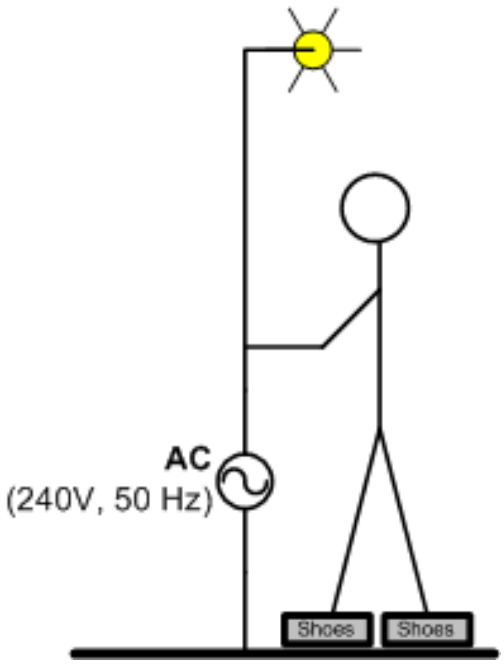
Touché is a Swept Frequency Capacitive Sensing technique that samples the return voltage many times of a capacitive touch surface whose operating frequency is being swept (changed) from a low frequency to a high frequency.

(a) If an AC base voltage applied to the touch surface ranges from **0.5 MHz** to **3.5 MHz**, what is the Nyquist frequency for sampling this in order to recover the base signal?

(b) If the signal also had high-frequency (**2.4 GHz**) noise, would the noise appear in the sampled signal? If so, what could be done to prevent this?

Q5. Mains Touch **[5 points]**

Uncle Robert wants to work on the mains (AC, 240 V, 50 Hz) to fix a light switch in the house. Since he has rubber shoes, he conjectures that he should be safe. To prove his point, he pulls out his multimeter and measures the resistance as 2MΩ and points out (according to Ohm's law) that this is much less than the 75mA that would cause fibrillation. Is he correct? Justify briefly.
(**Hint**: Consider the equivalent circuit between him and the floor)



Long Questions

Q6. Signals with Discreteness: **[20 points]**

Let x(t) be the continuous-time complex exponential signal:

$$x(t) = e^{jw_0 t}$$

With fundamental frequency, w_0, and fundamental period, $T_0 = 2\pi/w_0$. Consider the discrete-time signal obtained by taking equally spaced samples of x(t) that is:

$$x[n] = x(nT) = e^{jw_0 nT}$$

(a) Show that *x[n]* is periodic if and only if T/T_0 is a rational number – that is, if and only if some multiple of the sampling interval exactly equals a multiple of the period of *x(t)*.

(b) Suppose that *x[n]* is periodic – that is, that:

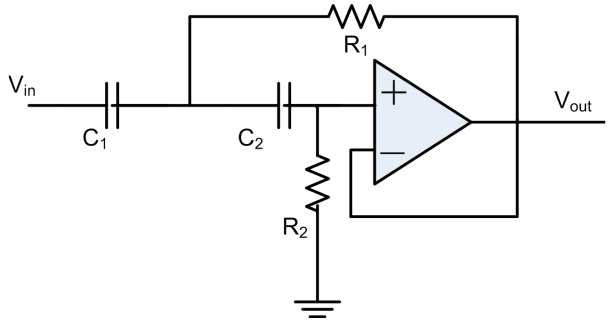
$$\frac{T}{T_0} = \frac{p}{q}$$

where *p* and *q* are integers. What are the fundamental period and frequency of *x[n]*?
Express the fundamental frequency as a fraction of $\boldsymbol{\omega_0 T}$.

(c) Again assuming that T/T_0 is rational, determine precisely how many periods of *x(t)* are needed to obtain the samples that form a single period.

(d) It has been proposed to arbitrarily space the samples of x(t) (asynchronous sampling). What might be (one) advantage and (one) disadvantage of this? Justify.
Shall we run to the patent office?

Q7. Linear Time-Invariant Modelling: Filtered and Distilled **[25 points]**



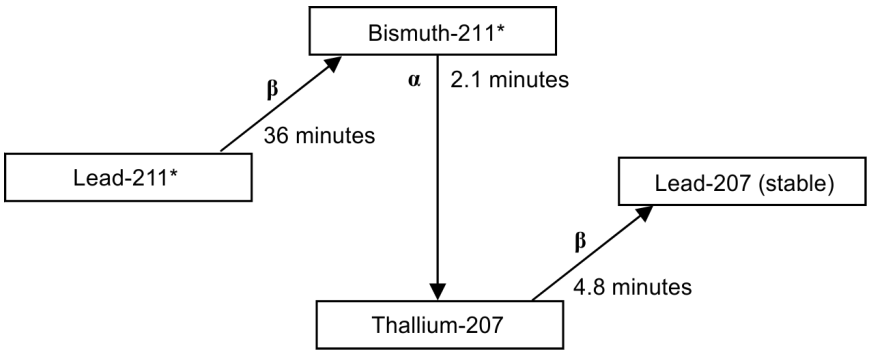
Consider the circuit shown in Figure above with input voltage V_{in} and output voltage V_{out}
(a) Derive the model for this circuit? (**Hint**: is it low-pass or high-pass?)
(b) For C_1=0.01 μF and R_1=10 kΩ, for what values of C_2 and R_2 is the circuit stable?
(c) For these values, draw the Root-Locus of this circuit (i.e., plot its zeros and poles on the s-plane).
(If you can not upload the figure, upload your **commented** MATLAB code for generating it)

