



This exam paper must not be removed from the venue

Venue

Seat Number

Student Number

Family Name

First Name

School of Information Technology and Electrical Engineering
EXAMINATION

Semester One Final Examinations, 2013

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Any unmarked paper dictionary is permitted

An unmarked Bilingual dictionary is permitted

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Rough Paper

Instructions To Students:

Please answer ALL questions. Thank you.

For Examiner Use Only

Question

Mark

[illegible]

Total

This exam has THREE (3) Sections for a total of 100 Points

Section 1: Linear Systems	30 %
Section 2: Signal Processing	30 %
Section 3: Digital Control.....	40 %

Please answer **ALL** questions

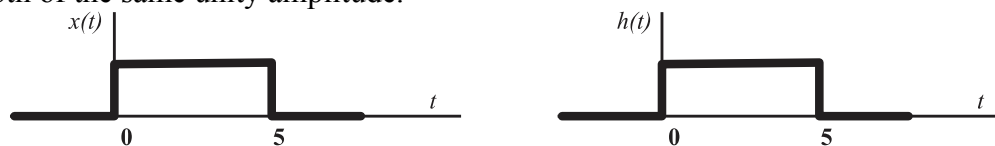
Section 1: Linear Systems

Please Record Answers in the **Answer Book**

(Total: **30%**)

1. **Convolution** (5%)

Determine the continuous time convolution of $\mathbf{x(t)}$ and $\mathbf{h(t)}$. Assume that they are both of the same unity amplitude.



2. **Does Linearity Imply Invertibility?** (5%)

IF $\mathbf{F}$ is a linear, time-invariant system such that $\mathbf{y=F[x]}$,
THEN is there another Linear System ($\mathbf{G}$) for which $\mathbf{x=G[y]}$?

3. **Causality and Stability** (5%)

A continuous-time LTI system has impulse response function

$$h(t) = \sum_{i=0}^{\infty} (0.5)^i \delta(t - i)$$

Is the system, $\mathbf{h(t)}$, **causal**? Is it BIBO **stable**? (Please briefly justify)

4. Linearity Ahoy!

(5%)

Dr Shilling pensively does tests on a new system $\mathbf{F}$ with inputs $\mathbf{x}[n]$ to get outputs $\mathbf{y}[n]$ and observes the following input/output signal pairs (for $n=1$ to 5):

i	Input ($\mathbf{x}_i[n]$)	Output ($\mathbf{y}_i[n]$)
1	[0, 0, 0, 1, 1]	[0,1,0,0,0]
2	[0, 1, 1, 1, 1]	[0,1,0,1,0]
3	[0, 1, 1, 2, 2]	[0,2,0,1,0]

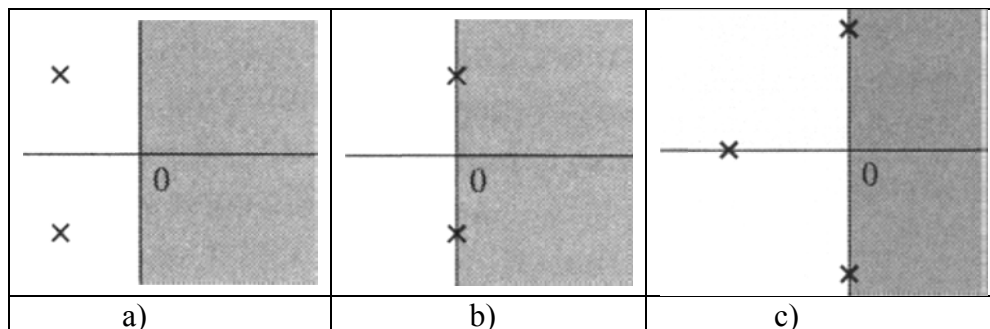
(In all cases below **please briefly justify** your answers)

- From this, can Dr Shilling conclude that the system $\mathbf{F}$ is linear?
- Assume that $\mathbf{F}$ is indeed linear, then is it possible to determine the **output** signal associated with an **input** of $\mathbf{x}=[0, 0, 0, 0, 0]$? If so, please compute it.
- Assume that $\mathbf{F}$ is indeed linear, then is it possible to determine the **input** signal associated with an **output** of $\mathbf{y}=[0, 0, 0, 0, 0]$? If so, please compute it.

5. Characteristic Roots and Characteristic Modes

(5%)

For systems having the following pole-zero plots (on the complex plane), please sketch the corresponding zero-input response.

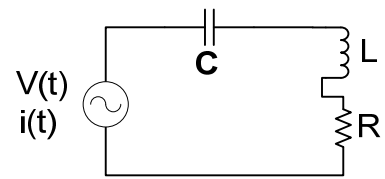


6. Second Order Systems

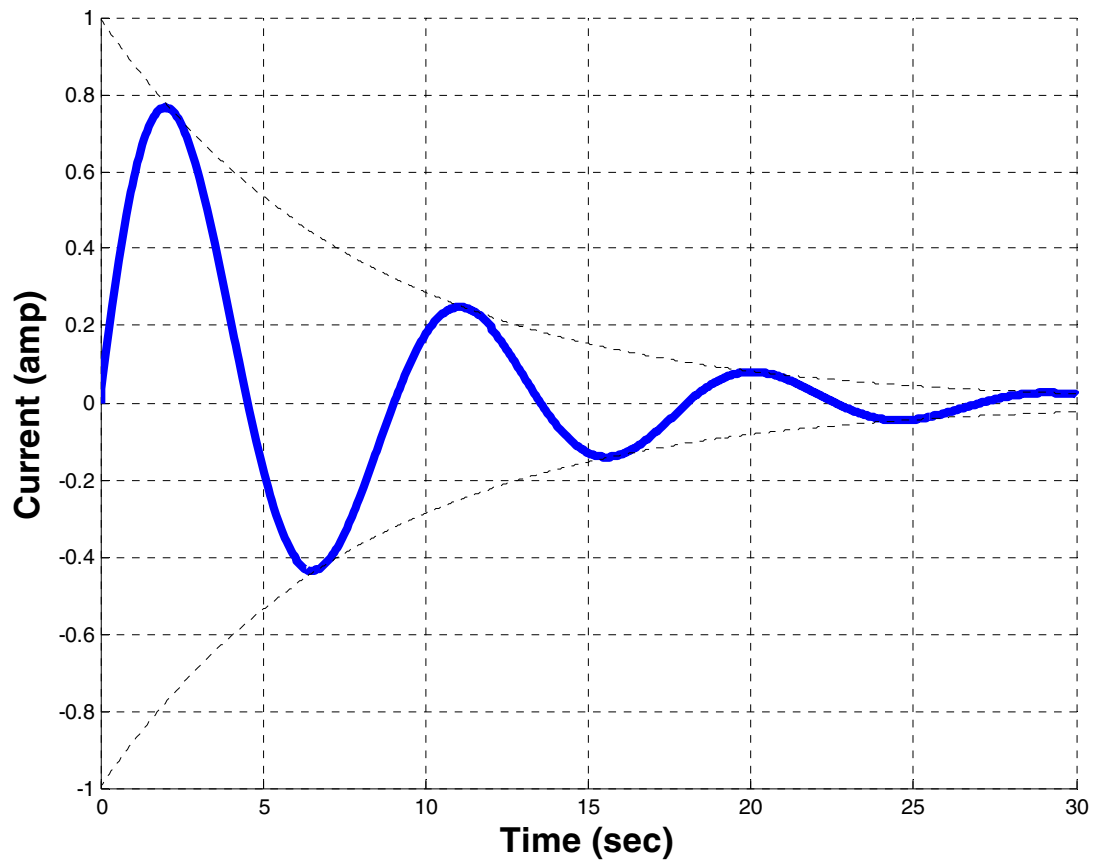
(5%)

Consider the following series RLC circuit system.