Q8. Systems Alchemy **[30 points]**

Nuclear decay is given by a first order linear differential model (the Bateman equation).
This is given as: $\frac{dN}{dt} = -\lambda N$, where N is the number of atoms and λ is the decay constant.
(n.b., $\tau = \frac{1}{\lambda}, t_{\frac{1}{2}} = \tau \cdot ln(2)$)

(a) How much (percent) is left after one lifetime (τ)?

(b) How long will it take (as a function of $t_{\frac{1}{2}}$) for concentrations to get to 10%?



Lead-211 Decay Series[4]

It has long been the goal to turn Lead (Pb) into Gold (Au). Thus, we consider the decay of Lead-211 (the "parent") to Bismuth-211, Thallium-207, and Lead-207 (the "children"). These are the latter parts of the Actinium decay series (which includes ^{235}U).

(c) Generate a plot of the concentrations (as a function of normalized atom count) of both isotopes as a function of time starting with one normalized quantity of Lead-211.

(d) Sadly, this is not a gilded pathway (it leads to Lead-207[5]). How long will it take for 1 kilogram of Lead-211 to turn into 900 grams of Lead-207 (you can neglect the mass loss associated with the alpha and beta emissions)? (**Hint**: assume you are starting with 100% pure ^{211}Pb)

(e) Determine and plot the ensemble radiation emission as a function of time for the duration determined above in part (d). (i.e., you may treat the α and β as equivalent "particles of radiation").

[4] Argonne National Laboratory, "Natural Decay Series: Uranium, Radium, and Thorium", Human Health Fact Sheet, August 2005. [http://www.ead.anl.gov/pub/doc/natural-decay-series.pdf]

[5] It is possible to go from Pb to Au via artificial nuclear transmutation (i.e., since ^{207}Pb is stable, it will not naturally decay). Ironically, it is easier to go from Au to Pb and more profitable to go from Ir to Pt. No, Chrysopoeia is not part of the exam.

For ELEC 7312 Only:

Q9. A Flood of Signals [* for ELEC 7312 Only *] **[20 points]**

Consider an idealised hydrology model for river height based on past rainfall in the region. Let u(t) denote the rainfall rate (in mm/hour) in a region at time t and y(t) denote the flood level (m), above a reference (non-flood) level. A simplified analysis gives the transfer function as

$$Y(s) = F(s)U(s),\ F(s) = \left[\frac{10}{(7s+1)(49s+1)}\right]$$

(**FYI**: The fast pole is from surface runoff, which is relatively quick, but shallow. The slow pole is from tributaries and groundwater, which contribute more water into the river over a much longer time scale)

Consider an intense rain after a long dry spell in which it which it rains 100 mm-per-hour for 30 minutes. This causes the river height to rise and then recede. Then:

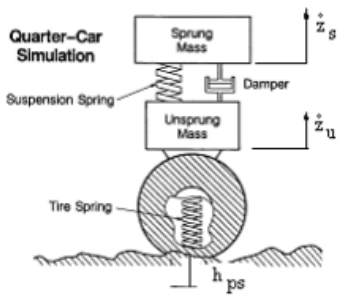
(a) How long does it take, after the beginning of the downpour, for the river to reach its maximum height? (Tidal effects can be neglected, consider positive time (t>0) measured in hours)

(b) Determine and plot the height of the river (in meters) as a function of time (in hours).

Q10. An Efficient Ride [* for ELEC 7312 Only *] **[30 points]**

Henry wants better fuel economy, so he increases the pressure in his tyres.

Note that a car's suspension can be viewed as a spring (for the tyre), a mass for the wheel (called the "unsprung mass"), the suspension spring and damper and mass for the car (typically ¼ the vehicle's mass). This is called the **quarter-car model**.



Quarter-Car Model (from Google Images c/o US DOT http://goo.gl/wOY5m)

(a) What is the overall order of the system as seen from the a passenger sitting in the car?

(b) Sketch (or plot) the expected signal for running over a pothole as seen by an accelerometer on the floor of the car and by an accelerometer on the seat. Consider the pothole as an impulse and the car's damper as overdamped (shocks are new).

(c) Is this wise? Discuss using your own research. (1 page maximum)