Recall that for such a circuit, the response is second order and that $\omega_0 = \frac{1}{\sqrt{LC}}$, $\alpha = \frac{R}{2L}$ and $\zeta = \frac{\alpha}{\omega_0}$



Determine the C for a circuit with $R=1\Omega$, $L=4H$ and the following impulse response:
(**hint:** the values might be larger than “common” parts)



Section 2: Signal ProcessingPlease Record Answers in the **Answer Book**(Total: **30%**)**7. What a Difference**

(5%)

Does the difference equation

$$y[k] + k^2 y[k-1] - 2 y[k-2] = x[k]$$

with initial conditions $y[-1] = y[-2] = 0$ define a linear system with input $x[k]$ and output $y[k]$? If so, what is the order of this system? **(Please briefly justify)**

8. Getting Sparse With Linearity

(5%)

Consider a novel sampling system that generates an output $f_c[k]$ by sampling an input signal $u[k]$ to give the output via the following function:

$$f_c[k] = \begin{cases} u[k], & \text{if } k \text{ is odd} \\ 0 & \text{if } k \text{ is even} \end{cases}$$

Is this system, $f_c[k]$, linear and time invariant? **(Please briefly justify)**

9. Shattering Past Nyquist?

(5%)

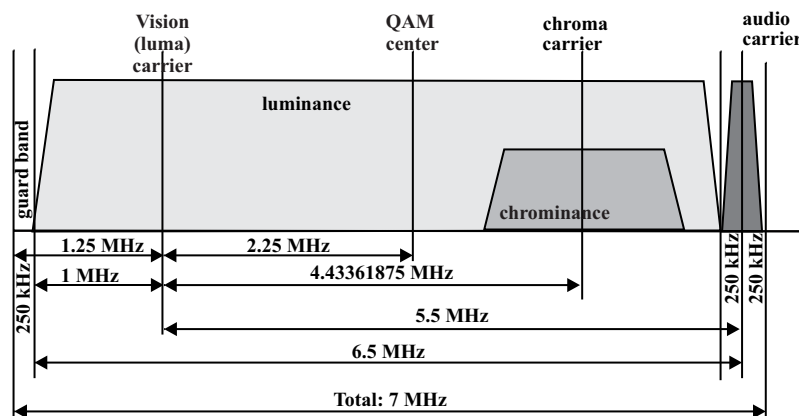
A crystal glass has a resonant frequency of 1100 Hz. A soprano sings this note exactly. If the note (treat it as a pure tone or sine wave) is amplified and transmitted using a digital amplifier that samples at 2000 Hz, will the glass shatter on the other side (assume arbitrarily high amplifier power)? **(Please briefly justify)**

10. Easy as ABC**(15%)**

Design a bandpass filter for tuning into TV content on Australian PAL B analog TV Channel 2. The **QAM center frequency** for this channel is **66.5 MHz**. The channel width is 7 MHz (so ± 3.5 MHz from the QAM center)¹. The highest frequency in the tuning range (f_h) is **257 MHz**. The filter should have minimum transmission (≤ -20 dB) at the neighbouring channel's QAM center frequencies (59.5 MHz and 73.5 MHz respectively)².

- What is the frequency range of the video signal and audio signal?
- What type of filter should we use?
- What order does this filter need to be?
- What is the sampling rate needed for the circuit?
- Please determine the transfer function and design this filter for these specifications.
- Please sketch the frequency response of this filter.

¹ Analog television is broadcast with two carrier frequencies: one for video and one for audio. In the case of ABC TV, the vision carrier (for the QAM video) is at **64.25 MHz** and the audio carrier (for the FM audio) is at **69.75 MHz**. A diagram showing the carrier frequencies within a PAL channel's 7MHz of bandwidth is below:



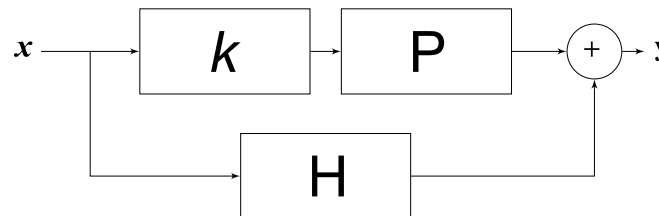
² Technically, Channel 3 has a QAM center of 88.5 MHz as it is in VHF Band II, whereas Channel 2 is in VHF Band I. Please design for the given specification as this is generalizable to other 7MHz wide channels.

Section 3: Digital ControlPlease Record Answers in the **Answer Book**(Total: **40%**)**11. Feedforward Block Diagram System Models**

(5%)

Please specify the transfer function ($T = \frac{y}{x}$) for the following system.

[Hint: Please carefully note the forward direction of the arrows and the location of the sum element.]

This is **not** the “most common” feedback control block diagram]**12. Digital Stability**

(5%)

The transfer function of a sampled system is:

$$T(z) = \frac{1}{z^2 + (K - 4)z + 0.8}$$

Please specify the range of K so that the system is stable.

13. Dynamic State-Space

(5%)

Consider the system described by the following differential equations:

$$\dot{x}_1(t) = a_1 x_1(t) + b_1 u(t)$$

$$\dot{x}_2(t) = a_2 x_2(t) + b_2 u(t)$$

$$y(t) = x_1(t) + x_2(t)$$

- a) Please write this system in the standard state-space matrix form, that is, for

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)$$

Determine the matrices **A**, **B**, **C**, and **D** in terms of the constant parameters a_1 , a_2 , b_1 , and b_2 .

- b) From this, please determine the system transfer function
- $H(s)$
- relating the output
- Y(s)**
- to the input
- U(s)**
- .

(hint: $H(s) = \frac{Y(s)}{U(s)}$)

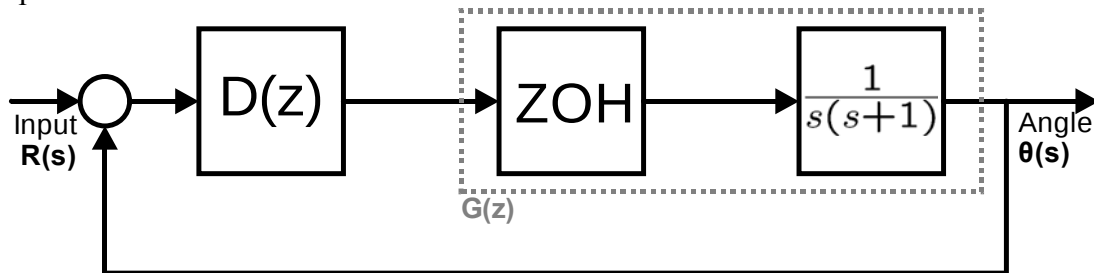
14. Steer-by-Wire

(10%)

A steer-by-wire system is proposed in which a DC motor regulates the hydraulic flow of a power steering system³. It has the following continuous time plant

$$P(s) = \frac{1}{s(s+1)}$$

The system is connected to a digital controller $D(z)$ by a ZOH process⁴ having a period of $T=0.1\text{sec}$.



- Determine $G(z)$
[**hint:** For this part you may leave it in terms of the z-Transform, $Z\{\bullet\}$ (i.e. $G(z)=Z\{G(s)\}$).]
- Sketch the impulse response of $G(z)$
- For a simple, proportional compensator, $D(z)=K$, please sketch the Root locus for this system.

³ Such systems are commercially available (often in very high-end vehicles) and should not be confused with all electric steer-by-wire systems (e.g. Nissan's Q50).

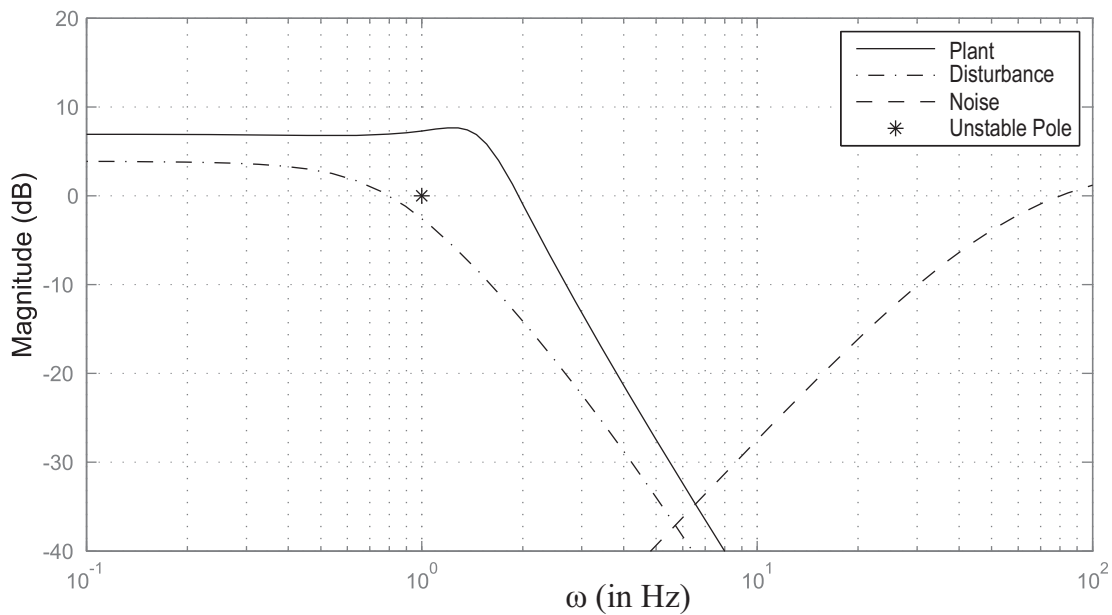
⁴ Zero Order Hold. Recall that a ZOH is modelled by $G_{ZOH} = \frac{1-e^{-sT}}{s}$

15. Lofty Controls!

(15%)

Dr Shilling has a new disposable, biodegradable, stealth flying machine⁵. However, this system, especially when disturbed by the wind, is open-loop unstable.

The professor consulted Prof Tjohi, who advises that the system needs a controller⁶. He suggests a microprocessor with sampling time of $T = 0.03$ sec. With this in mind, Jeeves, the professor's assistant, performed system identification on the system. The spectra of the plant, of the disturbance, and of the measurement noise⁷ are shown in the figure below. The plant has one unstable pole at $\pi = 1$. The reference for the control is $\mathbf{u}(t) = \mathbf{0}$.



- Draw the block diagram of a generic, complete discrete-time control system, including interfaces, filters, and all relevant signals.
- Can you do controller emulation in the case at hand?
(Please briefly justify)
- Is an anti-aliasing filter necessary?
(Please briefly justify)
- Based on these sections, Assess the stability of the discrete-time entire system. Will a simple, proportional controller be sufficient to regain stability?

END OF EXAMINATION — Thank you ☺

⁵ Specifications and a prototype are in the appendix for your reference (p.15).

⁶ In this way, it can be like the plane in "Paperman" (Pixar's Oscar-winning short film about lovely paper airplanes)

⁷ This may be considered as zero mean, white "Gaussian" noise. (FYI, \$10 IMUs are not exactly "tactical grade.")

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t} \tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1 - z^{-1})}{\Delta t(1 + z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi} \int_0^{\pi/\Delta t} H_d(\omega) \cos(n\omega\Delta t) d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	Periodic	Non-periodic	
	Discrete Fourier Transform	Discrete-Time Fourier Transform	
Discrete	$\tilde{X}[k] = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1} \tilde{X}[k]e^{j2\pi kn/N}$	$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega$	Periodic
	Complex Fourier Series	Fourier Transform	
Continuous	$X[k] = \frac{1}{T} \int_{-T/2}^{T/2} \tilde{x}(t)e^{-j2\pi kt/T} dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$ $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega$	Non-periodic
	Discrete	Continuous	Freq. Domain

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Table 3: Selected Fourier, Laplace and z -transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$\xleftrightarrow{DFT}$	$\tilde{X}[k] = \frac{1}{N}$	
$x[n] = \delta[n]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\xleftrightarrow{FS}$	$X[k] = \frac{1}{T}$	
$\delta_T[t] = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\xleftrightarrow{FT}$	$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
$\sin(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } t \leq T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\xleftrightarrow{FT}$	$X(j\omega) = \frac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \frac{1}{\pi t} \sin(\omega_c t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \begin{cases} 1 & \text{when } \omega \leq \omega_c , \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = 1$	
$x(t) = \delta(t - t_0)$	$\xleftrightarrow{FT}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$	
$x[n] = \frac{\omega_c}{\pi} \text{sinc } \omega_c n$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } \omega < \omega_c , \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s}$	
(unit ramp) $x(t) = t$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{s_0}{(s^2 + s_0^2)}$	
$x(t) = \cos(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{s}{(s^2 + s_0^2)}$	
$x(t) = e^{s_0 t} u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s - s_0}$	$\Re\{s\} > \Re\{s_0\}$
$x[n] = \delta[n]$	$\xleftrightarrow{z}$	$X(z) = 1$	all z
$x[n] = \delta[n - m]$	$\xleftrightarrow{z}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{z}{z - 1}$	
$x[n] = z_0^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$ z > z_0 $
$x[n] = -z_0^n u[-n - 1]$	$\xleftrightarrow{z}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$ z < z_0 $
$x[n] = a^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{z}{z - a}$	$ z < a $

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Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega}) X_2(e^{j\omega})$
Modulation	$x_1[n] x_2[n]$	$\frac{1}{2\pi} X_1(e^{j\omega}) \otimes X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\Im\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\Re\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) ^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\frac{d\tilde{x}(t)}{dt}$	$\frac{j2\pi k}{T} X[k]$
Time-shift	$\tilde{x}(t - t_0)$	$e^{-j2\pi k t_0 / T} X[k]$
Frequency-shift	$e^{j2\pi k_0 t / T} \tilde{x}(t)$	$X[k - k_0]$
Convolution	$\tilde{x}_1(t) \otimes \tilde{x}_2(t)$	$T X_1[k] X_2[k]$
Modulation	$\tilde{x}_1(t) \tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\Im\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\Re\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\frac{1}{T} \int_{-T/2}^{T/2} \tilde{x}(t) ^2 dt = \sum_{k=-\infty}^{\infty} X[k] ^2$	

Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j\omega} X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
Frequency-shift	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\Im\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\Re\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$	

Table 7: Properties of the z -transform.

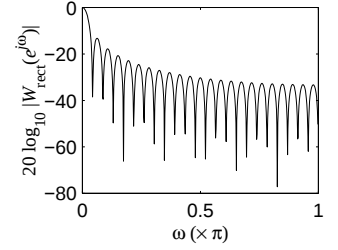
Property	Time domain	z -domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n - n_0]$	$z^{-n_0} X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$ z_0 R_x$
Differentiation in z	$nx[n]$	$-z \frac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\Im\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\Re\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim_{z \rightarrow \infty} X(z)$		

[†] $z = 0$ or $z = \infty$ may have been added or removed from the ROC.

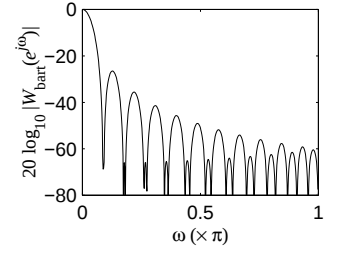
Table 8: Commonly used window functions.

Rectangular:

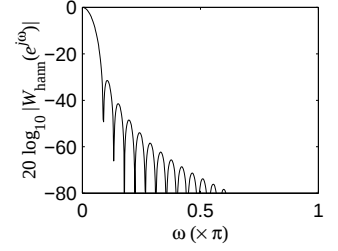
$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leq n \leq M, \\ 0 & \text{otherwise.} \end{cases}$$

*Bartlett (triangular):*

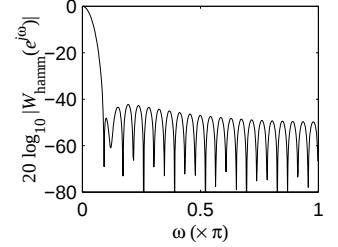
$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leq n \leq M/2, \\ 2 - 2n/M & \text{when } M/2 \leq n \leq M, \\ 0 & \text{otherwise.} \end{cases}$$

*Hanning:*

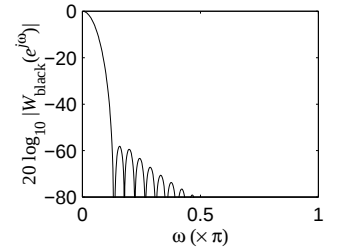
$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2} \cos(2\pi n/M) & \text{when } 0 \leq n \leq M, \\ 0 & \text{otherwise.} \end{cases}$$

*Hamming:*

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46 \cos(2\pi n/M) & \text{when } 0 \leq n \leq M, \\ 0 & \text{otherwise.} \end{cases}$$

*Blackman:*

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5 \cos(2\pi n/M) \\ \quad + 0.08 \cos(4\pi n/M) & \text{when } 0 \leq n \leq M, \\ 0 & \text{otherwise.} \end{cases}$$



Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20 \log_{10} \delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74

